1. Explain briefly the mechanism of conduction Convection and radiation heat transfer (1a,06 Dec2013/Jan2014, 1a,03,Dec16/Jan17,1b,06,June/July18)

2. State the laws governing three basic modes of heat transfer (1a 06 June/July 15, Dec 14/Jan 15)

3 State and explain the governing laws of conduction, convection and radiation heat transfer modes (1a,09,June/July2018)(1a,06,Dec18/Jan19,15 scheme,1a,10, Dec15/Jan16)

4. Derive an the general 3-D heat conduction equation in Cartesian coordinate system,state assumption made and hence Laplace and Poisson equations (1a. 10 Dec 2015/Jan 2016,1b,08, June/July14, 1b,08,June/July16,1b,08,Dec16/Jan17, 1c,8,Dec18/jan19,2a,08,June/July18 15ME63, 2a,08,Dec18/Jan19,15 scheme)

5 Write down 3 dimensional conduction equation in Cartesian coordinates. Explain meaning of each term 06 June/July 2013, Dec 14/Jan 15

6 What is thermal diffusivity? Explain its importance in heat conduction problems (1a, 04,June/July 2014)

7. Define thermal diffusivity (1a, 04 Dec17/Jan18)

8. Describe different types of boundary conditions applied to Heat conduction problems (1b, 04,June/July 2014)

9. What do you mean by initial conditions and boundary conditions I, II and III kind 06 June/July 2013,1a, June/July18)

10. What do you mean by boundary condition of one two and three kind (1b, 06 DEC18/Jan19, 1a,06,June/July16)

11. Explain the three types of boundary conditions used in conduction heat transfer (1a,06 June/July 17)

12. With sketches write down mathematical representation of three commonly used different types of boundary conditions for one dimensional heat equation in rectangular coordinates (1b,08 Dec13/Jan14)

13. Write a note on Thermal contact resistance (1b,3, June/July 2018)

14 A plate of thickness L whose one side is insulated and the other side is maintained at a temperature T_1 is exchanging heat by conduction to the surrounding area at a temperature T_2 with atmospheric air being the outside medium. Write mathematical formulation for one dimensional steady state heat transfer without heat generation. (1c,06 Dec 13/Jan14)

15 Consider a one dimensional steady state heat conduction in a plate with constant thermal conductivity in a region $0 \leq x \leq L$. A plate is exposed to uniform heat flux q W/m^2-K at x=0 and

dissipates heat Convection at X=L with heat transfer coefficient h in the surrounding air at T_∞. Write the mathematical formulation of this problem for the determination of one dimensional steady state temperature distribution within the wall (1c,04 June/July14)

Heat Transfer Mechanisms:-

There are modes of heat transfer- (i) conduction, (ii) convection and (iii) radiation

Conduction

In solids heat transfer takes place due to conduction.Conduction is the transfer of heat energy from one molecule to other adjacent molecule as a result of i)movement of free valence electrons from Higher energetic molecules of a substance to adjacent lower energetic molecules in the direction of decreasing temperature ii) vibration of molecules in the lattice

Fourier Law of Conduction

Conduction is governed by Fourier Law of Heat conduction

It states that the rate of heat transfer by conduction is directly proportional to area normal to the direction of heat transfer and temperature gradient in that direction

$$Q_x \propto -A\frac{dT}{dx}$$

$Q = -kA\frac{dT}{dx}$

Q_x is the rate of heat transfer in positive x-direction through area, A is the Area normal to the direction of heat transfer, $\frac{dT}{dx}$ is the temperature gradient and it is negative in positive x direction and k is the constant of proportionality and is a material property called *"thermal conductivity".* Therefore negative sign has to be introduced in equation to make Q_x positive in the direction of decreasing temperature

Convection

When a fluid moves over a solid body or inside a channel while temperature difference exists between solid surface and fluid , then heat transfer between the fluid and surface takes place due to movement of fluid molecules relative to the surface. This type of hat transfer due to motion of molecules is called convection.

In convection the total heat transfer is due to random motion of the fluid molecules together with the bulk motion of the fluid, the major contribution coming from the latter mechanism. Therefore bulk motion of the fluid is a necessary condition for convection heat transfer to take place in addition to the temperature gradient in the fluid.

If the motion of molecules is set by buoyancy effects due to the density difference caused by temperature difference in the fluid , the heat transfer is termed as Natural convection

If the motion of molecules in the fluid is set by external agency like fan or pump, the heat transfer is called as Forced convection

Law for Convection Heat Transfer

It is governed by Newton's Law which states that Heat flux directly proportional to temperature difference between the surface and the temperature of bulk fluid

$q \propto \Delta T;\quad q = h\Delta T\ W/m^2$ where h is the surface heat transfer coefficient due to convection either due to forced convection or natural convection

$Q = hA\Delta T$ watts

$Q = hA\left(T_w - T_f\right)$ Watts ie heat is transferred from the surface to fluid) (if $T_f < T_w$))

$Q = hA\left(T_f - T_w\right)$ Watts ie heat is transferred from the fluid to surface to fluid (if $T_{sf} > T_w$)

Radiation:

Thermal radiation is the energy emitted by matter (solid, liquid or gas) by virtue of its temperature. This energy is transported by electromagnetic waves (or alternatively, photons).While the transfer of energy by conduction and convection requires the presence of a material medium, radiation does not require any medium and it occurs most effectively in vacuum.

Law of Radiation:

Radiation Heat Transfer is governed by the Stefan-Boltzmans Law.

Stefan-Boltzman's law of radiation states that the emissive power of a black body is proportional to the fourth power of the absolute temperature of the body. Therefore if E_b is the emissive power of a black body at temperature T ^{0}K, then

E_b α T ^{4}; $E_b = \sigma T^4$ where σ is constant of proportionality and it is Stefan Boltzman constant =5.67x10^{-8}

Emissive power of any body E =∈ σT^4 wh ere ∈ is the Emissivity of body

Difference between Conduction ,Convection and Radiation

Conduction	Convection	Radiation
Heat transfer takes place due to movement of **free valence electrons** from one molecule to	Heat transfer takes place between the surface and fluid due to movement of **molecules**	Heat transfer takes place in vacuum due to **emission properties** of materials in the

other adjacent molecule whenever there is difference in energy level between molecules	set by buoyancy force caused by density difference (Natural Convection) Or due to external agency like fan , pump	form of electromagnetic waves (continuous) or Photons (Discreet)
It takes place in solids	It takes place in fluids	No media required
It is Governed by Fourier law of Heat Conduction $Q = -kA\frac{dT}{dx}$ Watts	This is Governed by Newton Law of Convection $Q = hA\Delta T\ Watts$	This is Governed by Stefan Law for Radiation $E_b = \sigma T^4$ Watts per m^2

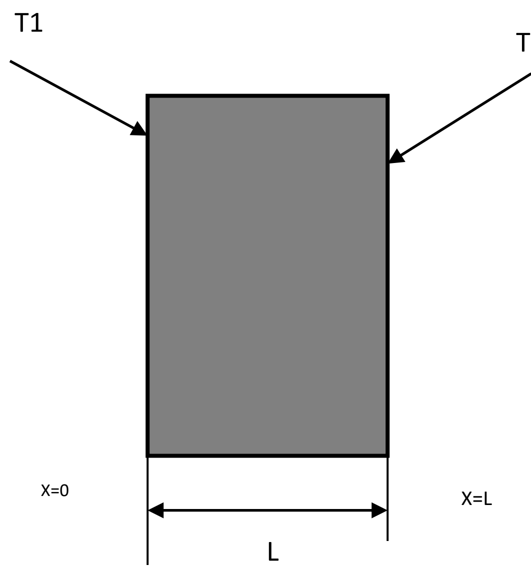
Thermal Conductivity

Thermal Conductivity of material is defines as the property of material by virtue of which it allows the heat flow through a body

If it does not vary with temperature it is called as uniform thermal conductivity

If it varies uniformly varies with temperature it is called as uniformly variable thermal conductivity

Fourier Law of Heat Conduction in Slab



Thermal conductivity =K

1. For constant thermal conductivity and Area

$$Q = -kA\frac{dT}{dx}$$

$$Q\,dx = -kA\,dT$$

Integrating Above equation

$$Q\int_0^L dx = -kA\int_{T_1}^{T_2} dT$$

$$Q(L-0) = -kA(T_2 - T_1)$$

$$QL = kA(T_1 - T_2)$$

$$Q = \frac{kA(T_1 - T_2)}{L}$$

$$Q = \frac{(T_1 - T_2)}{\frac{L}{kA}}$$

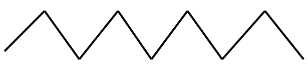
According to Electrical theory

$$I = \frac{V}{R}$$

Hence, Heat transfer can be treated Analogous to current

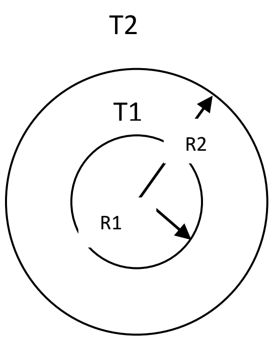
Hence Temperature difference $(T_1 - T_2)$ called as thermal potential

$\frac{L}{kA}$ is termed as thermal resistanceType equation here.

T_1  T_2

R= $\frac{L}{kA}$

<u>Fourier Law of Heat Conduction in in cylinder</u>



$$Q = -kA\frac{dT}{dr}$$

$$Q = -k2\pi rL\frac{dT}{dr}$$

$$Q\ \frac{dr}{r} = -k2\pi L\ dT$$

Integrating Above equation

$$Q\int_{r_1}^{r_2}\frac{dr}{r} = -2\pi LK\int_{T_1}^{T_2}dT$$

$$Q(ln\, r)_{r_1}^{r_2} = -2\pi LK(T_2 - T_1)$$

$$Q(ln\, r_2 - ln\, r_1) = 2\pi LK(T_1 - T_2)$$

$$Qln\frac{r_2}{r_1} = 2\pi LK(T_1 - T_2)$$

$$Q = \frac{2\pi LK(T_1 - T_2)}{ln\frac{r_2}{r_1}}$$

$$Q = \frac{(T_1 - T_2)}{\frac{ln\frac{r_2}{r_1}}{2\pi LK}}$$

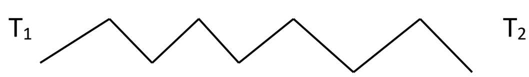
According to Electrical theory

$$I = \frac{V}{R}$$

Hence, Heat transfer can be treated Analogous to current

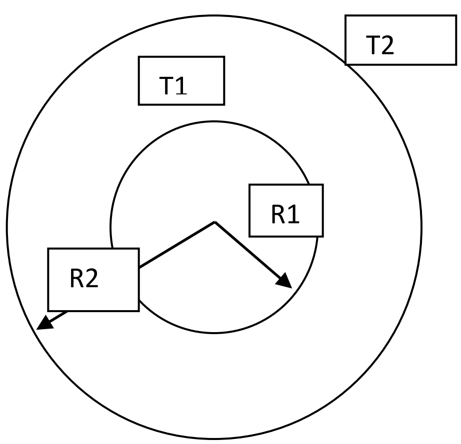
Hence Temperature difference $(T_1 - T_2)$ called as thermal potential

$\frac{ln\frac{r_2}{r_1}}{2\pi LK}$ is termed as thermal resitance



$$R = \frac{ln\frac{r_2}{r_1}}{2\pi LK}$$

Fourier Law of Heat Conduction in in sphere



$$Q = -kA\frac{dT}{dr}$$

$$Q = -k4\pi r^2\frac{dT}{dr}$$

$$Q\ \frac{dr}{r^2} = -k4\pi\ dT$$

Integrating Above equation

$$Q\int_{r_1}^{r_2}\frac{dr}{r^2} = -4\pi K\int_{T_1}^{T_2}dT$$

$$Q\left(\frac{r^{-2+1}}{-2+1}\right)_{r_1}^{r_2} = -4\pi K(T_2 - T_1)$$

$$-Q(r^{-2+1})_{r_1}^{r_2} = 4\pi K(T_1 - T_2)$$

$$-Q\left(\frac{1}{r}\right)_{r_1}^{r_2} = 4\pi K(T_1 - T_2)$$

$$-Q\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = 4\pi K(T_1 - T_2)$$

$$Q\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = 4\pi K(T_1 - T_2)$$

$$Q = \frac{4\pi K(T_1 - T_2)}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

$$Q = \frac{(T_1 - T_2)}{\frac{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}{4\pi K}}$$

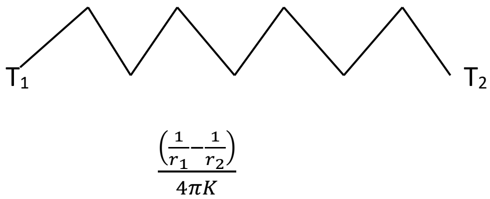
According to Electrical theory

$$I = \frac{V}{R}$$

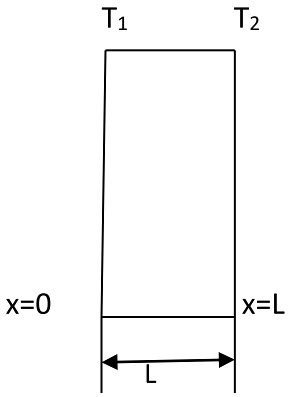
Hence, Heat transfer can be treated Analogous to current

Hence Temperature difference $(T_1 - T_2)$ called as thermal potential

$\frac{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}{4\pi K}$ is termed as thermal resitance



2. For Uniformly variable thermal conductivity $k = k_0(1+\alpha T)$ and Constant Area



$$Q = -kA\frac{dT}{dx}$$

$$Q\,dx = -k_0(1+\alpha T)A\,dT$$

$$Q\int_0^L dx = -k_0A\int_{T_1}^{T_2}(1+\alpha T)dT$$

$$Q(L-0) = -k_0A\left(T+\frac{\alpha T^2}{2}\right)_{T_1}^{T_2}$$

$$QL = k_0A\left(T+\frac{\alpha T^2}{2}\right)_{T_2}^{T_1}$$

$$QL = k_0A\left((T_1-T_2)+\frac{\alpha(T_1^2-T_2^2)}{2}\right)$$

$$QL = k_0A\left((T_1-T_2)+\frac{\alpha(T_1-T_2)(T_1+T_2)}{2}\right)$$

$$QL = k_0A(T_1-T_2)\left(1+\frac{\alpha(T_1+T_2)}{2}\right)$$

$$QL = k_0A(T_1-T_2)(1+\alpha T_m)$$

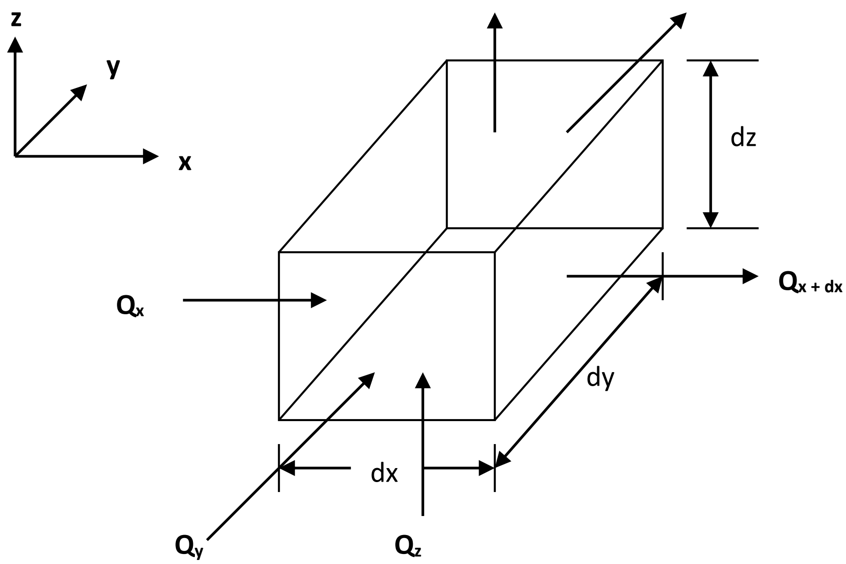
$$Q = \frac{k_0(1+\alpha T_m)A(T_1 - T_2)}{L}$$

$$Q = \frac{k_m A(T_1 - T_2)}{L}$$

$$Q = \frac{(T_1 - T_2)}{\frac{L}{k_m A}}$$

$\frac{L}{k_m A}$ is called as thermal resistance

3D Heat Conduction Equation in Cartesian Coordinates



Conduction heat transfer across the six faces of a volume element

let us consider a volume element of the solid of dimensions dx, dy dz in x y and z direction respectively

According to heat balance

$$Q_x + Q_y + Q_z + Q_g = Q_{x+dx} + +Q_{z+dz} + \Delta E$$

$$(Q_x - Q_{x+dx}) + \left(Q_y - Q_{y+dy}\right) + (Q_z - Q_{z+dz}) + Q_g = \Delta E$$

$$Q_{x+dx} = Q_x + \frac{\partial Q}{\partial x}dx$$

$$Q_x - Q_{x+dx} = -\frac{\partial Q}{\partial x}dx$$

$$Q_x = -k_x A \frac{\partial T}{\partial x}$$

$$Q_x = -k_x dydz \frac{\partial T}{\partial x}$$

$$\frac{\partial Q}{\partial x} dx = \frac{\partial}{\partial x}\left(-k_x dydz \frac{\partial T}{\partial x}\right) dx$$

$$\frac{\partial Q}{\partial x} dx = -\frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right) dxdydz$$

$$-\frac{\partial Q}{\partial x} dx = +\frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right) dxdydz$$

Hence,

$$Q_x - Q_{x+dx} = +\frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right) dxdydz$$

Similarly

$$Q_y - Q_{y+dy} = +\frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y}\right) dxdydz$$

$$Q_z - Q_{z+dz} = +\frac{\partial}{\partial z}\left(k_z \frac{\partial T}{\partial z}\right) dxdydz$$

$$Q_g = q''' \; dxdydz$$

Change Internal Energy $\Delta E = mC \frac{\partial T}{\partial \tau}$

$$\Delta E = \rho VC \frac{\partial T}{\partial \tau}$$

$$\Delta E = \rho dxdydzC \frac{\partial T}{\partial \tau}$$

$$(Q_x - Q_{x+dx}) + (Q_y - Q_{y+dy}) + (Q_z - Q_{z+dz}) + Q_g = \Delta E$$

$$+\frac{\partial}{\partial x}\left(k_y \frac{\partial T}{\partial x}\right) dxdydz + \frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y}\right) dxdydz + \frac{\partial}{\partial z}\left(k_z \frac{\partial T}{\partial z}\right) dxdydz + q''' \; dxdydz = \rho dxdydzC \frac{\partial T}{\partial \tau}$$

$$\frac{\partial}{\partial y}\left(k_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial T}{\partial z}\right) + q''' = \rho C \frac{\partial T}{\partial \tau}$$

If material is isotropic $k_x = k_y = k_z = constant = k$

$$k \frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) + k \frac{\partial}{\partial y}\left(\frac{\partial T}{\partial y}\right) + k \frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right) + q''' = \rho C \frac{\partial T}{\partial \tau}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right)+\frac{q'''}{k}=\frac{\rho C}{k}\frac{\partial T}{\partial \tau}$$

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{q'''}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

Where α is defined as thermal diffusivity $=\frac{k}{\rho C}$

If there is no Heat generation 3D equation reduces as given below

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}=\frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

If there is no Heat generation and steady state 3D equation reduces as given below

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}=0$$

Unsteady with no heat generation 3D equation reduces to

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}=\frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

For 1D steady state with no heat generation governing equation reduces to

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right)=0$$

If K is constant

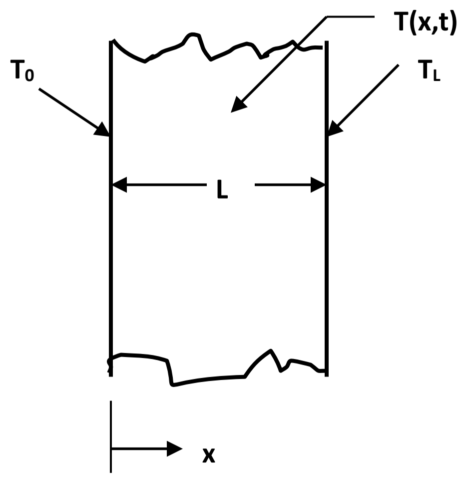
$$K\frac{d^2 T}{dx^2}=0; \qquad \frac{d^2 T}{dx^2}=0$$

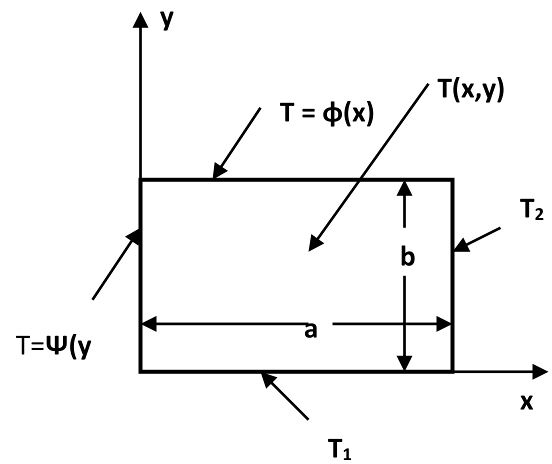
Boundary and Initial Conditions

Initial conditions specifies temperature distribution at the origin of time coordinate ie $\tau=0$

The boundary condition specifies thermal condition at the boundary surfaces of the region. Boundary conditions normally encountered in practice are, At the boundary surface I) distribution of temperature may be specified – Boundary condition of first Kind , ii) distribution of heat flux may be specified – Boundary condition of second Kind , iii)heat transfer by convection from the boundary surface into ambient or vice versa may be specified with known heat transfer coefficient – Boundary condition of third kind

2.4.1. Specified Temperatures at the Boundary:- Consider a plane wall of thickness L whose outer surfaces are maintained at temperatures T_0 and T_L as shown in Fig.2.6. For one-dimensional unsteady state conduction the boundary conditions can be written as





Boundary condition of First Kind

$$\frac{d^2T}{dx^2} = 0$$

Boundary conditions are

(i) at x = 0, $T(0,t) = T_0$; (ii) at x = L, $T(L,t) = T_L$.

Consider another example of a rectangular plate as shown in Fig. The boundary conditions for the four surfaces to determine two-dimensional steady state temperature distribution T(x,y) can be written as follows.

(i) at x = 0, $T(0,y) = \Psi(y)$; (ii) at y = 0, $T(x,0) = T_1$ for all values of y

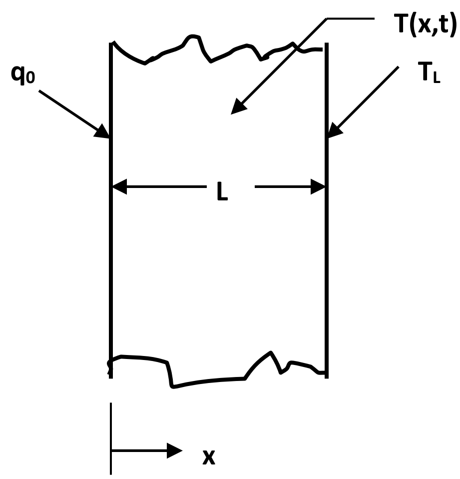
(iii) at x = a, $T(a,y) = T_2$ for all values of y; (iv) at y = b, $T(x,b) = \phi(x)$

2.4.2. Specified heat flux at the boundary:-

Consider a plane wall of thickness L whose outer surfaces are maintained at $q_0 at\ x = 0$and T_L as shown in Fig. For one-dimensional unsteady state conduction the boundary conditions can be written as

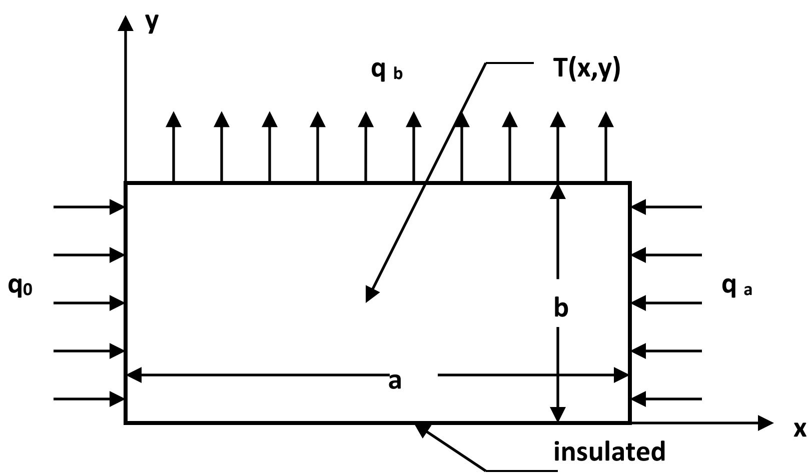
Governing equation

At $x = 0,\ q_0 = -kA\frac{dT}{dx}$ and $x = L,\quad \mathrm{T(L, \tau)\ =\ T_L}$



Boundary condition of second Kind

Consider a rectangular plate as shown in Fig. 2.8 and whose boundaries are subjected to the prescribed heat flux conditions as shown in the figure. Then the boundary conditions can be mathematically expressed as follows.



Prescribed heat flux boundary conditions

(i) at $x = 0$, $-k\,(\partial T / \partial x)|_{x=0} = q_0$ for $0 \le y \le b$;

(ii) at $y = 0$, $(\partial T / \partial y)|_{y=0} = 0$ for $0 \le x \le a$;

(iii) at $x = a$, $k\,(\partial T / \partial x)|_{x=a} = q_a$ for $0 \le y \le b$;

(iv) at $y = b$, $-k\,(\partial T / \partial y)|_{y=b} = 0$ for $0 \le x \le a$;

Boundary surface subjected to convective heat transfer:- Fig. 2.9 shows a plane wall whose outer surfaces are subjected to convective boundary conditions.The surface at $x = 0$ is in contact with a fluid which is at a uniform temperature T_i and the surface heat transfer coefficient is h_i. Similarly the other surface at $x = L$ is in contact with another fluid at a uniform temperature T_0 with a surface heat transfer coefficient h_0. This type of boundary condition is encountered in heat exchanger wherein heat is transferred from hot fluid to the cold fluid with a metallic wall separating the two fluids. This type of boundary condition is normally referred to as the boundary condition of third kind. The mathematical representation of the boundary conditions for the two surfaces of the plane wall can be written as follows.

(i) at $x = 0$, $q_{convection} = q_{conduction}$; i.e., $h_i[T_i - T|_{x=0}] = -k(dT / dx)|_{x=0}$

(ii) at x = L, $-k(dT / dx)|_{x = L} = h_0 [T|_{x = L} - T_0]$

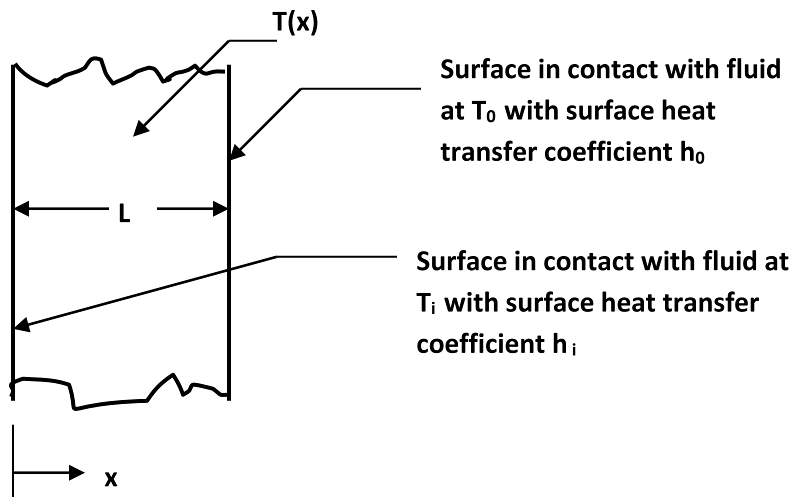


Fig. 2.9: Boundaries subjected to convective heat transfer

2.4.4.Radiation Boundary Condition: Fig. 2.10 shows a plane wall whose surface at x =L is having an emissivity 'ε' and is radiating heat to the surroundings at a uniform temperature T_s. The mathematical expression for the boundary condition at x = L can be written as follows:

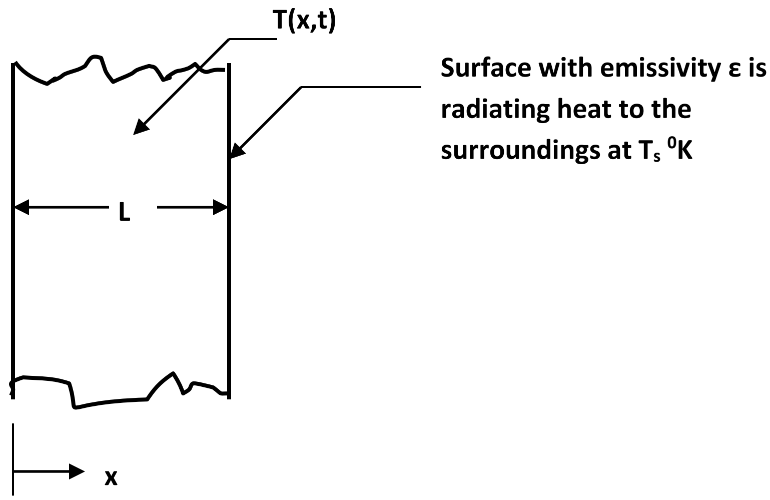
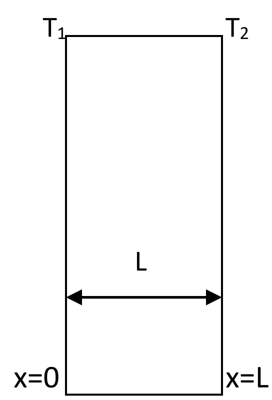


Fig. 2.10: Boundary surface at x = L subjected to radiation heat transfer

(i) at x = L, $q_{conduction} = q_{radiation}$; i.e., $-k\,(dT / dx)|_{x = L} = \sigma\,\varepsilon\,[(T|_{x = L})^4 - T_s^{\,4}]$

In the above equation both $T|_{x = L}$ and T_s should be expressed in degrees Kelvin

<u>Temperature Distribution in ID heat flow for slab with constant thermal conductivity</u>



$\frac{d}{dx}\left(k\frac{dT}{dx}\right) = 0$

If K is constant

$$K\frac{d^2T}{dx^2} = 0;$$

$$\frac{d^2T}{dx^2} = 0$$

Integrating above equation

$\frac{dT}{dx} = C_1$; $dT = C_1 dx$

Integrating again

$T = C_1 x + C_2$---A

Boundary conditions are i) $x = 0\,, T = T_1 and\ ii)\ x = L\,, T = T_2$

Boundary condition i) in A

$T_1 = 0 + C_2;\ C_2 = T_1$

ii) in A

$T_2 = C_1 L + C_2$

$T_2 = C_1 L + T_1$

$\frac{T_2 - T_1}{L} = C_1$

$T = \frac{T_2 - T_1}{L} x + T_1$

Above is the temperature distribution equation

Rate of Heat Transfer

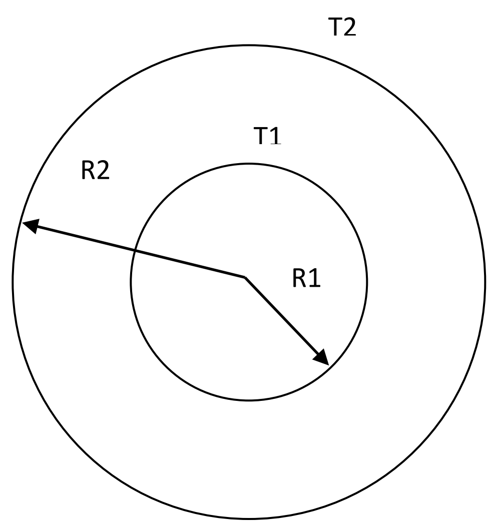
$Q = -KA \frac{dT}{dx}$

$Q = -KAC_1$

$Q = -KA \frac{T_2 - T_1}{L}$

$Q = \frac{KA(T_1 - T_2)}{L}$

Temperature Distribution in ID heat flow for cylinder with constant thermal conductivity



$\frac{d}{dr}\left(kr \frac{dT}{dr}\right) = 0$

If K is Constant

$k \frac{d}{dr}\left(r \frac{dT}{dr}\right) = 0$

$\frac{d}{dr}\left(r \frac{dT}{dr}\right) = 0$

Integrating above equation

$r \frac{dT}{dr} = C_1;$ $\frac{dT}{dr} = \frac{C_1}{r}$

$dT = C_1 \frac{dr}{r}$

Integrating above equation

$T = C_1 lnr + C_2$ ---------A

Boundary conditions are $i)\ at\ r = r_1, T = T_1\ ii)\ at\ r = r_2, T = T_2$

$i)\ in\ A$

$T_1 = C_1 lnr_1 + C_2$-------1

$ii)\ in\ A$

$T_2 = C_1 lnr_2 + C_2$ ------------2

Eqn 1- Eqn 2

$$T_1 - T_2 = C_1 lnr_1 - C_1 lnr_2$$

$$T_1 - T_2 = C_1 ln\frac{r_1}{r_2}$$

$$\frac{T_1 - T_2}{ln\frac{r_1}{r_2}} = C_1$$

$$T_1 = \frac{T_1 - T_2}{ln\frac{r_1}{r_2}} lnr_1 + C_2$$

$$T_1 - \frac{T_1 - T_2}{ln\frac{r_1}{r_2}} lnr_1 = C_2$$

$$T = \frac{T_1 - T_2}{ln\frac{r_1}{r_2}} lnr + T_1 - \frac{T_1 - T_2}{ln\frac{r_1}{r_2}} lnr_1$$

$$T - T_1 = (T_1 - T_2)\left(\frac{lnr - lnr_1}{ln\frac{r_1}{r_2}}\right)$$

$$\frac{T - T_1}{T_1 - T_2} = \frac{ln\frac{r}{r_1}}{ln\frac{r_1}{r_2}}$$

$$\frac{T - T_1}{T_1 - T_2} = -\frac{ln\frac{r}{r_1}}{ln\frac{r_2}{r_1}}$$

$$\frac{T_1 - T}{T_1 - T_2} = \frac{ln\frac{r}{r_1}}{ln\frac{r_2}{r_1}}$$

Above is the temperature distribution

Rate of Heat transfer

$$Q = -KA\frac{dT}{dr}$$

$$Q = -K2\pi rL\frac{dT}{dr}$$

$$Q = -K2\pi rL\frac{C_1}{r}$$

$$Q = -2\pi LKC_1$$

$$Q = -2\pi LK\frac{T_1 - T_2}{ln\frac{r_1}{r_2}}$$

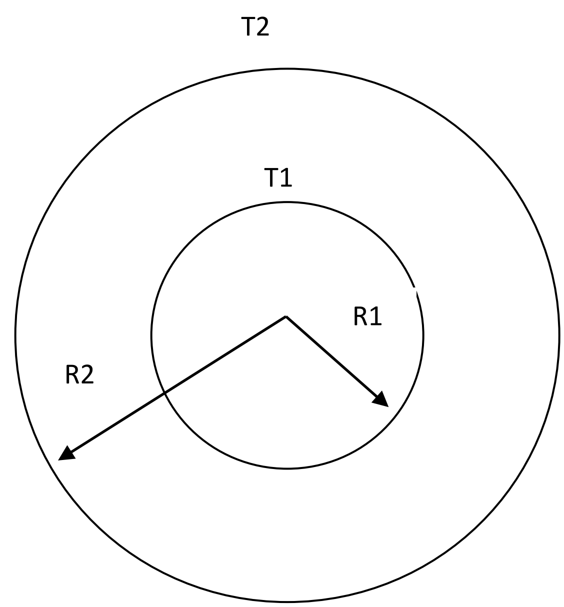
$$Q = 2\pi LK\frac{T_1 - T_2}{ln\frac{r_2}{r_1}}$$

Temperature Distribution in ID heat flow for sphere with constant thermal conductivity

$$\frac{d}{dx}\left(kr^2\frac{dT}{dr}\right) = 0$$

$$k\frac{d}{dx}\left(r^2\frac{dT}{dr}\right) = 0$$

$$\frac{d}{dx}\left(r^2\frac{dT}{dr}\right) = 0$$



Integrating above equation

$$r^2\frac{dT}{dr} = C_1$$

$$\frac{dT}{dr} = \frac{C_1}{r^2}$$

$$dT = \frac{C_1}{r^2}\ dr$$

Integrating above equation

$$T = C_1\frac{r^{-2+1}}{-2+1} + C_2$$

$$T = -C_1r^{-1} + C_2$$

$T = \frac{-C_1}{r} + C_2$--------A

Boundary conditions are $i)\ at\ r = r_1, T = T_1\ ii)\ at\ r = r_2, T = T_2$

$i)\ in\ A$

$T_1 = \frac{-C_1}{r_1} + C_2$-----1

$ii)\ in\ A$

$T_2 = \frac{-C_1}{r_2} + C_2$--------------2

Eqn 1— Eqn 2

$$T_1 - T_2 = \frac{-C_1}{r_1} - \frac{-C_1}{r_2}$$

$$T_1 - T_2 = \frac{-C_1}{r_1} + \frac{C_1}{r_2}$$

$$T_1 - T_2 = -C_1\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$-\frac{T_1 - T_2}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)} = C_1$$

Substituting $C_1 in$ 1

$$T_1 = \frac{T_1 - T_2}{r_1\left(\frac{1}{r_1} - \frac{1}{r_2}\right)} + C_2$$

$$C_2 = T_1 - \frac{T_1 - T_2}{r_1\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

Substituting C_1 and C_2inA

$$T = \frac{T_1 - T_2}{r\left(\frac{1}{r_1} - \frac{1}{r_2}\right)} + T_1 - \frac{T_1 - T_2}{r_1\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

$$T - T_1 = \frac{T_1 - T_2}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}\left(\frac{1}{r} - \frac{1}{r_1}\right)$$

$$\frac{T - T_1}{T_1 - T_2} = \frac{\left(\frac{1}{r} - \frac{1}{r_1}\right)}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

$$\frac{T - T_1}{T_1 - T_2} = -\frac{\left(\frac{1}{r_1} - \frac{1}{r}\right)}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

$$\frac{T_1 - T}{T_1 - T_2} = \frac{\left(\frac{1}{r_1} - \frac{1}{r}\right)}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

Above is the temperature distribution equation

Rate of Heat transfer

$$Q = -KA\frac{dT}{dr}$$

$$Q = -K4\pi r^2\frac{dT}{dr}$$

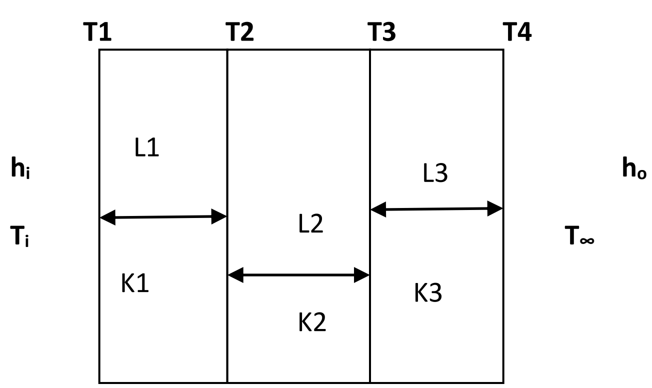
$$Q = -K4\pi r^2\frac{C_1}{r^2}$$

$$Q = -K4\pi C_1$$

$$Q = -4\pi K\left(-\frac{T_1 - T_2}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}\right)$$

$$Q = 4\pi K\ \frac{T_1 - T_2}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

Composite Slab



Convective Thermal resistance from fluid at T_i to Inner surface $= \frac{1}{h_1 A}$

Conductive Thermal resistance in inner first layer $= \frac{L_1}{k_1 A}$

Conductive Thermal resistance in second layer $= \frac{L_2}{k_2 A}$

Conductive Thermal resistance in third layer $= \frac{L_3}{k_3 A}$

Convective Thermal resistance from outer surface to Surrundings at $T_\infty = \frac{1}{h_0 A}$

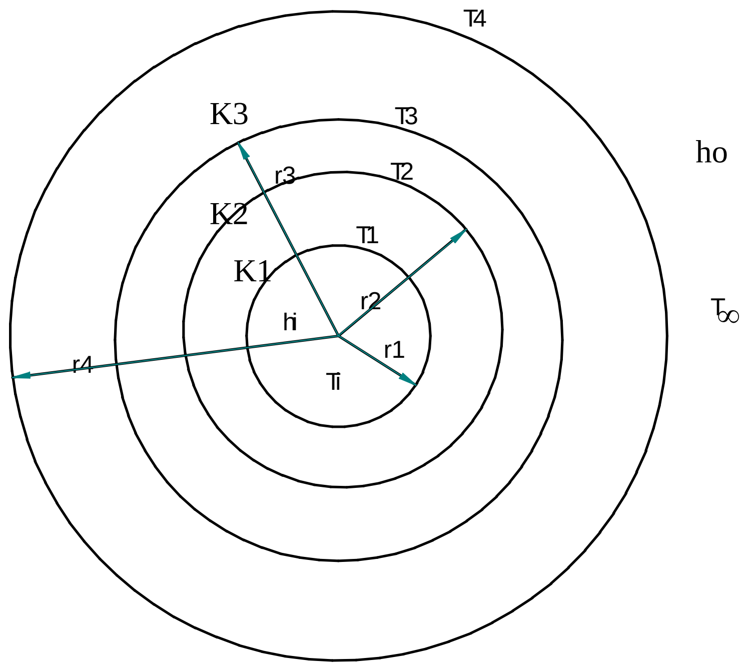
Total resistance from $T_i \; to \; T_\infty = \frac{1}{h_1 A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{L_3}{k_3 A} + \frac{1}{h_0 A}$

Hence Heat transfer rate $Q = \frac{T_i - T_\infty}{\frac{1}{h_1 A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{L_3}{k_3 A} + \frac{1}{h_0 A}}$

$$Q = \frac{T_i - T_\infty}{\frac{1}{A}\left(\frac{1}{h_1} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3} + \frac{1}{h_0}\right)}$$

$$Q = \frac{T_i - T_1}{\frac{1}{h_1 A}} \; ; \; Q = \frac{T_1 - T_2}{\frac{L_1}{k_1 A}} \; ; \; Q = \frac{T_2 - T_3}{\frac{L_2}{k_2 A}} \; ; \; Q = \frac{T_3 - T_4}{\frac{L_3}{k_3 A}} \; Q = \frac{T_4 - T_\infty}{\frac{1}{h_0 A}}$$

Composite cylinder



Convective Thermal resistance from fluid at T_i to Inner surface $= \frac{1}{h_1 2\pi r_1 L}$

Conductive Thermal resistance in inner first layer $= \frac{ln\frac{r_2}{r_1}}{2\pi L k_1}$

Conductive Thermal resistance in second layer $= \frac{ln\frac{r_3}{r_2}}{2\pi L k_2}$

Conductive Thermal resistance in third layer $= \frac{ln\frac{r_4}{r_3}}{2\pi L k_3}$

Convective Thermal resistance from outer surface to Surrundings at $T_\infty = \frac{1}{h_0 2\pi r_4 L}$

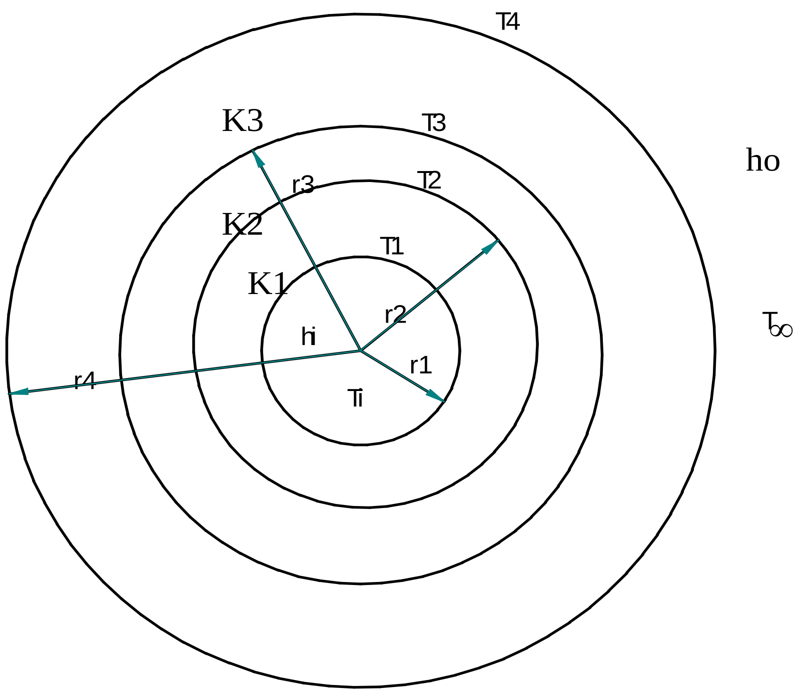
Total resistance from T_i *to* $T_\infty = \frac{1}{h_1 2\pi r_1 L} + \frac{ln\frac{r_2}{r_1}}{2\pi L k_1} + \frac{ln\frac{r_3}{r_2}}{2\pi L k_2} + \frac{ln\frac{r_4}{r_3}}{2\pi L k_3} + \frac{1}{h_0 2\pi r_4 L}$

Hence Heat transfer rate $Q = \frac{T_i - T_\infty}{\frac{1}{h_1 2\pi r_1 L} + \frac{ln\frac{r_2}{r_1}}{2\pi L k_1} + \frac{ln\frac{r_3}{r_2}}{2\pi L k_2} + \frac{ln\frac{r_4}{r_3}}{2\pi L k_3} + \frac{1}{h_0 2\pi r_4 L}}$

$$Q = \frac{T_i - T_\infty}{\frac{1}{2\pi L}\left(\frac{1}{h_1 r_1} + \frac{ln\frac{r_2}{r_1}}{k_1} + \frac{ln\frac{r_3}{r_2}}{k_2} + \frac{ln\frac{r_4}{r_3}}{k_3} + \frac{1}{h_0 r_4}\right)}$$

$$Q = \frac{T_i - T_1}{\frac{1}{h_1 2\pi r_1 L}}\ ;\ Q = \frac{T_1 - T_2}{\frac{ln\frac{r_2}{r_1}}{2\pi L k_1}}\ ;\ Q = \frac{T_2 - T_3}{\frac{ln\frac{r_3}{r_2}}{2\pi L k_2}}\ ;\ Q = \frac{T_3 - T_4}{\frac{ln\frac{r_4}{r_3}}{2\pi L k_3}}\ Q = \frac{T_4 - T_\infty}{\frac{1}{h_0 2\pi r_4 L}}$$

Composite sphere



Convective Thermal resistance from fluid at T_i to Inner surface $= \frac{1}{h_i 4\pi r_1^2}$

Conductive Thermal resistance in inner first layer $= \frac{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}{4\pi k_1}$

Conductive Thermal resistance in second layer $= \frac{\left(\frac{1}{r_2} - \frac{1}{r_3}\right)}{4\pi k_2}$

Conductive Thermal resistance in third layer $= \frac{\left(\frac{1}{r_3} - \frac{1}{r_2}\right)}{4\pi k_3}$

Convective Thermal resistance from outer surface to Surrundings at $T_\infty = \frac{1}{h_o 4\pi r_4^2}$

Total resistance from $T_i\ to\ \ T_\infty = \frac{1}{h_i 4\pi r_1^2} + \frac{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}{4\pi k_1} + \frac{\left(\frac{1}{r_2} - \frac{1}{r_3}\right)}{4\pi k_2} + \frac{\left(\frac{1}{r_3} - \frac{1}{r_2}\right)}{4\pi k_3} + \frac{1}{h_o 4\pi r_4^2}$

Hence Heat transfer rate $Q = \frac{T_i - T_\infty}{\frac{1}{h_i 4\pi r_1^2} + \frac{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}{4\pi k_1} + \frac{\left(\frac{1}{r_2} - \frac{1}{r_3}\right)}{4\pi k_2} + \frac{\left(\frac{1}{r_3} - \frac{1}{r_2}\right)}{4\pi k_3} + \frac{1}{h_o 4\pi r_4^2}}$

$$Q = \frac{T_i - T_\infty}{\frac{1}{4\pi}\left(\frac{1}{h_1 r_1^2} + \frac{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}{k_1} + \frac{\left(\frac{1}{r_2} - \frac{1}{r_3}\right)}{k_2} + \frac{\left(\frac{1}{r_3} - \frac{1}{r_2}\right)}{k_3} + \frac{1}{h_0 r_4^2}\right)}$$

$$Q = \frac{T_i - T_1}{\frac{1}{h_i 4\pi r_1^2}} \; ; \; Q = \frac{T_1 - T_2}{\frac{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}{4\pi k_1}} \; ; \; Q = \frac{T_2 - T_3}{\frac{\left(\frac{1}{r_2} - \frac{1}{r_3}\right)}{4\pi k_2}} \; ; \; Q = \frac{T_3 - T_4}{\frac{\left(\frac{1}{r_3} - \frac{1}{r_2}\right)}{4\pi k_3}} \; Q = \frac{T_4 - T_\infty}{\frac{1}{h_o 4\pi r_4^2}}$$

Module 2

Critical Thickness of Insulation for Sphere and Cylinder

VTU Questions (Theory)

1. Define critical thickness of insulation and explain its significance (2a, June/July 17, 04 Marks)
2. What is physical significance of critical thickness of insulation? Derive an expression for critical thickness of insulation for a cylinder
3. What is critical thickness of insulation on small diameter wire or pipe? Explain its physical significance and derive an expression for the same
4. Derive an expression for critical thickness of insulation for a cylinder. Discuss the design aspects for providing insulation scheme for cable wires and steam pipes 06 June/July 13, Dec14/Jan15, small diameter wire or pipe

5 Derive an expression for critical thickness of insulation for a sphere

<u>Critical radius of Insulation</u>

It is radius of insulation in cylinder or sphere is the radius at which heat transfer maximum of insulation for which heat transfer is maximum.

<u>Critical thickness of Insulation</u>

It is thickness of insulation in cylinder or sphere is the thickness at which heat transfer maximum of insulation for which heat transfer is maximum

Cylinder

$$Q = \frac{T_s - T_\infty}{\frac{ln\frac{r_2}{r_1}}{2\pi LK} + \frac{1}{h2\pi r_1 L}}$$

maximum Heat Transfer is constant . Hence differentiation of Q at maximum value with respect to $r_2 = 0$

$\frac{dQ}{dr_2} = 0;$

$$\frac{d}{dr_2}\left(\frac{T_s - T_\infty}{\frac{ln\frac{r_2}{r_1}}{2\pi LK} + \frac{1}{h2\pi r_2 L}}\right) = 0$$

$$\frac{d}{dr_2}\left(\frac{ln\frac{r_2}{r_1}}{2\pi LK}+\frac{1}{2\pi r_2 L}\right)=0$$

$$\frac{d}{dr_2}\left(\frac{ln\frac{r_2}{r_1}}{2\pi LK}\right)+\frac{d}{dr_2}\left(\frac{1}{h2\pi r_2 L}\right)=0$$

$$\frac{1}{2\pi LK}\frac{d}{dr_2}\left(ln\frac{r_2}{r_1}\right)+\frac{1}{h2\pi L}\frac{d}{dr_2}\left(\frac{1}{r_2}\right)=0$$

$$\frac{1}{2\pi LK}\frac{d}{dr_2}\left(ln\frac{r_2}{r_1}\right)+\frac{1}{h2\pi L}\frac{d}{dr_2}\left(\frac{1}{r_2}\right)=0$$

$$\frac{1}{2\pi LK}\frac{d}{dr_2}(lnr_2-lnr_1)+\frac{1}{h2\pi L}\frac{d}{dr_2}(r_2^{-1})=0$$

$$\frac{1}{2\pi LK}\left(\frac{1}{r_2}-0\right)+\frac{1}{h2\pi L}(-1r_2^{-2})=0$$

$$\frac{1}{2\pi LKr_2}-\frac{1}{h2\pi L}r_2^{-2}=0$$

$$\frac{1}{2\pi LKr_2}-\frac{1}{h2\pi Lr_2^2}=0$$

$$\frac{1}{2\pi LKr_2}=\frac{1}{h2\pi Lr_2^2}$$

$$\frac{1}{Kr_2}=\frac{1}{hr_2^2}$$

$$r_2=\frac{K}{h}$$

Ie for maximum heat transfer value 0f radius of insulation is equal to $\frac{K}{h}$

Hence critical radius of insulation for cylinder is $r_c=\frac{K}{h}$

Critical thickness of Insulation for cylinder is $\frac{K}{h}-r_1$

Sphere

$$Q=\frac{T_s-T_\infty}{\frac{\frac{1}{r_1}-\frac{1}{r_2}}{4\pi K}+\frac{1}{h4\pi r_2^2}}$$

maximum Heat Transfer is constant . Hence differentiation of Q at maximum value with respect to $r_2 = 0$

$\frac{dQ}{dr_2} = 0;$

$$\frac{d}{dr_2}\left(\frac{T_s - T_\infty}{\frac{\frac{1}{r_1}-\frac{1}{r_2}}{4\pi K} + \frac{1}{h4\pi r_2^2}}\right) = 0$$

$$\frac{d}{dr_2}\left(\frac{\frac{1}{r_1}-\frac{1}{r_2}}{4\pi K} + \frac{1}{h4\pi r_2^2}\right) = 0$$

$$\frac{d}{dr_2}\left(\frac{\frac{1}{r_1}-\frac{1}{r_2}}{4\pi K}\right) + \frac{d}{dr_2}\left(\frac{1}{h4\pi r_2^2}\right) = 0$$

$$\frac{1}{4\pi K}\frac{d}{dr_2}\left(\frac{1}{r_1}-\frac{1}{r_2}\right) + \frac{1}{h4\pi}\frac{d}{dr_2}\left(\frac{1}{r_2^2}\right) = 0$$

$$\frac{1}{4\pi K}\frac{d}{dr_2}(r_1^{-1} - r_2^{-1}) + \frac{1}{h4\pi}\frac{d}{dr_2}(r_2^{-2}) = 0$$

$$\frac{1}{4\pi K}\left(0 - (-1r_2^{-2})\right) + \frac{1}{h4\pi K}(-2r_2^{-3}) = 0$$

$$\frac{1}{4\pi K}\left(0 + \frac{1}{r_2^2}\right) + \frac{1}{h4\pi K}\left(-2\frac{1}{r_2^3}\right) = 0$$

$$\frac{1}{4\pi K}\frac{1}{r_2^2} = \frac{2}{h4\pi K}\frac{1}{r_2^3}$$

$$\frac{1}{Kr_2^2} = \frac{2}{h}\frac{1}{r_2^3}$$

$$r_2 = \frac{2K}{h}$$

Ie for maximum heat transfer value 0f radius of insulation is equal to $\frac{2K}{h}$

Hence critical radius of insulation for cylinder is $r_c = \frac{2K}{h}$

Critical thickness of Insulation for cylinder is $\frac{2K}{h} - r_1$

6. A wire of 8 mm diameter at a temperature of 60°C is to be insulated by a material having K=0.174 W/m° C. Heat transfer Coefficient h_a= 8 W/m^{2}K and ambient temperature T_a= 25°C.For maximum heat loss , find the minimum thickness of insulation. Find increase in heat dissipation due to insulation

Solution

Take length of wire is 1m

$d_1 = 8mm = 0.008m;\ r_1 = 0.004;\ T_s = 60^oC;\ \ k = 0.174W/mK; h = 8W/m^2K; T_\infty = 25^0C;$

With bare wire

$Q = hA(T_s - T_\infty);\quad Q = h2\pi r_1 L(T_s - T_\infty)\ ;$

$$Q = 8*2*\pi*0.004*1*(60-25);\ Q = 0.8796\ watts$$

For critical radius $= \frac{K}{h}$

$$r_c = \frac{0.174}{8} = 0.02175m$$

Heat Transfer in wire with insulation with critical radius

$$Q_{max} = \frac{T_s - T_\infty}{\frac{ln\frac{r_c}{r_1}}{2\pi LK} + \frac{1}{h2\pi r_1 L}}$$

$$Q_{max} = \frac{60-25}{\frac{ln\frac{0.02175}{0.004}}{2\pi*1*0.174} + \frac{1}{8*2\pi*0.004*1}}$$

$$Q_{max} = \frac{60-25}{\frac{ln\frac{0.02175}{0.004}}{2\pi*1*0.174} + \frac{1}{8*2\pi*0.004*1}}$$

Q_{max} = 5.366Watts

Increase in Heat transfer due to insulation of critical radius $= 5.366 - 0.8976 = 4.468\ Watts$

Percentage in increase in Heat Transfer =$\frac{Q_{max}-Q}{Q}$x100

$$\frac{5.366-0.8976}{0.8976}x100 = 497.81\%$$

7. An electric cable of 10 mm diameter is to be laid in atmosphere at 20 °C .The estimated surface temperature of the cable due to heat generation is 65 °C. Find the maximum percentage increase in heat dissipation when the wire is insulated with rubber having a 0.155 W/m°C and h= 88.5 W/m² °C

$d_1 = 0.01m;\ r_1 = 0.005m;\ \ T_s = 65^oC;\ \ k = 0.155W/mK;\ h = 88.5W/m^2K;\ T_\infty = 20^0C$

With bare wire

$$Q = hA(T_s - T_\infty);\quad Q = h2\pi r_1 L(T_s - T_\infty)$$

For critical radius $= \frac{K}{h}$

$$Q_{max} = \frac{T_s - T_\infty}{\frac{ln\frac{r_c}{r_1}}{2\pi LK} + \frac{1}{h2\pi r_1 L}}$$

Percentage increase in Heat Transfer $=\frac{Q_{max}-Q}{Q}$x100

8. A Copper pipe carrying refrigerant -20°C is 10mm in outer diameter and is exposed to ambient 25°C with convective heat transfer coefficient of 50W/m²K. It is proposed to apply the insulation material having thermal conductivity of 0.5W/mK. Determine the thickness beyond which the heat gain will be reduced. Calculate the heat loss for 2.5mm and 7.5mm thick layer of insulation over 1m length.

$d_1 = 10mm = 0.01m;\ r_1 = 5mm = 0.005m;\ \ T_s = -20^oC;\ \ k = 0.5W/mK;\ \ \ h = 50W/m^2K\ ;\ T_\infty = 25^0C$

For critical radius $= \frac{K}{h}$

$r_c = \frac{0.5}{50} = 0.01m$

Critical thickness of insulation is $r_c - r_1 = 0.01 - 0.005 = 0.005m = 5mm$

Heat gain will be reduce beyond 5 mm thickness of insulation

Heat Transfer with critical thickness of Insulation

$$r_c = 0.01m$$

$$Q = \frac{T_s - T_\infty}{\frac{\ln\frac{r_c}{r_1}}{2\pi LK} + \frac{1}{h2\pi r_1 L}}$$

$$Q = \frac{-20 - 25}{\frac{\ln\frac{0.01}{0.005}}{2\pi * 1 * 0.5} + \frac{1}{50 * 2 * \pi * 0.005 * 1}}$$

$Q = -30.12Watts$

Heat Transfer with 2.5mm thickness of Insulation

$r_2 = r_1 + thickness;\ r_2 = 5 + 2.5 = 7.5mm; r_2 = 0.0075$

$$Q = \frac{T_s - T_\infty}{\frac{\ln\frac{r_c}{r_1}}{2\pi LK} + \frac{1}{h2\pi r_1 L}}$$

$$Q = \frac{-20 - 25}{\frac{\ln\frac{0.0075}{0.005}}{2\pi * 1 * 0.5} + \frac{1}{50 * 2 * \pi * 0.005 * 1}}$$

$Q = -58.77Watts$

Heat Transfer with 7.5mm thickness of Insulation

$r_2 = r_1 + thickness;\ r_2 = 5 + 7.5 = 12.5mm; r_2 = 0.0125m$

$$Q = \frac{T_s - T_\infty}{\frac{\ln\frac{r_2}{r_1}}{2\pi LK} + \frac{1}{h2\pi r_1 L}}$$

$$Q = \frac{-20 - 25}{\frac{\ln\frac{0.0125}{0.005}}{2\pi * 1 * 0.5} + \frac{1}{50 * 2 * \pi * 0..005 * 1}}$$

$Q = -48.47Watts$

Negative sign indicates heat is gained to the refrigerant ie from atmosphere to refrigerant

9. A small electric heating application uses 1.82mm diameter wire with 0.71mm thickness K (insulation) =0.118W/mK and h_o=34.1W/m^{2}K. Determine the critical thickness of insulation for

this case and change in heat transfer rate if critical thickness was used. Assume the temperature difference between surface of wire and surrounding air remain unchanged

10. A sphere of 8 mm diameter at a temperature of 60°C is to be insulated by a material having K=0.174 W/m° C. Heat transfer Coefficient h_a= 8 W/m²K and ambient temperature T_a= 25°C.For maximum heat loss , find the minimum thickness of insulation. Find increase in heat dissipation due to insulation

$d_1 = 10mm = 0.01m;\ r_1 = 5mm = 0.005m;\ T_s = -20^oC;\ k = 0.5W/mK;\ h = 50W/m^2K\ ;\ T_\infty = 25^0C$

For critical radius $= \frac{2K}{h}$

$r_c = \frac{2*0.5}{50} = 0.02m$

Critical thickness of insulation is $r_c - r_1 = 0.02 - 0.005 = 0.015m = 15mm$

Heat gain will be reduce beyond 5 mm thickness of insulation

With bare wire

$Q = hA(T_s - T_\infty);\quad Q = h * 4 * \pi * r_1^2 * (T_s - T_\infty)\ ;$

$$Q = 8 * 4 * \pi * 0.005^2 * (60 - 25);\ Q = 0.08796\, watts$$

For critical radius $= \frac{K}{h}$

$$r_c = \frac{0.174}{8} = 0.02175m$$

Heat Transfer in wire with insulation with critical radius

$$Q = \frac{T_s - T_\infty}{\frac{\frac{1}{r_1} - \frac{1}{r_2}}{4\pi K} + \frac{1}{h4\pi r_2^2}}$$

$$Q = \frac{60 - 25}{\frac{\frac{1}{0.005} - \frac{1}{0.02}}{4\pi * 0.174} + \frac{1}{8 * 4\pi * 0.02^2}}$$

$Q_{max} =$

Increase in Heat transfer due to insulation of critical radius $= 5.366 - 0.8976 = 4.468\, Watts$

Percentage in increase in Heat Transfer $=\frac{Q_{max} - Q}{Q}$x100

11. Obtain an expression for temperature distribution and heat flow through a fin of uniform cross section with the end is insulated

11. Obtain an expression for temperature distribution and heat flows through a rectangular fin when the end of the fin is insulated

12. Derive an expression for temperature distribution for a pinfin when the tip of the film is insulated

13. Derive an expression for temperature distribution for a short fin of uniform cross section without insulated tips starting from fundamental energy balance equation

14. Differentiate between effectiveness and efficiency of fins

15. Define fin effectiveness. When the use of fin is not justified

Extended Surfaces (Fins)

Uses of fin

Fins are used to increase the heat transfer from the surface so that the surface temperature of fin can be maintained at designed value

Applications

1. Automobile radiators ,
2. Compressors and IC engines
3. CPU of computers to dissipate the heat
4. Hydrogen fuel cells

Rectangular fin:

Assumption in Heat transfer of Fin

1. Steady state heat transfer in fins
2. Properties of material of fins is constant
3. No internal Heat generation

4. One dimensional conduction
5. Uniform convection across the surface are h=constant

Consider a rectangular protruding from the surface.

Consider a small element in a rectangular fin of elemental length dx at a distance x from the base of fin. Let T_o be the temperature of the base of the fin , 'T' be the temperature of fin at x , h is the surface heat transfer coefficient , P is the perimeter of fin and A_c cross sectional area of element

Heat balance applied to the element

Heat inflow to the element by conduction at X =Amount of leaving the element at X+dx +Amount of heat convected from the surface to surroundings

$Q_x = Q_{x+dx} + Q_{con}$ -------------1

$Q_x - Q_{x+dx} - Q_{con} = 0$

$$Q_x = -KA_c\frac{dT}{dx}$$

$$Q_{x+dx} = Q_x + \frac{dQ_x}{dx}dx$$

$Q_x - Q_{x+dx} = -\frac{dQ_x}{dx}dx$ -------------2

Substituting 2 in 1

$-\frac{dQ_x}{dx}dx - Q_{con} = 0$-------3

$$Q_x = -KA_c\frac{dT}{dx}$$

$$\frac{dQ_x}{dx}dx = \frac{d\left(-KA_c\frac{dT}{dx}\right)}{dx}dx$$

$$\frac{dQ_x}{dx}dx = -KA_c\frac{d^2T}{dx^2}dx$$

$$-\frac{dQ_x}{dx}dx = KA_c\frac{d^2T}{dx^2}dx$$

Heat from fin to surroundings by Convection $Q_{con} = hA_{convection}(T - T_\infty)$

$A_{convection} = Pdx$ where P is the perimeter of element and dx is the length of element

$Q_{con} = hPdx(T - T_\infty)$-------------5

Substituting 4 and 5 in3

$$KA_c\frac{d^2T}{dx^2}dx - hPdx(T - T_\infty) = 0$$

$$KA_c\frac{d^2T}{dx^2} - hP(T - T_\infty) = 0$$

$\frac{d^2T}{dx^2} - \frac{hP}{KA_c}(T - T_\infty) = 0$ -------------6

Let $\theta = (T - T_\infty)$; differentiating equation $\frac{dT}{dx} = \frac{d\theta}{dx} - 0$ hence, $\frac{dT}{dx} = \frac{d\theta}{dx}$

$$\frac{hP}{KA_c} = Constant = m^2$$

Substituting above values in equation 6

$\frac{d^2\theta}{dx^2} - m^2\theta = 0$

Above equation is second order homogeneous differential equation and solution for above equation is

$\theta = Ae^{-mx} + Be^{mx}$ ---------------A

$e^{-mx} = Coshmx + Sinhmx;\ e^{mx} = Coshmx - Sinhmx$

Substituting above values in equation A

$\theta = (A + B)Coshmx + (A - B)Sinhmx$

$\theta = CCoshmx + DSinhmx$ -------------B where $C = A + B \ and \ D = A - B$

Case 1

End of the fin is insulated;

$\theta = CCoshmx + DSinhmx$ -------------B

Boundary conditions are i) at $= 0$, $\theta = \theta_0$ ii) at $x = L$, $\frac{d\theta}{dx} = 0$ (since end is insulated $\theta \ is \ constant$)

Boundary condition i) in B

$\theta_0 = CCosh(0) + DSinh(0)$

$\theta_0 = C$-------1

Differentiating equation B

$$\frac{d\theta}{dx} = Cmsinhmx + DmCoshmx$$

$$\left(\frac{d\theta}{dx}\right)_{x=L} = \ CmsinhmL + DmCoshmL$$

$$0 = \ \theta_0 msinhmL + DmCoshmL$$

$$DmCoshmL = -\theta_0 msinhmL$$

$D = -\theta_0 tanhmL$-----2

Substituting C and D in equation B

$$\theta = \theta_0 Coshmx - \theta_0 tanhmLSinhmx$$

$$\theta = \theta_0 Coshmx - \theta_0 \frac{SinhmL}{CoshmL} Sinhmx$$

$$\theta = \theta_0 \left(\frac{CoshmLCoshmx - SinhmLSinhmx}{CoshmL}\right)$$

$$\theta = \theta_0 \left(\frac{Coshm(L-x)}{CoshmL}\right)$$

$$T - T_\infty = (T_o - T_\infty)\left(\frac{Coshm(L-x)}{CoshmL}\right)$$

$$\frac{T - T_\infty}{T_o - T_\infty} = \frac{Coshm(L-x)}{CoshmL}$$

Above is the Temperature equation for end of the fin is insulated

Rate of Heat Transfer

$$Q = -KA_c\left(\frac{dT}{dx}\right)_{x=0}$$

$$Q = -KA_c\left(\frac{d\theta}{dx}\right)_{x=0}$$

$$\frac{d\theta}{dx} = Cmsinhmx + DmCoshmx$$

$$\left(\frac{d\theta}{dx}\right)_{x=0} = Dm$$

$$Q = -KA_c Dm$$

$$Q = -KA_c(-\theta_0 tanhmL)m$$

$Q = KA_c m\theta_0 tanhmL$

Case 2 : very long fin L=∞ so that $T_L = T_\infty$ at $x = L$ $\theta = T_\infty - T_\infty = 0$

$\theta = Ae^{-mx} + Be^{mx}$ ---------------A

Boundary conditions are Boundary conditions are i) at $= 0$, $\theta = \theta_0$ ii) at $x = L$, $\theta = 0$

Boundary Condition 1 in A

$\theta_0 = Ae^{-0} + Be^{0}$

$\theta_0 = A + B$ -----1

Boundary Condition 2 in B

$0 = Ae^{-m\infty} + Be^{m\infty}$

$0 = A(0) + B(\infty)$

$0 = 0 + B(\infty)$

Hence B must be 0 ie B=0

Substituting B =0 in 1

$\theta_0 = A + 0$ hence $A = \theta_0$ and B= 0

Hence Equation B becomes

$\theta = \theta_0 e^{-mx}$

$$\frac{\theta}{\theta_0} = e^{-mx}$$

$$\frac{T - T_\infty}{T_o - T_\infty} = e^{-mx}$$

Above the temperature distribution equation for very long fin

Rate of Heat Transfer

$$Q = -KA_c \left(\frac{dT}{dx}\right)_{x=0}$$

$$Q = -KA_c \left(\frac{d\theta}{dx}\right)_{x=0}$$

$\theta = \theta_0 e^{-mx}$

$\frac{d\theta}{dx} = \theta_0(-m)e^{-mx}$

$\frac{d\theta}{dx} = -m\theta_0 e^{-mx}$

$\left(\frac{d\theta}{dx}\right)_{x=0} = -m\theta_0$

$Q = -KA_c(-m\theta_0)$

$Q = KA_c m\theta_0$

<u>Case 3 : Short fin with end is not insulated</u>

$\theta = CCoshmx + DSinhmx$ -------------B

Boundary conditions are i) at $x = 0$, $\theta = \theta_0$

ii) at $x = L$, Heat transfer by conduction =Heat is transferred from the end surface by convection

Boundary condition i) in B

$\theta_0 = CCosh(0) + DSinh(0)$

$\theta_0 = C$-------1

Boundary condition 2

$-KA_c\left(\frac{d\theta}{dx}\right)_{x=L} = hA_c(\theta)_{x=L}$

$-K\left(\frac{d\theta}{dx}\right)_{x=L} = h(\theta)_{x=L}$ -------------------1

$\theta = CCoshmx + DSinhmx$

$\frac{d\theta}{dx} = CmSinhmx + DmCoshmx$

$(\theta)_{x=L} = \theta_0\, CoshmL + DSinhmL$ ------------------------2

$\left(\frac{d\theta}{dx}\right)_{x=L} = \theta_0 mSinhmL + DmCoshmL$ ------------------3

$-K(\theta_0 mSinhmL + DmCoshmL) = h(\theta_0\, CoshmL + DSinhmL)$

$-K\theta_0 mSinhmL - h\theta_0\, CoshmL = K\, DmCoshmL + h\, DSinhmL$

$-\theta_0(KmSinhmL + h\, CoshmL) = D(KmCoshmL + h\, SinhmL)$

$$-\theta_0 \frac{KmSinhmL + h\ CoshmL}{KmCoshmL + h\ SinhmL} = D$$

Substituting C and D in equation B

$$\theta = \theta_0 Coshmx - \theta_0 \frac{KmSinhmL + h\theta_0\ CoshmL}{KmCoshmL + h\ SinhmL} Sinhmx$$

$$\theta = \theta_0 \frac{Coshmx(KmCoshmL + h\ SinhmL) - Sinhmx(KmSinhmL + h\theta_0\ CoshmL)}{KmCoshmL + h\ SinhmL}$$

$$\theta = \theta_0 \frac{(KmCoshmLCoshmx + h\ SinhmLCoshmx) - (KmSinhmLSinhmx + h\ CoshmLSinhmx)}{KmCoshmL + h\ SinhmL}$$

$$\theta = \theta_0 \frac{h(\ SinhmLCoshmx - CoshmLSinhmx) - Km(SinhmLSinhmx - CoshmLCoshmx)}{KmCoshmL + h\ SinhmL}$$

$$\theta = \theta_0 \frac{h(\ SinhmLCoshmx - CoshmLSinhmx) + Km(CoshmLCoshmx - SinhmLSinhmx)}{KmCoshmL + h\ SinhmL}$$

$$\theta = \theta_0 \frac{h\big(\ Sinhm(L-x)\big) + Km\big(Coshm(L-x)\big)}{KmCoshmL + h\ SinhmL}$$

$$\theta = \theta_0 \frac{\frac{h}{mk}\big(\ Sinhm(L-x)\big) + \big(Coshm(L-x)\big)}{CoshmL + \frac{h}{mk}\ SinhmL}$$

$$\frac{T - T_\infty}{T_o - T_\infty} = \frac{\frac{h}{mk}\big(\ Sinhm(L-x)\big) + \big(Coshm(L-x)\big)}{CoshmL + \frac{h}{mk}\ SinhmL}$$

Above is the temperature distribution equation for short fin with no insulation at the end surface

Rate of Heat Transfer

$$Q = -KA_c\left(\frac{dT}{dx}\right)_{x=0}$$

$$Q = -KA_c\left(\frac{d\theta}{dx}\right)_{x=0}$$

$$\frac{d\theta}{dx} = CmSinhmx + DmCoshmx$$

$$\left(\frac{d\theta}{dx}\right)_{x=0} = Dm$$

$$Q = -KA_c Dm$$

$$Q = -KA_c mD$$

$$Q = -KA_c m\left(-\theta_0 \frac{KmSinhmL + h\ CoshmL}{KmCoshmL + h\ SinhmL}\right)$$

$$Q = KA_c m\theta_0 \left(\frac{KmSinhmL + h\ CoshmL}{KmCoshmL + h\ SinhmL}\right)$$

$$Q = KA_c m\theta_0 \left(\frac{tanhmL + \frac{h}{mk}CoshmL}{1 + \frac{h}{mk}\ SinhmL}\right)$$

<u>Case 4; Fin is connected to two surfaces maintained at two Different temperature</u>

$\theta = CCoshmx + DSinhmx$ -------------B

Boundary conditions are i) when $x = 0, \theta = \theta_1$ ii) $x = L, \theta = \theta_2$

Boundary condition i) in B

$$\theta_1 = CCosh(0) + DSinh(0)$$

$\theta_1 = C$-------1

Boundary condition ii) in B ie ii) $x = L, \theta = \theta_2$ in B

$$\theta_2 = CCoshmL + DSinhmL$$

$$\theta_2 = \theta_1 CoshmL + DSinhmL$$

$$\frac{\theta_2 - \theta_1 CoshmL}{SinhmL} = D$$

Substituting c and D in equation B

$$\theta = \theta_1 Coshmx + \frac{\theta_2 - \theta_1 CoshmL}{SinhmL} Sinhmx$$

$$\theta = \frac{\theta_1 CoshmxSinhmL + \theta_2 Sinhmx - \theta_1 CoshmLSinhmx}{SinhmL}$$

$$\theta = \frac{\theta_1 (CoshmxSinhmL - CoshmLSinhmx) \quad + \theta_2 Sinhmx}{SinhmL}$$

$$\theta = \frac{\theta_1 sinhm(L - x) \quad + \theta_2 Sinhmx}{SinhmL}$$

$$\frac{\theta}{\theta_1} = \frac{sinhm(L - x) + \frac{\theta_2}{\theta_1} sinhmx}{SinhmL}$$

$$\frac{T - T_\infty}{T_1 - T_\infty} = \frac{sinhm(L - x) + \frac{T_2 - T_\infty}{T_1 - T_\infty} sinhmx}{SinhmL}$$

Above equation is temperature distribution equation

$$Q = -KA_c \left(\frac{dT}{dx}\right)_{x=0}$$

$$Q = -KA_c \left(\frac{d\theta}{dx}\right)_{x=0}$$

$$\theta = \frac{\theta_1 sinhm(L - x) \;\; + \theta_2 Sinhmx}{SinhmL}$$

$$\frac{d\theta}{dx} = \frac{\theta_1(-m)cos\,hm(L - x) \;\; + \theta_2 mcoshmx}{SinhmL}$$

$$\left(\frac{d\theta}{dx}\right)_{x=0} = \frac{-m\theta_1 coshmL + m\theta_2}{SinhmL}$$

$$Q = -KA_c \left(\frac{d\theta}{dx}\right)_{x=0}$$

$$Q = -KA_c \left(\frac{-m\theta_1 coshmL + m\theta_2}{SinhmL}\right)$$

$$Q = KA_c \left(\frac{m\theta_1 coshmL - m\theta_2}{SinhmL}\right)$$

Effectiveness of fin :

It is the ratio of actual rate of heat transfer from the fin to the rate of heat transfer that would dissipated from the same surface area without fin

$$\varepsilon = \frac{Heat\ transfer\ from\ the\ fin}{Heat\ transfer\ from\ the\ same\ surface\ area\ without\ fin}$$

For fin with end of fin insulated

$$\varepsilon = \frac{KA_c m\theta_0 tanhmL}{hA_c\theta_0}$$

$$\varepsilon = \frac{KmtanhmL}{hA_c}$$

$$\varepsilon = \frac{K\sqrt{\frac{hP}{KA_c}}tanhmL}{hA_c}$$

If $\varepsilon > 1$ indicates fins are enhancing the heat transfer to maintain the surface temperature to designed value.

If $\varepsilon < 1$ indicates fins are acting as insulator and it reduces the dissipation of heat to environment

If $\varepsilon = 1$ indicates addition of fins have no use ie it neither enhances or reduces the heat transfer from the surface

Efficiency of fin:

It is defined as the ratio of the actual heat transferred by fin to the maximum heat transfer from the fin if the whole surface of fin is maintained at base temperature

$$\eta = \frac{Q_{fin}}{Q_{max}}$$

$$\eta = \frac{Q_{fin}}{hPL\theta_0}$$

If end of the fin is insulated

$$\eta = \frac{KA_c m\theta_0 tanhmL}{hPL\theta_0}$$

$$\eta = \frac{KA_c\sqrt{\frac{hP}{KA_c}}\theta_0 tanhmL}{hPL\theta_0}$$

$$\eta = \frac{KA_c\sqrt{\frac{hP}{KA_c}}tanhmL}{hPL}$$

$$\eta = \frac{\sqrt{\frac{KA_c}{hP}}tanhmL}{L}$$

$$\eta = \frac{\frac{1}{m}tanhmL}{L}$$

$$\eta = \frac{tanhmL}{mL}$$

Biot number in fin:

It is defined as the ratio of ratio of heat transfer by convection and convection to specific heat transfer by conduction

$B_i = \frac{h}{\frac{k}{\delta}}$; $B_i = \frac{h\delta}{K}$ where h is the surface heat transfer coefficient , δ is thickness of the fin and K is the thermal conductivity of material

If $B_i < 1\ then\ effectiveness\ \varepsilon\ > 1$ desirable

If $B_i > 1\ then\ effectiveness\ \varepsilon\ < 1$ fin acts as insulator and it decreases hrate of heat transfer

If $B_i = 1\ then\ effectiveness\ \varepsilon = 1$ Fin neither it will enhance or decrease the heat transfer It as good as no fin

Case 3 and case 4 is less important

A rod (K= 200 W/mK) 5 mm in diameter and 5 cm long has its one end maintained at 100 °C. The surface of the rod is exposed to ambient air at 25 °C with convection heat transfer coefficient of 100 W/m^{2}K. Assuming the **other end is insulated**. Determine i)the temperature of the rod at 20 mm distance from the end at 100 °C ii) Heat dissipation rate from the surface of the rod iii) effectiveness iv) Efficiency of fin

$K = 200W/mK,\quad d = 5mm = 0.005m, T_b = 100^0C;\ T_\infty = 25^0C; h = 100W/m^2K,$ other end is insulated i) T=? at x=20mm=0.02m ii) Q=? iii) ϵ =?

$\boldsymbol{End\ is\ Insulated}$

i) $From\ Data\ Hand\ Book, Page\ 48\ ,\quad \frac{T-T_\infty}{T_b-T_\infty} = \frac{coshm(L-x)}{coshmL}$

$m = \sqrt{\frac{hP}{kA_c}};\qquad For\ circular\ rod\ \ m = \sqrt{\frac{h\pi d}{k\frac{\pi d^2}{4}}};\qquad m = \sqrt{\frac{4h}{kd}}$

$m = \sqrt{\frac{4*100}{200*0.005}}\ ;\quad m = 20$

$mL = 20 * 0.05;\ \ mL = 1$

$\frac{T-T_\infty}{T_b-T_\infty} = \frac{coshm(L-x)}{coshmL};\quad \frac{T-25}{100-25} = \frac{cosh20(0.05-0.02)}{coshmL};\quad T = 94.64^oC$

ii) Rate of Heat Transfer

$From\ Data\ Hand\ Book, Q = \sqrt{hPKA_c}(T_b - T_\infty)tanhmL$

$P = \pi d = \pi * 0.005 = 0.0157m;\quad A_c = \frac{\pi d^2}{4} = \frac{\pi 0.005^2}{4} = 1.963 * 10^{-5}m^2$

$Q_{with\ fin} = \sqrt{100 * 0.0157 * 200 * 1.963 * 10^{-5}}(100 - 20)tanh(1); Q = 4.4844W$

Rate of Heat transfer =4.4844 Watts

iii) Effectiveness of fin

$$\epsilon = \frac{Heat\ Transfer\ with\ fin}{Heat\ Transfer\ with\ out\ fin}$$

Heat Transfer without fin= $hA_c(T_b - T_\infty)$

$Q_{without\ fin} = 100 * 1.963 * 10^{-5} * (100 - 20);\ Q_{without\ fin} = 0.0147225Watts$

$\epsilon = \frac{Q_{with\ fin}}{Q_{without\ fin}};\ \epsilon = \frac{4.4844}{0.0147225}\ ;\ \epsilon = 30.45$

iv) Efficiency of fin

$$\eta = \frac{Heat\ Transfer\ with\ fin}{Heat\ Transfer\ with\ fin\ maintained\ at\ base\ temperature}$$

From Data Hand Book Page 49

$\eta = \frac{tanhmL}{mL}; \quad \eta = \frac{tanh(1)}{1}; \quad \eta = 0.7615$

2. A steel rod (K= 30 W/mK) 1 cm diameter and 5 cm long with **insulation end** is to be used as a spine. It is exposed to the surrounding temperature of 65 °C , and heat transfer coefficient of 50 W/m²K. The temperature of the base is 98°C. Determine i) Fin efficiency ii)Temperature at the end of spine iii) Heat dissipation from spine

3. A very long rod, 25 mm in diameter has one end maintained at 100 °C. The surface of the rod is exposed to ambient air at 25°C with Convection coefficient of 10 W/m²K .What are the heat losses from the rods, constructed of pure copper with K= 398 W/mK and stainless steel with K=14 W/mK ? Also, estimate how long the rods must be considered infinite

A very Long fin

Case I : Pure Copper

$m = \sqrt{\frac{hP}{kA_c}};\quad For\ circular\ rod\ \ m = \sqrt{\frac{h\pi d}{k\frac{\pi d^2}{4}}};\quad m = \sqrt{\frac{4h}{kd}};\ m = \sqrt{\frac{4*10}{398*0.025}} = 2.005$

$P = \pi d = \pi * 0.025 = 0.0785m;\quad A_c = \frac{\pi d^2}{4} = \frac{\pi 0.005^2}{4} = 4.9087 * 10^{-4}m^2$

For Long fin, $Q = \sqrt{hPKA_c}(T_b - T_\infty);$

$Q = \sqrt{10 * 0.0785 * 398 * 4.9087 * 10^{-4}}\ (100 - 25); Q = 29.37Watts$

how long the rods must be considered infinite

To calculate length to be considered to be infinite

Take T is 0.01°C more than surrounding temperature ie 25.01°C

$\frac{T-T_\infty}{T_b-T_\infty} = e^{-mx}$; $\frac{25.01-25}{100-25} = e^{-2.005x}$; $ln\frac{0.01}{75} = e^{-2.005x}$; $-8.92 = 2.005x$; $x = 4.44m$

$$Length\ of\ the\ rod\ must\ be\ more\ than\ 4.44m\ to\ consider\ rod\ as\ infinite$$

Case I : Stainless Steel K=14W/mK

$m = \sqrt{\frac{hP}{kA_c}}$; $For\ circular\ rod\ \ m = \sqrt{\frac{h\pi d}{k\frac{\pi d^2}{4}}}$; $m = \sqrt{\frac{4h}{kd}}$; $m = \sqrt{\frac{4*10}{14*0.025}} = 10.69$

$P = \pi d = \pi * 0.025 = 0.0785m$; $A_c = \frac{\pi d^2}{4} = \frac{\pi 0.005^2}{4} = 4.9087 * 10^{-4}m^2$

For Long fin, $Q = \sqrt{hPKA_c}(T_b - T_\infty)$;

$Q = \sqrt{10 * 0.0785 * 14 * 4.9087 * 10^{-4}}\,(100 - 25)$; $Q = 5.5Watts$

how long the rods must be considered infinite

To calculate length to be considered to be infinite

Take T is 0.01°C more than surrounding temperature ie 25.01°C

$\frac{T-T_\infty}{T_b-T_\infty} = e^{-mx}$; $\frac{25.01-25}{100-25} = e^{-2.005x}$; $ln\frac{0.01}{75} = e^{-2.005x}$; $-8.92 = 10.69x$; $x = 0.8346m$

$$Length\ of\ the\ rod\ must\ be\ more\ than\ 0.8346m\ to\ consider\ rod\ as\ infinite$$

4. A very long rod, 25 mm in diameter (K=380W/mK) rod extends from a surface at 120 °C. The temperature of surrounding air at 25°C and the heat transfer over the rod is 10 W/m^{2}K .calculate the heat losses from the rods,

5. The Aluminum square fins(0.6 mmx 0.6mm), 12 mm long are provided on the surface of a semiconductor electronics device to carry 2W of energy generated. The temperature at the surface of the device should not exceed 85 °C, when the surrounding is at 35°C .Given K= 200 W/mK h =15 W/m^{2}K. Determine the number of fins required to carry out the about duty. Neglect the heat loss from the end of the fin

6. Find the amount of heat transferred through an iron fin of thickness of 5mm, height 50 mm and width 100 cm. Also, determine the temperature difference $'\theta'$ at the tip of the fin assuming atmospheric temperature of 28 °C and base temperature of fin to be 108 °C. Take K_{fin}=50 W/mK

Short rectangular fin End is not insulated

h =10 W/m^{2}K $t = 5mm = 0.005m; L = 50mm = 0.05m: w = 100cms = 1m;$

$i)\ Q = ?\ ii)\ Temperature\ at\ the\ end\ of\ fin = ?\ T = ?\ at\ x = L = 0.05m$

$K = 50W/mK; h = 8W/m^2K; T_b = 108^oC;\ T_\infty = 28^oC$

$m = \sqrt{\frac{hP}{kA_c}};$

$P = 2(W + t);\quad P = 2(1 + 0.005)\ ;\ \ P = 2.01m$

$A_c = W * t = 1 * 0.005 = 0.005mm^2$

$$m = \sqrt{\frac{10 * 2.01}{50 * 0.005}} = 8.966;\quad mL = 8.966 * 0.05 = 0.4483$$

$$tanhmL = tanh0.4483 = 0.4205$$

Page 49 Short fin not Insulated

For Long fin, $Q = \sqrt{hPKA_c}(T_b - T_\infty)\frac{\tan hmL + \frac{h_L}{mK}}{1 + \frac{h_L}{mK} tanhmL}$

$$\frac{h_L}{mK} = \frac{10}{8.966 * 50} = 0.0223$$

$$Q = \sqrt{10 * 2.01 * 50 * 0.005}(108 - 28)\frac{0.4206 + 0.0023}{1 + (0.0023 * 0.4205)}$$

$$Q = 7868Watts$$

ii) Temperature at the end of the fin

$$\frac{T - T_\infty}{T_b - T_\infty} = \frac{coshm(L - x) + \frac{h_L}{mK} sinhm(L - x)}{coshmL + \frac{h_L}{mK} sinhmL}$$

$$\frac{T - T_\infty}{T_b - T_\infty} = \frac{coshm(L - L) + \frac{h_L}{mK} sinhm(L - L)}{coshmL + \frac{h_L}{mK} sinhmL}$$

$$\frac{T - T_\infty}{T_b - T_\infty} = \frac{1 + \frac{h_L}{mK}(0)}{coshmL + \frac{h_L}{mK} sinhmL}$$

$$\frac{T - T_\infty}{T_b - T_\infty} = \frac{1}{coshmL + \frac{h_L}{mK} sinhmL}$$

$$\frac{T - 28}{108 - 28} = \frac{1}{cosh(0.4483) + 0.0223 sinh(0.4483)}$$

$$T = 71.94^{o}C$$

Temperature at the end of the fin is $71.94^{o}C$

Temperature at the tip of fin$T_L - T_\infty = 71.94 - 28 = 43.94^{o}C$

7. Find the amount of heat transferred to an iron fin of thickness of 5 mm, height 50 mm and width 100 cm. Also determine the temperature difference θ at the tip of fin assuming atmospheric temperature of 28 °C and base temperature of fin to be 108 °C. Take K_{fin} =50 W/mK , h=10 W/m^2 K

8. Determine the amount of heat transferred through an iron fin of thickness 5 mm, height 50 mm and width 100 centimeters. Also determine the temperature at the fin end of the tips of fin. Assuming atmospheric temperature of 28 °C. Take K=508 W/m°C h=8 W/m^2 °C . Base fin temperature =108°C .

At x=L

10. A motor Body is 360 mm in diameter (OD) and 240 mm long. Its surface temperature should not exceed 55°C when dissipating 340 watts. Longitudinal fins of 15 mm thickness and 40 mm height are produced. The convection heat transfer coefficient is 40 W/m^2 °C . Determine the number of fins required. Assume, the atmospheric temperature is 30°C for a finite fin

Figure

Short fin end is not insulated

h =40 W/m^{2}K $t = 15mm = 0.015m; L = 40mm = 0.04m: w = 240mm = 0.24m;$

$Q_{Total} = 340\ Watts\ \ T_b = 55^oC;\ T_\infty = 30^oC$

Assume K=43 W/mK since it is not given This value has taken from Data Book from properties of material

$i)\ Number\ of\ fin = ?$

$First\ we\ have\ to\ find\ Rate\ of\ Heat\ transfer\ thrrough\ per\ fin\ by\ susing\ appropriate\ formula$

$$Q_{Total} = nQ_{fin}$$

$m = \sqrt{\frac{hP}{kA_c}};$

$P = 2(W + t);\quad P = 2(0.24 + 0.015)\ ;\ \ P = 0.51m$

$A_c = W * t = 0.24 * 0.015 = 3.6 * 10^{-3}mm^2$

$$m = \sqrt{\frac{40 * 0.51}{43 * 0.015}} = 11.47;\quad mL = 11.47 * 0.04 = 0.459$$

$$tanhmL = tanh0.459 = 0.429$$

Page 49 Short fin not Insulated

For Long fin, $Q = \sqrt{hPKA_c}(T_b - T_\infty)\frac{\tan hmL + \frac{h_L}{mK}}{1 + \frac{h_L}{mK}tanhmL}$

$$\frac{h_L}{mK} = \frac{40}{11.47 * 43} = 0.081$$

$$Q_{fin} = \sqrt{40 * 0.51 * 43 * 3.6 * 10^{-3}}(55 - 30)\frac{0.429 + 0.081}{1 + (0.081 * 0.429)}$$

$$Q_{fin} = 38.91Watts$$

$Q_{Total} = nQ_{fin};\ 340 = n * 38.91;\ \ n = 8.7$

Hence number of fin required is 9

11. A set of aluminium fins (K= 180 W/m^o C) that are to be fitted to a small air compressor. The device dissipates 1 kW by convecting to the surrounding air which is at 20^o C. Each fin is 100 mm long 30 mm high and 5 mm thick. The tip of each fin may be assumed to be adiabatic and a heat transfer coefficient of 15 W/m$^{2\,o}$ C acts over the remaining surfaces. Estimate the number of fins required to ensure the base temperature does not exceed 120 ^{o}C

Hint: End of the fin is insulated

$Q_{total} = 1kW = 1000Watts$

$Q_{Total} = nQ_{fin}$

$For\ Heat\ Transfer\ through\ fin\ with\ end\ of\ the\ fin\ is\ insulated$

$Q_{fin} = \sqrt{hPKA_c}(T_b - T_\infty)tanhmL$

Number of fin= $\frac{Q_{Total}}{Q_{fin}}$

12. In order to reduce the thermal resistance at the surface of a vertical plane wall (50x50cm) , 100 fins (1cm diameter, 10cm long) are attached. If the pin fins are made of copper having a thermal conductivity of 300W/mK and the value of the surface heat transfer coefficient is 15 W/m^{2}K, calculate the decrease in the thermal resistance. Also calculate the consequent increase in heat transfer rate from the wall if it is maintained at a temperature of 200^{o}C and the surroundings are at 30^{o}C

D=1cm=0.01m; $= 10cm = 0.1m; no\ of\ fins\ n = 100; K = 300W/mK; h = 15W/m^2K;$

$T_b = 200^oC;\ T_\infty = 30^oC$

i) $From\ Data\ Hand\ Book, Page\ 48\ ,\quad \frac{T-T_\infty}{T_b-T_\infty} = \frac{coshm(L-x)}{coshmL}$

$m = \sqrt{\frac{hP}{kA_c}};\qquad For\ circular\ rod\ \ m = \sqrt{\frac{h\pi d}{k\frac{\pi d^2}{4}}};\qquad m = \sqrt{\frac{4h}{kd}}$

$m = \sqrt{\frac{4*15}{300*0.01}}\ ;\quad m = 4.47$

$mL = 4.47 * 0.1;\ \ mL = 0.447$

ii) Rate of Heat Transfer

$From\ Data\ Hand\ Book, Q = \sqrt{hPKA_c}(T_b - T_\infty)tanhmL$

$P = \pi d = \pi * 0.005 = 0.0157m; \quad A_c = \frac{\pi d^2}{4} = \frac{\pi 0.005^2}{4} = 1.963 * 10^{-5} m^2$

$Q_{with\ fin}\, for\ one\ fin = \sqrt{15 * 0.0157 * 300 * 1.963 * 10^{-5}}(200 - 30) tanh(0.447); Q = 7.48 Watts$

Rate of Heat transfer/fin =7.48$Watts$

Heat transfer for 100fins from vertical plane =748 Watts

Heat transfer through unfinned area from the vertical plane (5ocmx50cm) =$hA_{unfinned\ area}\,(T_b - T_\infty)$

$A_{unfinned\ area} = A - A_f; A_{unfinned\ area} = (L * W) - n\left(\frac{\pi d^2}{4}\right);$

$A_{unfinned\ area} = (0.5 * 0.5) - 100\left(\frac{\pi * 0.01^2}{4}\right)$

$A_{unfinned\ area} = 0.2421 m^2$

$Q_{unfinned} = 8.5 * 0.2421 * (200 - 30);\ Q_{unfinned} = 349.83 Watts$

Total Heat transfer from vertical plane = Heat transfer from vertical plane through 100fins+ Heat transfer from unfinned area of vertical palne

$Q = 748 + 349.83 = 2069.83 Watts$

Thermal resistance =$\frac{\Delta T}{Q}$

Thermal resistance $with\ fin\ attached\ R_1 = \frac{200-30}{2069.83} = 0.0821$

Heat Transfer from the surface without fins attached $= hA(T_b - T_\infty)$

Heat Transfer from the surface without fins attached $= 8.5 * (0.5 * 0.5) * (200 - 30)$

Heat Transfer from the surface without fins attached=361.25Watts

Thermal resistance $without\ fin\ attached\ R_2 = \frac{200-30}{361.25} = 0.469$

Decrease in thermal resistance due to 100 fins attached =$0.469 - 0.821 =$

13. A thin rod of copper K=100 W/mK , 12.5 mm in diameter spans between two parallel plates 150 mm apart .Air Flows over the rod providing a heat transfer coefficient of 50 W/m^{2}K. The surface temperature of the plate exceeds air by 40 ^{o}C .Determine the excess temperature at the centre of the rod over that of air and ii)Heat lost from the rod in in watts.

Unsteady Heat Transfer

1. What is lumped system analysis? Derive an expression for temperature distribution and rate of heat transfer in case of lumped system analysis (3a, 08, June/July18,3a, 10, June/July18)
2. Derive an expression for temperature distribution in lumped system (3b,08/June/July17)
3. Obtain an expression for instantaneous heat transfer and total heat transfer for lumped heat analysis treatment heat conduction problems (3a, 08M June/July13)
4. Derive the expressions of temperature variation, instantaneous heat transfer and total heat transferred for one dimensional transient heat conduction
5. Derive the expressions of temperature variation, heat flow using Lumped Parameter Analysis (4a, 6,June/July 18)
6. Obtain an expression for instantaneous heat transfer and total heat transfer for lumped heat analysis treatment of heat conduction problem (3a, 10,June/July 16)
7. Obtain an expression for instantaneous heat transfer and total heat transfer using lumped heat analysis for unsteady state heat transfer from a body to the surroundings (3a, 10,Dec16/Jan17
8. Derive an expression for instantaneous heat transfer and total heat transfer in terms of product of Biot number and Forier number is one dimensional transient heat conduction. (3b,08, Dec14/Jan15)
9. Show that the temperature distribution under lumped analysis is given by initial temperature ambient temperature $\frac{T-T_a}{T_i-T_a}$ = $e^{-B_iF_n}$ where $T_i = Initial\ temperature$ and $T_a = Atmospheric\ temperature$ (3a,10m,Dec15/Jan16, 4a,08, Dec18/Jan19)

10. Show that the temperature distribution in a body during Newtonian heating or cooling is given by $\frac{T-T_a}{T_i-T_a}$=$\frac{\theta}{\theta_i} = exp\left\{\frac{-hA_st}{\rho CV}\right\}$ (3a,6,Dec13/Jan14)

11. Explain physical significance of Biot and fourier numbers 06 M June/July13, Fourier number (3a, 04, June/July17)

12. Define Biot number and Fourier number s (3a,02,Dec14/Jan15,3a,03,Dec17/Jan18, 4b, June/July18)

13. Write a short note on Biot number and Fourier number (3b, 04, June/July16)

14. What are Biot and Fourier numbers? Explain the physical significance of Biot number and fourier number (3b, 06, June/July13) (3a,04,June/July15)(3a,06,June/July14)(3b, 04,Dec18/Jan19)

Transient Heat Conduction

Unsteady conduction heat transfer is the conduction heat transfer in which heat transfer varies with respect to the time. Here temperature varies with the time

$Q = f(x, \tau)$ is the mathematical representation of One Dimensional unsteady Heat transfer where τ is the time

$T = f(x, \tau)$

Transient heat conduction problems can be divided into periodic and non periodic heat flow problems

Periodic heat flow problems are those in which the temperature varies on a regular basis,ex: the variation of temperature of the surface of the earth during a 24 hrs

Tin the Non periodic type the temperature at any point varies non linearly with time.

Lumped Analysis: (Applicable only if $\boldsymbol{B_i} < 0.1$)

Heat transfer in heating or cooling of a body is dependent upon both the internal resistance $\frac{L}{kA}$ and surface resistances $\frac{1}{hA}$

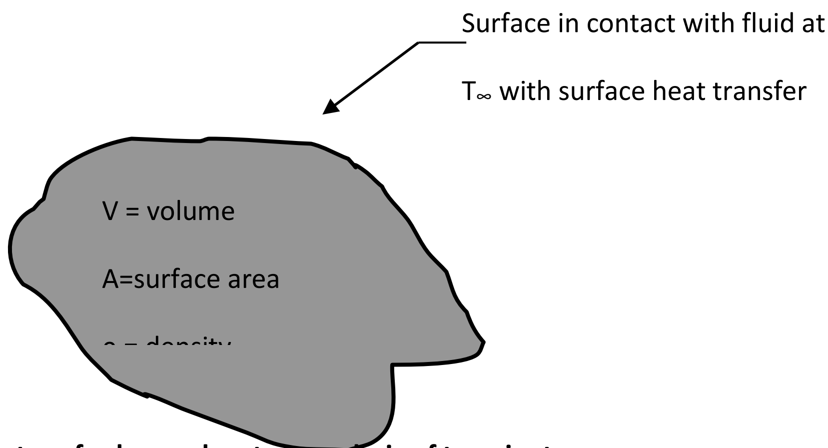
When the body material is having large thermal conductivity , surface resistances $\frac{1}{hA}$ is larger compared Internal resistance $\frac{L}{kA}$. Hence Internal resistance $\frac{L}{kA}$ is negligible. In this case there is no variation of temperature inside the solid. Hence Temperature is function of time only

Ie $T = f(\tau)$

The process in which the internal resistance $\frac{L}{kA}$ is ignored being negligible in comparison with its surface resistance is called **Newtonian heating or cooling process**. Such analysis is also called **Lumped heat capacity analysis**

Derivation for Lumped heat analysis:

Consider a solid Area A m^2 , with initial temperature T_o throughout is suddenly placed in a an environment T_∞ as shown in figure



Nomenclature for lumped system analysis of transient

Conduction heat transfer

According to energy balance

Amount of Heat convected to the body from environment = Increase in internal energy of the solid

$hA(T_\infty - T) = mC\frac{dT}{d\tau}$ where m , C ,T , τ are the mass , Specific heat, Temperature and time respectively

$$hA(T_\infty - T) = \rho VC\frac{dT}{d\tau}$$

$$-\frac{hA}{\rho VC}(T - T_\infty) = \frac{dT}{d\tau}$$

$$\frac{dT}{d\tau} + \frac{hA}{\rho VC}(T - T_\infty) = 0$$

$\frac{hA}{\rho VC} = \ m$ where m is the constant and let $\theta = T - T_\infty$ then $\frac{d\theta}{d\tau} = \frac{dT}{d\tau} - 0$ ie $\frac{dT}{d\tau} = \frac{d\theta}{d\tau}$

Hence eqn 1 can be written as

$$\frac{d\theta}{d\tau} + m\theta = 0$$

Above is the first order homogeneous differential equation . Hence solution for above equation is

$\theta = Ce^{-m\tau} - - - - - - - - 1$

Using initial condition ie At $= 0$, $\theta = \theta_o = T_o - T_\infty$

Substituting above initial condition in 1

$\theta_o = Ce^o$

Ie $C = \theta_o$

Substituting C in 1

$\theta = \theta_o e^{-m\tau}$

$$T - T_\infty = (T_o - T_\infty)e^{-\frac{hA}{\rho VC}\tau}$$

$$\frac{T - T_\infty}{T_o - T_\infty} = e^{-\frac{hA}{\rho VC}\tau}$$

$$\frac{hA\tau}{\rho VC} = \frac{h\tau}{\rho LC} = \frac{h\frac{L}{K}\tau}{\rho LC\frac{L}{K}} = \frac{h\frac{L}{K}\tau}{L^2\frac{\rho C}{K}} = \frac{B_i\tau}{L^2\frac{1}{\alpha}} = \frac{B_i\alpha\tau}{L^2} = B_iF_n$$

$$\frac{T - T_\infty}{T_o - T_\infty} = e^{-B_iF_n}$$

Above Equation is Temperature distribution equation in terms of Fourier Number and Biot Number

<u>Instantaneous Heat Transfer Rate in Lumped Analysis</u>

$$Q_i = mC\frac{dT}{d\tau}$$

$$Q_i = \rho VC\frac{dT}{d\tau}$$

$$\frac{T - T_\infty}{T_o - T_\infty} = e^{-\frac{hA}{\rho VC}\tau}$$

$$T - T_\infty = (T_o - T_\infty)e^{-\frac{hA}{\rho VC}\tau}$$

Differentiating above equation

$$\frac{dT}{d\tau} - 0 = (T_o - T_\infty)\left(-\frac{hA}{\rho VC}\right)e^{-\frac{hA}{\rho VC}\tau}$$

$$\frac{dT}{d\tau} = (T_o - T_\infty)\left(-\frac{hA}{\rho VC}\right)e^{-\frac{hA}{\rho VC}\tau}$$

$$Q_i = \rho VC(T_o - T_\infty)\left(-\frac{hA}{\rho VC}\right)e^{-\frac{hA}{\rho VC}\tau}$$

$$Q_i = -(T_o - T_\infty)hAe^{-\frac{hA}{\rho VC}\tau}$$

Total Heat transfer

$$Q_t = \int_0^\tau Q_i \, d\,\tau$$

$$Q_t = -(T_o - T_\infty)hA\int_0^\tau e^{-\frac{hA}{\rho VC}\tau}\, d\,\tau$$

$$Q_t = -(T_o - T_\infty)hA\left(\frac{e^{-\frac{hA}{\rho VC}\tau}}{-\frac{hA}{\rho VC}}\right)_0^\tau$$

$$Q_t = \rho VC(T_o - T_\infty)\left(e^{-\frac{hA}{\rho VC}\tau} - 1\right)$$

Mixed Boundary Condition in Lumped Analysis

Consider one part of the body is subjected to Convection while remainder part is subjected to heat flux as shown in fig

Consider a slab of thickness initially at T_0 . For $\tau > 0$ heat is supplied $q\ W/m^2$ at the boundary at x=0 while heat is dissipated by convection from other boundary at x=L with heat transfer coefficient h and Exposed to environment T_∞. Let T be the temperature of the body for $\tau > 0$

According to Heat balance

Heat supplied at heat flux side + Heat supplied by convection at the other boundary surface= Change in Internal Energy of the body

Consider Area equal on both sides (Heat flux side and Convection Heat Transfer side)

$$Aq + hA(T_\infty - T) = mC\frac{dT}{d\tau}$$

$Aq + hA(T_\infty - T) = \rho VC\frac{dT}{d\tau}$ for $\tau > 0$ with initial Condition $T(\tau) = T_o$ for $\tau = 0$

$$\frac{Aq}{\rho VC} - \frac{hA}{\rho VC}(T - T_\infty) = \frac{dT}{d\tau}$$

$\frac{Aq}{\rho VC} = \frac{dT}{d\tau} + \frac{hA}{\rho VC}(T - T_\infty)$------1

Let $\theta = (T - T_\infty)$ hence $\frac{d\theta}{d\tau} = \frac{dT}{d\tau} - 0;\quad \frac{d\theta}{d\tau} = \frac{dT}{d\tau}$

$\frac{Aq}{\rho VC} = Q$ and $\frac{hA}{\rho VC} = m$

Hence equation can be written as

$Q = \frac{d\theta}{d\tau} + m\theta$ for $\tau > 0$ with initial condition $\theta = \theta_0 = T_o - T_\infty$ at $\tau = 0$

Above equation is first order homogeneous differential equation and solution for above equation is

$\theta = Ce^{-m\tau} + \frac{Q}{m}$ ---------------2

initial condition $\theta = \theta_0 = T_o - T_\infty$ at $\tau = 0$

substituting initial condition in equation 2

$$\theta_0 = Ce^{-0} + \frac{Q}{m}$$

$$\theta_0 = C + \frac{Q}{m}$$

$$C = \theta_0 - \frac{Q}{m}$$

Substituting C in equation2

$$\theta = \left(\theta_0 - \frac{Q}{m}\right)e^{-m\tau} + \frac{Q}{m}$$

$$\theta = \theta_0 e^{-m\tau} + \frac{Q}{m}(1 - e^{-m\tau})$$

Where $\theta = T - T_\infty$; $\theta_0 = T_o - T_\infty$; $\frac{Aq}{\rho VC} = Q$ and $\frac{hA}{\rho VC} = m$

$\frac{Q}{m} = \frac{\frac{Aq}{\rho VC}}{\frac{hA}{\rho VC}}$; $\frac{Q}{m} = \frac{q}{h}$

$$T - T_\infty = (T_o - T_\infty)e^{-m\tau} + \frac{q}{h}(1 - e^{-m\tau})$$

Above is the temperature distribution equation

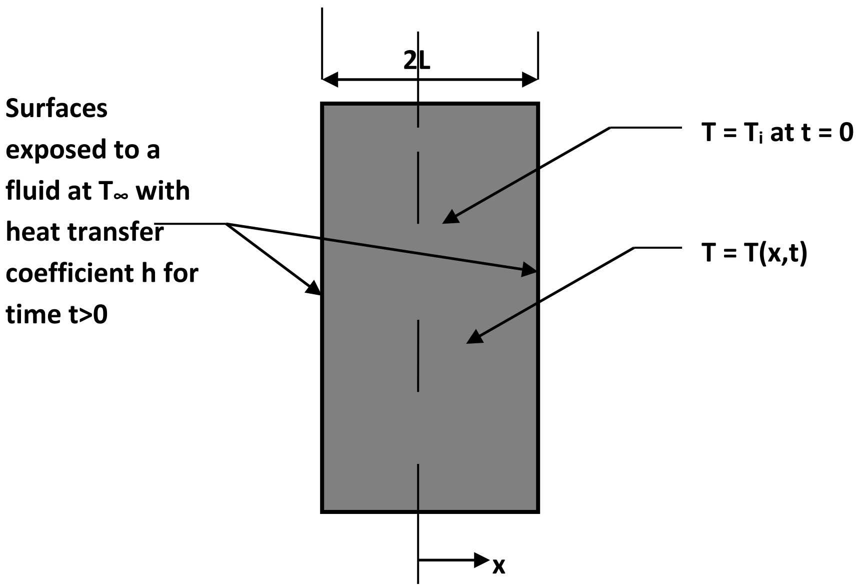
Above all analysis is Applicable only if $B_i < 0.1$

One-dimensional Transient Conduction (Use of Heissler's Charts):

There are many situations where we cannot neglect internal temperature gradients in a solid while analyzing transient conduction problems. Then we have to determine the temperature distribution within the solid as a function of position and time and the analysis becomes more complex. However the problem of one-dimensional transient conduction in solids without heat generation can be solved readily using the method of separation of variables. The analysis is illustrated for solids subjected to convective boundary conditions and the solutions were presented in the form of transient – temperature charts

by **Heissler.** These charts are now familiarly known as "Heissler's charts".

consider a slab of thickness 2L, which is initially at a uniform temperature T_i. Suudenly let the solid be exposed to an environment which is maintained at a uniform temperature of T_∞ with a surface heat transfer coefficient of h for time t > 0.Fig.4.3 shows the geometry , the coordinates and the boundary conditions for the problem. Because of symmetry in the problem with respect to the centre of the slab the 'x' coordinate is measured from the centre line of the slab as shown in the figure.



Geometry, coordinates and boundary conditions for transient

The mathematical formulation of this transient conduction problem is given as follows:

Governing differential equation: $\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}$ -----------------1

Initial condition : at t = 0, $T = T_i$ in $0 < x < L$

Boundary conditions are :

(i) at x = 0, $\frac{\partial T}{\partial x} = 0$ (axis of symmetry) for all t > 0)

(ii) at x = L, $-k\left(\frac{\partial T}{\partial x}\right)_{x=L} = h(T|_{x=L} - T_\infty)$ for all t > 0

It is more convenient to analyze the problem by using the Dimentionless variables

As given

$\theta(x,t), = \frac{T(x,\tau)-T_\infty}{T_i-T_\infty}$ Dimensionless temperature

$X = \frac{x}{L}$ Dimensionless length

$B_i = \frac{hL}{K}$ Biot Number

$F_n = \frac{\alpha\tau}{L^2}$ Dimensionless time or Fourier Number

The heat conduction equation becomes

$\frac{\partial^2\theta}{\partial X^2} = \frac{\partial\theta}{\partial F_n} \quad for\ 0 < X < 1\ for\ F_n > 0$

$\frac{\partial\theta}{\partial X} = 0$ at $X = 0$ and $for\ F_n > 0$

$\frac{\partial\theta}{\partial X} + B_i\,\theta = 0$ at $X = 0$ and $for\ F_n > 0$

$\theta = 1\ in\ 0 \le X \le 1\ \ for\ F_n = 0$

In above dimensionless equations the temperature depends only X, B_i, F_n

And dimensionless equations are solved by using separating variables and presented in the form of Charts which is called as Heissler's Charts

Transient-Temperature charts for Long cylinder and sphere: The dimensionless transient-temperature distribution and the heat transfer results for infinite cylinder and sphere can also be represented in the form of charts as in the case of slab. For infinite cylinder and sphere the radius of the outer surface R is used as the characteristic length so that the Biot number is defined as Bi = hR / k and the dimensionless distance from the centre is r/R where r is any radius (0 ≤ r ≤ R).These charts are illustrated in Data Hand Book

Transient conduction in semi-infinite solids:-

A semi-infinite solid is an idealized body that has a *single plane surface* and extends to infinity in all directions. The earth for example, can be considered as a semi-infinite solid in determining the variation of its temperature near its surface

Theare three cases as follows

Case 1:- The solid is initially at a uniform temperature T_i and suddenly at time $\tau > 0$, the boundary-surface temperature of the solid is changed to and maintained at a uniform temperature T_0 which may be greater or less than the initial temperature T_i.

Case2:- The solid is initially at a uniform temperature T_i and suddenly at time $\tau > 0$ the boundary surface of the solid is subjected to a uniform heat flux of q_0 W/m^2.

Case 3:- The solid is initially at a uniform temperature T_i. Suddenly at time t>0 the boundary surface is exposed to an ambience at a uniform temperature T_∞ with the surface heat transfer coefficient h. T_∞ may be higher or lower than T_i.

Solution to Case 1:- The schematic for problem 1 is shown in Fig. 4.10. The mathematical formulation of the problem to determine the unsteady temperature distribution in an infinite solid T(x,t) is as follows:

The governing differential equation is

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

The initial condition is at time t = 0, $T(x,0) = T_i$

and the boundary condition is at x = 0, $T(0,t) = T_0$

It is convenient to solve the above problem in terms of the dimensionless temperature

$\theta(x,t), = \frac{T(x,\tau)-T_\infty}{T_i-T_\infty}$

$\frac{\partial^2\theta}{\partial x^2} = \frac{1}{\alpha}\frac{\partial\theta}{\partial\tau} \quad for\ 0 < x < \infty\ for\ \tau > 0$

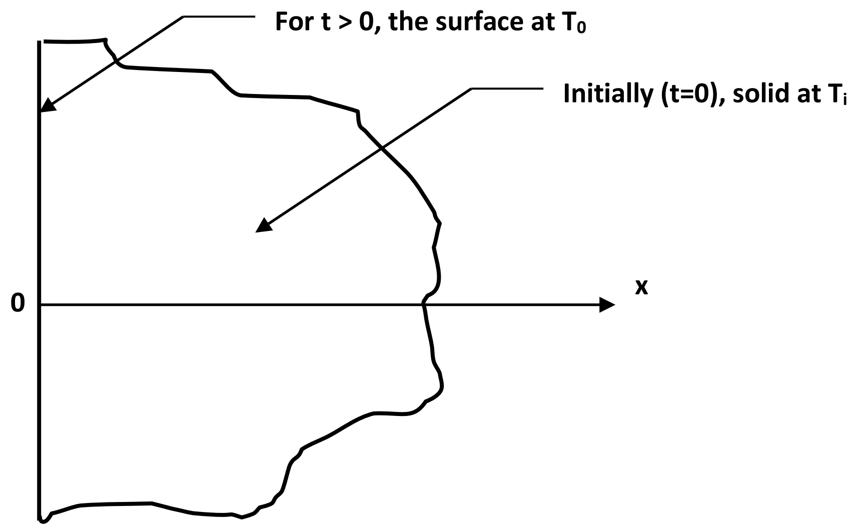


Fig. 4.10: Semi-infinite solid with specified surface temperature T_0 for t > 0

The initial condition will be at time t = 0, $\theta(x,0) = T_i - T_\infty$

And the boundary condition will be at x = 0, $\theta(0,t) = T_0 - T_\infty$

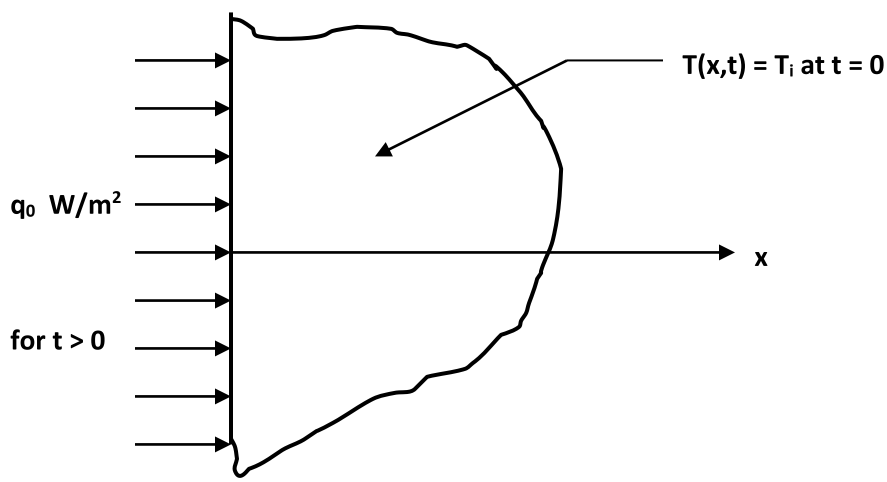
This problem has been solved analytically and the solution θ(x,t) is represented graphically as θ(x,t) as a function of the dimensionless variable $\frac{x}{2\sqrt{\alpha\tau}}$ as shown in Data Hand Book

In engineering applications, the heat flux at the boundary surface x = 0 is also of interest. The analytical expression for heat flux at the surface is given by

$$q_s(\tau) = \frac{K(T_o-T_i)}{\sqrt{\pi\alpha\tau}}$$

Dimensionless temperature θ(x,t), $= \frac{T(x,\tau)-T_\infty}{T_i-T_\infty}$ is plotted against $\xi = \frac{x}{2\sqrt{\alpha\tau}}$ and given in Data Hand Book

Solution to case2:- The schematic for this problem is shown in Fig.



An infinite solid subjected to a constant heat flux at x = 0 for t > 0

Governing differential equation in terms of T(x,t) and the initial condition are same that for case 1

The boundary condition at x=o is $-k\left(\frac{\partial T}{\partial x}\right)_{x=L} = q_o$

The temperature distribution within the solid T(x,t) is given by

T(x, t) = $T_i + \frac{2q_o}{K}\sqrt{\alpha\tau}\left(\frac{1}{\sqrt{\pi}}exp(-\xi^2) + \xi erf(\xi) - \xi\right)$

where ξ = $\frac{x}{2\sqrt{\alpha\tau}}$ and erf (ξ)= $\frac{2}{\sqrt{\pi}}\int_0^{\xi} e^{-y^2}\,dy$

Here erf (ξ) is called the *"error function"* of argument ξ and its values for different values of ξ are tabulated.

Solution to Problem 3 :- The solid is initially at a uniform temperature T_i and suddenly for t >0 the surface at x = 0 is brought in contact with a fluid at a uniform temperature T_∞ with a surface heat transfer coefficient h. For this problem the solution is represented in the form of a plot where the dimensionless temperature [1 − θ(x,t)] is plotted against dimensionless distance $\frac{x}{\sqrt{\alpha\tau}}$, using $\frac{h\sqrt{\alpha\tau}}{k}$ as the parameter. It can be noted that the case h → ∞ is equivalent to the boundary surface ay x = 0 maintained at a constant temperature T_∞.

Dimensionless Numbers in Transient Heat Transfer

Biot Number : I tis defined the ratio of heat transfer coefficient at the surface of solid to specific conductance of the solid

$$B_i = \frac{h}{\frac{K}{L}}$$

$B_i = \frac{hL}{K}$ where h is the heat transfer coefficient at the surface of the body , K thermal conductivity of the solid body , L is the Characterstic length.

Biot Number signifies whether internal resistance $\frac{L}{KA}$ is negligible in transient heat conduction or not negligible

If Biot Number less than 0.1 , transient heat conduction can be analyzed as Lumped Analysis or else surface resistance

Fourier Number: is Dimensionless time : It is defined as the rate of heat conduction in Volumeacross chercterstic length to rate of heat storage in Volume

$$F_n = \frac{\text{rate of heat conduction in Volume across chercterstic length}}{\text{rate of heat storage in Volume}}$$

$$F_n = \frac{\frac{KA}{L}}{\frac{\rho VC}{\tau}}$$

$$F_n = \frac{\frac{KL^2}{L}}{\frac{\rho L^3 C}{\tau}} = \frac{K\tau}{\rho C L^2} = \frac{\alpha\tau}{L^2}$$

$F_n = \frac{\alpha\tau}{L^2}$ where $\alpha\ is\ thermal\ diffusivity$, L is the Charecterstic length , τ is the time period

Fourier number is the measure of heat transfer due to conduction in comparison with the rate of heat storage in volume. Larger the Fourier number deeper the penetration into the solid over a given period

Charecterstic Length

For Slab

$L = \frac{Volume}{Heat\,Transfer\,Area}$ where V is the Volume of the body A is the heat transfer area

For slab if Both area is exposed to environment Heat Transfer Area 2A where is the Area of one surface

If Thickness is 2L , then Volume = Surface Area x thickness ie V=2L*A

$$L_C = \frac{2AL}{2A}$$

$$L_C = L = \frac{Thickness}{2}$$

For slab if one surface area is exposed to environment Heat Transfer Area A where A is the Area of one surface

If Thickness is 2L , then Volume = Surface Area x thickness ie V=2L*A

$$L_C = \frac{AL}{2A}$$

$$L_C = \frac{L}{2} = Thickness$$

For Long Cylinder

$$L_C = \frac{Volume}{Heat\,Transfer\,Area}$$

$$L_C = \frac{\pi R^2 L}{2\pi R L}$$

$$L_C = \frac{R}{2}$$

Note In lumped Analysis $L_C = \frac{R}{2}$

But for Heissler Charts $L_C = R$

Small Cylinder where Length is Known

$$L_C = \frac{Volume}{Heat\ Transfer\ Area}$$

$$L_C = \frac{\pi R^2 L}{2\pi RL + 2\pi R^2}$$

For Sphere

$$L_C = \frac{Volume}{Heat\ Transfer\ Area}$$

$$L_C = \frac{\frac{4}{3}\pi R^3}{4\pi R^2}$$

$$L_C = \frac{R}{3}$$

For Lumped Analysis $L_C = \frac{R}{3}$

Note that for Heissler Chart $L_C = R$

For Cube

$$L_C = \frac{Volume}{Heat\ Transfer\ Area}$$

$$L_C = \frac{a^3}{6a}$$

$$L_C = \frac{a^3}{6a^2}$$

$L_C = \frac{a}{6}$ where a is the side of Cube

1. A steel ball of 5 cm diameter at 450°C is suddenly placed in a controlled environment of 100 °C, Considering the following data, find the time required for the ball to attain a temperature of 150 °C Take C_p=450 J/Kg K, K= 35 W/mK h =10 W/m^{2}K ρ =8000 kg/m^3 (3b,06,June/July15)

2. A 15 mm diameter mild steel Sphere (K =42 W/m° C) is exposed to cooling air flow at 20 ° C resulting in the convective heat transfer Coefficient at h=120 W/m^2 ° C. Determine the following i) Time required to cool the sphere from 550°C to 90°C .ii) Instantaneous heat transfer rate 2 minutes after start of cooling iii) Total energy stored from the sphere during the first 2 minutes For mild steel take : ρ =7850 kg /m^3 C_p=475J/ kg K, α=0.045 m^2/hr.(3b, 10, Dec15/Jan16)

Add question iv) What is the Rate of cooling after 2 min

$B_i = \frac{hL_c}{K}; L_c = \frac{V}{A}$ where V is the Volume , A = Area of Heat Transfer

$For\ Sphere\ , V = \frac{4}{3}\pi R^3; A = Surface\ Area = 4\,\pi R^2;$ Hence $L_c = \frac{\frac{4}{3}\pi R^3}{4\,\pi R^2} = \frac{R}{3}$

$R = \frac{D}{2} = \frac{15}{2} = 7.5mm = 0.0075m; L_c = \frac{0.0075}{3}$=0.0025

$B_i = \frac{120}{42} * \frac{0.075}{3}; B_i = 0.00714$

$B_i < 0.1$; Hence Lumped Analysis equation is to be used

$\frac{T-T_\infty}{T_0-T_\infty} = exp\left\{\frac{-hA_s\tau}{\rho CV}\right\}$; where T_0 $is\ the\ Initial\ temperature$, τ is the time in sec

$\frac{T-T_\infty}{T_0-T_\infty} = exp\left\{\frac{-h\tau}{\rho L_c V}\right\}$ since $\frac{A}{V} = \frac{1}{L_c}$

$$T_0 = 550^oC; T = 90^oC;\ T_\infty = 20^oC;\ h = \frac{120W}{m^2C};\ \tau = ?$$

$\frac{90-20}{550-20} = e^{-\left(\frac{120*\tau}{7850*475*0.025}\right)}$; $ln\frac{70}{530} = -\left(\frac{120*\tau}{7850*475*0.0025}\right)$; $-2.0243 = -1.287 * 10^{-2}\tau$

$\tau = 157.288\ sec;$

ii) $Instantaneous\ \ heat\ transfer\ Q_i = ?\ after\ 2min\ \tau = 2min$

Temperature after 2 min T=?

$\frac{T-T_\infty}{T_0-T_\infty} = exp\left\{\frac{-h\tau}{\rho L_c V}\right\};$ $\frac{T-20}{550-20} = e^{-\left(\frac{120*120}{7850*475*0.025}\right)};$ $T = 133.08^oC$

$$Instantaneous\ \ heat\ transfer\ Q_i = hA_s(T - T_\infty)$$

$$Instantaneous\ \ heat\ transfer\ after2min\quad Q_i = 120 * (4 * \pi * 0.0075^2)(133.08 - 20)$$

$$Instantaneous\ \ heat\ transfer\ after2min\quad Q_i = 9.591Watts$$

iii) Total Heat transfer after 2min

$Q_T = \rho CV(T - T_0)$ Page 57 equation in Transient Heat conduction equation

$Q_T = 7850 * 475 * \left(\frac{4}{3}\pi * 0.0025^3\right)(133.08 - 550); Q_T = -107.747\ watts$

Negative sign indicates heat is transferred from body to environment

Rate of cooling after 2 minute

$$\frac{T - T_\infty}{T_0 - T_\infty} = exp\left\{\frac{-hA_s\tau}{\rho CV}\right\}$$

$$T - T_\infty = (T_0 - T_\infty)\ exp\left\{\frac{-hA_s\tau}{\rho CV}\right\}$$

Differentiating above equation with respect to time $\frac{dT}{d\tau} - 0 = (T_0 - T_\infty)\frac{-hA_s}{\rho CV}\ exp\left\{\frac{-hA_s\tau}{\rho CV}\right\}$

$$\frac{dT}{d\tau} = (550 - 20)\frac{-\left(120 * (4 * \pi * 0.0075^2)\right)}{7850 * 475 * \left(\frac{4}{3}\pi * 0.0025^3\right)}\ exp\left\{\frac{-(120 * (4 * \pi * 0.0075^2) * 120)}{7850 * 475 * \left(\frac{4}{3}\pi * 0.0025^3\right)}\right\}$$

$$\frac{dT}{d\tau} = 530 * (-0.01287)\ exp(-0.01287 * 120)$$

$\frac{dT}{d\tau} = -1.455^0 C/sec;\ \frac{dT}{d\tau} = -1.455 * 60\ C/min\ \frac{dT}{d\tau} = -87.32^o C/min$

3. A steel ball bearings (K=50W/mK, α=1.3 x$10^{-5}$$m^2$/sec.), 40mm in diameter are heated to a temperature of 650°C. It is then quenched in a oil bath at 50°C, where the heat transfer coefficient is estimated to be 300W/m^2K. Calculate

   i) Time required for bearing to reach 200°C
   ii) The total amount of heat removed from a bearing during this time and
   iii) The instantaneous heat transfer rate from the bearings , when they are first immersed in oil bath and whey reach 200°C (3b, 14, Dec13/Jan14)

   Hint

   i) Time required to reach 200^0C
   Ball is sphere
   First find $B_i = \frac{hL_c}{K}\ L_c = \frac{R}{3}$ as explained earlier problem Bi=$4.166x10^{-3}$

4. A steel ball 5 cm diameter and initially at 900^o C is placed in still air at 30 ^{o} C. Find i)temperature of the ball after 30 seconds ii) the rate of cooling in ^{o} C /min after 30secs
Assume h=20 W/m^2 ^{o}C , K (Steel)=40 W/m^{o}C, $\rho(Steel)$ =7800 kg/m^3 C_p(Steel)=460 J/kg (3c, 08 June/July17)

5. A hot mild steel Sphere(K=43 W/mK) having 10 mm diameter is planned to be cooled by an air flow at 25 ^{o}C. The Convection heat transfer coefficient is 115 W/m^{2}K. Calculate the following i) time required to cool the sphere from 600 ^{o}C to 100 ^{o}C.ii) Instantaneous heat transfer rate 1.5 minute after the start of Cooling iii) Total energy transferred from the sphere during the first 1.5 min (3c, 8 June/July 16)
6. A 15 mm diameter mild steel Sphere(K =42 W/mK) is exposed to cooling air flow at 20^{o}C resulting in the heat transfer Coefficient h= 120 W/m^{2}K. Determine the following i) time required to cool the sphere from 550^{o}C to 290 ^{o}C ii)Instantaneous heat transfer rate for 2 minutes after starts of cooling iii)Total energy transferred from the sphere during the first 2 minutes. Take for mild steel ρ =7850 kg/m^3 ,C= 475J/ kgK α =0.045 m^2/hr (4b, 08Dec18/Jan19 15me,3b,10 Dec15/16)

7. An aluminium sphere weighing 6 kg and initially at temperature of 350 ^{o}C is immersed in a fluid at 30 ^{o}C with convection coefficient of 60 W/m^{2}K. Estimate the time required to cool the sphere to 100^{o}C take the thermo physical properties as C= 900 J/ KgK ρ =2700 kg/m^3 K=205 W/mK (3c,08 Dec18/Jan19)

   Hint : $\boldsymbol{m = Density * Volume = \rho * \frac{4}{3}\pi R^3}$

   $\boldsymbol{6 = 2700 * \frac{4}{3}\pi R^3 \; ; R = \sqrt[3]{\frac{6*3}{2700*4*\pi}} \; ; \; R = 0.081m}$
8. An Aluminum sphere weighing 6 kg and initially at a temperature of 420 ^{o}C is suddenly immersed in a fluid at 18^{o}C. The convective heat transfer coefficient is 45 W/m^{2}K. Estimate the time required to cool the sphere to 120 ^{o}C. Also find the total heat flow from the sphere to the surrounding when it cools from 300 ^{o}C to 120^{o}C (For aluminum ρ = 2700 kg/m^3 C=900 J/kgK ,K= 200 W/mK) (3b,10 Dec2016/Jan17)

   Hint : $\boldsymbol{m = Density * Volume = \rho * \frac{4}{3}\pi R^3}$

   $\boldsymbol{6 = 2700 * \frac{4}{3}\pi R^3 \; ; R = \sqrt[3]{\frac{6*3}{2700*4*\pi}} \; ; \; R = 0.081m}$

9. The temperature of a gas stream is measured with a thermocouple. The junction may be approximated as a sphere of diameter 1mm K=25W/m^{o}C, ρ =8400 kg /m^3 , C_p=0.4kJ/ kg K, h= 560W/m^{2}K. How long will it take for the thermocouple to record 99% of the applied difference?. (3c, 08, June/July14)
   Hint:

Thermocouple approximated as sphere

thermocouple to record 99% of the applied difference

ie $T - T_\infty = (1 - 0.99)(T_0 - T_\infty)$;

$$\frac{T - T_\infty}{T_0 - T_\infty} = \frac{(1 - 0.99)(T_0 - T_\infty)}{(T_0 - T_\infty)}$$

$$Hence\ \frac{T - T_\infty}{T_0 - T_\infty} = 0.01$$

Then use

$$\frac{T - T_\infty}{T_0 - T_\infty} = exp\left\{\frac{-hA_s\tau}{\rho CV}\right\}$$

And proceed

10. A Thermocouple is used to measure the temperature in a gas stream. The junction is approximated as a sphere with thermal conductivity of 25 W/mK. The properties of the junctions are ρ =9000 kg /m^3 C=0.35 kJ/ kg K, h =250 W/m^{2}K. Calculate the diameter of the junction if Thermocouple measures 95% of the applied temperature difference in 3 seconds (3b,04 June/July 2018)

    Thermocouple approximated as sphere

    thermocouple to record 99% of the applied difference

    ie $T - T_\infty = (1 - 0.95)(T_0 - T_\infty)$;

$$\frac{T - T_\infty}{T_0 - T_\infty} = \frac{(1 - 0.95)(T_0 - T_\infty)}{(T_0 - T_\infty)}$$

$$Hence\ \frac{T - T_\infty}{T_0 - T_\infty} = 0.05$$

    Then use

$$\frac{T - T_\infty}{T_0 - T_\infty} = exp\left\{\frac{-hA_s\tau}{\rho CV}\right\};\ 0.05 = exp\left\{\frac{-hA_s\tau}{\rho CV}\right\}$$

    And proceed

11. The average heat transfer coefficient for flow of 100^{o}C air over a flat plate is measured by the temperature time history of a 3 cm thick copper slab exposed to 100 ^{o}C air, in one test run, the initial temperature of slab was 210^{o}C and in 5 minute the temperature is decreased by 40 ^{o}C. Calculate the heat transfer coefficient for this case. Assume ρ =9000 kg/m^{3}C=0.38 kJ/ kg K, K= 370 W/mK (4c,06 June/July2018)

$T_\infty = 100^0C$; Thickness of slab 2L $= 3cm = 0.03m$

the temperature is decreased by 40 ^{o}C ie $T_o - T = 40^oC$

Hence $T = T_o - 40 = 210 - 40 = 170C$

$B_i = \frac{hL_c}{K}$; Here h is not given Hence assume $B_i < 0.1\ Lumped\ analysis$

$L_c = \frac{V}{A_{HT}}$; where V= surface area x thickness $= A * 2L$;

$A_{HT} = 2A$ since both area are expsed to Heat transfer

$Hence\ L_c = \frac{V}{A_{HT}} = \frac{A*2L}{2A};\ L_c = L = \frac{thickness}{2} = \frac{0.03}{2} = 0.015$

$\frac{T-T_\infty}{T_0-T_\infty} = exp\left\{\frac{-hA_s\tau}{\rho CV}\right\};\ \frac{170-100}{210-100} = exp\left\{\frac{-h*5*60}{9000*380*0.015}\right\};\ \frac{70}{110} = e^{-5.85*10^{-3}h}$

$ln\frac{70}{110} = -5.85*10^{-3}h;\quad -0.4520 = -h*5.85*10^{-3};\ h = 77.26\frac{W}{m^2C}$

$B_i = \frac{hL_c}{K};\ B_i = \frac{77.26*0.015}{370}$; $B_i = 3.13*10^{-3}$; $B_i < 0.1$ Hence lumped is to be applied

$Hence\ assumption\ is\ correct$

12 A household electric iron (ρ =2700 kg /m^3 , C_p= 0.896 kJ/kgK and K=200 W/m^{o}C) and weighs 1.5 kg .The total area of iron is 0.06m^2 and it is heated with 500 W heating element. Initially the iron is at 25^{o}C (ambient temperature) . How long it takes for the iron to reach 100^{o}C and take h_a =15 W/m^{2o}C (3c,06M June/July13)

13. What are Heisler charts? Explain their significance in solving transient convection problems (3b, 06, June/July14)

14. A long cylindrical shaft 20cm in diameter is made of steel K= 14.9 W/mK ρ =7900 kg/m^3 , C=477 J/kgK and α =3.95 X 10^{-6} m/s. It comes out an oven at a uniform temperature of 600^{o}C .The shaft is then allowed to cool slowly in an environment at 200 ^{o}C with an average heat transfer coefficient of 80 W/m^{2}K . Calculate the temperature at the centre of the shaft , 45 min after the start of cooling process. Also calculate heat transfer per unit length of the shaft during this period (3c,08 Dec17/Jan18

$B_i = \frac{hL_c}{K}; L_c = \frac{V}{A}$ where V is the Volume , A = Area of Heat Transfer

$For\ Cylinder\ , V = \pi R^2 L;\ A = Heat\ Transfer\ \ Area = 2\ \pi RL;$ Hence $L_c = \frac{\frac{4}{3}\pi R^3}{2\ \pi RL} = \frac{R}{2};$

$R = \frac{D}{2};\qquad R = \frac{0.2}{2};\quad R == 0.1m$

$L_c = \frac{R}{2} = \frac{0.1}{2}; L_c = 0.05$

$B_i = \frac{80*0.05}{14.9} = 0.268;\ B_i > 0.1$ Hence we cannot apply Lumped Analysis

$$Hence\ Hiesler\ Chart\ for\ cylinder\ is\ to\ be\ used$$

For Hiesler Charts 2 parameter required to centre temperature

$i)\ \frac{\alpha\tau}{R^2}$ ii) $\frac{hR}{K}$

$\frac{\alpha\tau}{R^2} = \frac{3.95*10^{-6}*45*60}{0.1^2};\ \frac{\alpha\tau}{R^2} = 1.0665$

$\frac{hR}{K} = \frac{80*0.1}{14.9};\qquad \frac{hR}{K} = 0.5369$

$$From\ First\ chart\ for\ cylinder\ ie\ Ch\ art\ for\ centre\ tempearture\ of\ cylinder$$

For the values ; $\frac{\alpha\tau}{R^2} = 1.0665; \frac{hR}{K} = 0.5369$

In Hiesler Chart $\frac{T_o - T_\infty}{T_i - T_\infty} = 0.44;$ Here $T_o = centre\ temperature\ ;\ T_i = Initial\ Temperature$

$\frac{T_o - 200}{600-200} = 0.44;\quad T_o = 376^oC$

$$Rate\ of\ Heat\ Transfer\ after\ 45\ min$$

Third chart of Hiesler for cylinder is to be used

$B_i^2 F_n = 0.5369^2 * 1.0665;\ \boldsymbol{B_i = \frac{hR}{K} = 0.5368}$

$\frac{Q}{Q_0} = 0.78$ where Q is the rate of Heat transfer and Q_0 is the maximum ossible Heat Transfer

$$Q = 0.78Q_0$$

$Q_0 = mC_p(T_i - T_\infty);$

$Q_0 = \rho V C_p(T_i - T_\infty)$

$V\ for\ cylinder$ is $\pi R^2 L;\ V = \pi * 0.1^2 * 1$ =0.03141

$Q_0 = \rho V C_p(T_i - T_\infty);\ Q_0 = 0.78 * 7900 * 477*(600 - 200) = 47.35 * 10^6 watts$

$Q = 0.78Q_0; Q = 0.78 * 47.35 * 10^6 = 36.93 * 10^6$ Watts

Please note that to refer Hiesler chart For cylinder and Sphere $\boldsymbol{B_i = \frac{hR}{K}}$ and Not $\boldsymbol{\frac{hL_c}{K}}$

Add the question : Find the temperature at the depth of thf 5 cm from the surface

To find the temperature at any radius refer second chart for cylinder

Parameter required $\frac{r}{R}$ and $\boldsymbol{B_i} = \frac{\boldsymbol{hR}}{\boldsymbol{K}}$

$r = R - depth; r = 0.1 - 0.05;\ \ r = 0.1 - 0.05 = 0.05; \frac{r}{R} = \frac{0.05}{0.2};\ \frac{r}{R} = 0.5$

For $\frac{r}{R} = 0.5\ and\ \boldsymbol{B_i} = \frac{\boldsymbol{hR}}{\boldsymbol{K}} = \ \boldsymbol{0.5368}$

$\frac{T-T_\infty}{T_i-T_\infty} = 0.85;\qquad \frac{T-200}{600-200} = 0.85; T = 363.2^oC$

If Surface temperature is to be determined

$\frac{r}{R} = \frac{R}{R} = 1;$

$For\ \frac{r}{R} = 1; and\ \boldsymbol{B_i} = \frac{\boldsymbol{hR}}{\boldsymbol{K}}$ **from 2nd chart** $\frac{T-T_\infty}{T_i-T_\infty}$

$then\ find\ T$

15. Aluminum rod of 5 cm diameter and 1meter long at 200°C is suddenly exposed to a convective environment 70°C . Calculate the temperature of a radius of 1cm and heat lost per meter length of the rod 1 minute after the cylinder exposed to the environment – properties of Al ρ =2700 kg /m^3 C_p=900J/ kg K, h= 500W/m^{2}K, α=8.5x10^{-5}m^2/s (3c, 10, Dec14/Jan15)

16. An aluminium, wire 1 mm in diameter at 200° C is suddenly exposed to an environment at 30 ° C with h= 85.5 W/m^{2}K. Estimate the time required to cool the wire to 90° C. If the same wire to place in a stream at (h= 11. 65 W/m^{2}K),what would be time required to reach it to 90 ° C. Assume thermo-physical properties C =900 J/kgK, ρ =2700 kg /m^3 K=204 W/mK (3b,09 Dec17/jan18)

17. A long 15 cm diameter cylindrical shaft made of SS 314 (K= 14.9 W/mK, ρ =7900 kg/m^3) allowed to cool slowly in a Chamber of 150°C with an average heat transfer coefficient of 85 W/m^{2}K .Determine the temperature of the centre of the shaft 25 minutes after the start of cooling process ii) Surface temperature at that time iii) heat transfer per unit length of shaft during this time period (3c, 10 June/July2015)

ii) For surface temperature

$\frac{r}{R} = \frac{R}{R} = 1;$

$For\ \frac{r}{R} = 1; and\ \boldsymbol{B_i} = \frac{\boldsymbol{hR}}{\boldsymbol{K}}$ **from 2nd chart note the value of** $\frac{T-T_\infty}{T_i-T_\infty}$

$then\ find\ T$ after substituting $T_i and\ T_\infty$

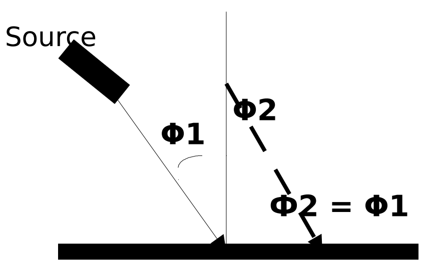
$$T\ is\ the\ durface\ temperature$$

18. Water pipes are to be buried underground in a wet soil (α =2.78x10^{-5} m^2/hr) which is initially at 4.5^{o}C. The soil surface temperature suddenly drops to -5^{o}C and remains at this value for 10hrs. Calculate the minimum depth at which the pipes are laid if the surrounding soil temperature is to be maintained above 0^{o}C. The soil may be considered as semi-infinite solid. Treat the present conditions as the condition as the condition of an semi infinite solid. (3c,06, June/July2018)

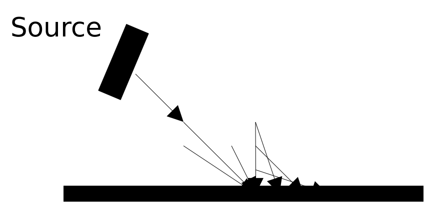
MODULE 3
RADIATION

VTU theory Questions

1. Define i) black body ii) Plank’s law iii) Wien displacement law iv) Lambert’s cosine law, v) Kirchoffffs law
2. Explain the following terms i) black body and grey body ii) Radiosity and irradiation
3. Explain i) Stefan – Boltzman Law ii) Wien displacement law iii) Radiation shield iv) Radiosity v) Black body
4. State and explain kirchoffs law.
5. State and Prove Kirchoffs law of radiation
6. State and prove wien's displacement law of radiation
7. Explain briefly the concept of black body
8. Prove that emissive power of a black body in a hemispherical enclosure is π times intensity of radiation
9. For a black body show that intensity of radiation is $\frac{1}{\pi}$ times Emissive power
10. Derive an expression for a radiation shape factor and show that it is a function of geometry only



(a) Specular Radiation



(b) Diffuse Radiation

***Specular Radiation and Diffuse Radiation*:**

Specular and Diffuse Radiation

When radiation strikes a surface, two types of reflection phenomena may be observed. If the angle of incidence is equal to the angle of reflection, the radiation is called Specular. On the other hand, when an incident beam is distributed uniformly in all directions after reflection, the radiation is called Diffuse Radiation. The two types of radiation are depicted in Fig. 10.3. Ordinarily, no real surface is either specular or diffuse. An ordinary mirror is specular for visible light, but would not necessarily be specular over the entire wavelength range. A rough surface exhibits diffuse behaviour better than a highly polished surface. Similarly, a highly polished surface is more specular than a rough surface.

Emissive Power: E (W/m²)

The emissive power of a surface is the total energy emitted by a surface at a given temperature per unit time per unit area for the entire wavelength range, from **$\lambda = 0$** to **$\lambda = \infty$**.

Monochromatic Emissive Power: E_λ (W/m²/unit wavelength)
The emissive power of a surface is the energy emitted by a surface at a given temperature per unit time per unit area per unit wavelength at given wavelength Total Emissive Power E can be calculated by integrating Monochromatic Emissive Power from **$\lambda = 0$** to **$\lambda = \infty$**.

$$\mathbf{E = \int_0^\infty E_\lambda d\lambda}$$

Emissivity ε
The emissivity of a surface is the ratio of the emissive power of the surface to the emissive power of a black surface at the same temperature. It is denoted by the symbol **ε**.

Monochromatic Emissivity

The Monochromatic emissivity of a surface is the ratio of the Monochromatic emissive power of the surface to the Monochromatic emissive power of a black surface at the same temperature and at same vave length . It is denoted by the symbol **ε_λ**

Solid Angle (Steridians)

It is defined as ratio of spherical surface enclosed by a cone ,with its vertex at the centre of the sphere , to the square of radius of the sphere. Unit of Solid Angle is steredians

Intensity of radiation

It is defined as the total energy emitted by a surface at a given temperature per unit time per unit area per unit solid angle W/m²/steredian

Black Body
A body which absorbs all incident radiation falling on it is called a blackbody. For a blackbody, $\alpha = 1$, $\rho = \tau = 0$. For a given temperature and wavelength, Emissive Power of black body at given temperature and wavelength is

greater than any other body. It is hypothetical body. It is a standard with which the radiation characteristics of other media are compared

Gray Body:

A gray body is a body having the same value of monochromatic emissivity at all wavelengths. i.e.

$$\varepsilon = \varepsilon_{\lambda,} \text{ for a gray body}$$

Radiosity of a Surface (J):

This is defined as the total energy leaving a surface per unit time per unit area of the surface. This definition includes the energy reflected by the surface due to some radiation falling on it.

Irradiation of a surface(G):

This is defined as the radiant energy falling on a surface per unit time, per unit area of the surface.

Therefore if E is the emissive power, J is the radiosity, ε is the irradiation and ρ the reflectivity of a surface, then,

$$\mathbf{J = E + rG}$$

For an opaque surface, $\boldsymbol{\rho + \alpha = 1}$ or $\mathbf{r = (1 - \alpha)}$

$$\mathbf{J = E + (1\text{-}\alpha)G}$$

STEFAN – BOLTZMANN LAW

This law states that the emissive power of a blackbody is directly proportional to the fourth power of the absolute temperature of the body.

i.e., $\mathbf{E_b \; \alpha \; T^4}$

Or $\mathbf{E_b = \sigma T^4}$ ---------------------------------- **(10.5)**

where σ is called the Stefan – Boltzmann constant.

In SI units $\sigma = 5.669 \times 10^{-8}\ W/(m^2\text{-}K^4)$.

PLANCK'S LAW:

This law states that the monochromatic power of a blackbody is given by

$$E_{b\lambda} = \frac{C_1}{\lambda^5\left(e^{\frac{C_2}{\lambda T}} - 1\right)}$$

where C_1 and C_2 are constants whose values are found from experimental data;

$C_1 = 3.7415 \times 10^{-16}\ Wm^2$ and $C_2 = 1.4388 \times 10^{-2}$ m-K. λ is the wavelength and T is the absolute temperature in K.

<u>10.2.3 *WEIN'S DISPLACEMENT LAW:*</u>

It can be seen from Planc law that at a given temperature, $E_{b\lambda}$ depends only on **λ.** Therefore the value of **λ** which gives maximum value of $E_{b\lambda}$ can be obtained by differentiating Plancs law w.r.t **λ** and equating it to zero.

$$\text{Planc law } E_{b\lambda} = \frac{C_1}{\lambda^5\left(e^{\frac{C_2}{\lambda T}} - 1\right)}$$

$$\frac{d\left(\dfrac{C_1}{\lambda^5\left(e^{\frac{C_2}{\lambda T}} - 1\right)}\right)}{d\lambda} \quad \text{¿}0$$

$\lambda_m T$ = 0.002898 m-K

λ_m is wavelength for Maximum emissive power at given temperature

From the Wein Displacement law, it can be seen that the wavelength at which the monochromatic emissive power is a maximum decreases with increasing temperature.

Proof:

$$\text{Planc law } E_{b\lambda} = \frac{C_1}{\lambda^5\left(e^{\frac{C_2}{\lambda T}} - 1\right)}$$

$$\frac{d\left(\dfrac{C_1}{\lambda^5\left(e^{\frac{C_2}{\lambda T}} - 1\right)}\right)}{d\lambda} \quad \text{¿}0$$

$$\frac{d\left(\lambda^5\left(e^{\frac{C_2}{\lambda T}} - 1\right)\right)}{d\lambda} = 0$$

Let $C_2/\lambda T = y$ **.** Then Eq. (10.6) reduces to

$$\frac{d\left(\lambda^5\left(e^y-1\right)\right)}{d\lambda} = 0$$

$$\lambda^5\left(y e^y \frac{dy}{d\lambda}\right) + 5\lambda^5\left(e^y-1\right) = 0$$

Where $\frac{dy}{d\lambda} = \frac{d\left(\frac{C_2}{\lambda T}\right)}{d\lambda}$

Solution for above equation is $e^y(5 - y) = 5$

By trial and error , y = 4.965

$C_2/\lambda_m T = 4.965$

$\lambda_m T = C_2/4.965 = 1.4388\times10^{-2} /4.965$

$\lambda_m T = 0.002898$ m-K

Lamberts Cosine Law

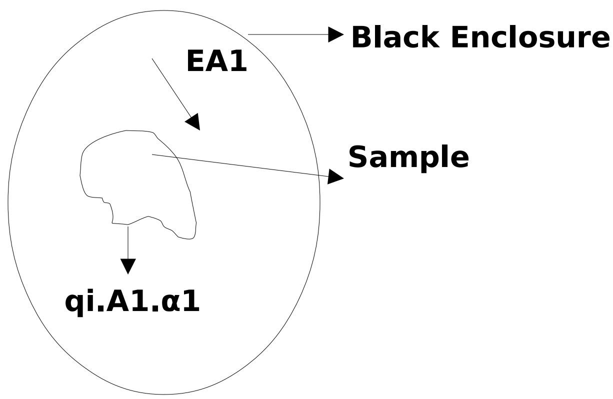
It states that A diffuse surface radiates wnergy in such a manner that the rate of energy radiated in any particular direction is proportional to the cosine of the angle between the direction under any consideration and normal to the surface.

$$E_\theta = E_n cos\theta$$

KIRCHOFF'S LAW:

This law states that at any temperature the ratio of total emissive power to absorptivity is constant for all substances which are in thermal equilibrium with their surroundings.

Proof: Consider a perfect black enclosure i.e. the one which absorbs all the incident radiation falling on it (see Fig 10.5). Now let the radiant flux from this enclosure per unit area arriving at some area be $\mathbf{q_i}$ W/m^2



Model used for deriving Kirchoff law

Consider a body is placed inside the enclosure and allowed to come to thermal equilibrium with it. At equilibrium, the energy absorbed by the body must be equal to the energy emitted;

E is Emissive Power , q_i is radiation heat flux in W/m^2 falling on the body,

At thermal equilibrium we may write

$$\mathbf{E_1\ A_1 = q_i\ A_1\ \alpha_1}$$

$\mathbf{E_1 = q_i\ \alpha_1}$ **.. 1**

If the body in the enclosure with a another body 2 and allow it to come to thermal equilibrium with the

$\mathbf{E_2\ A_2 =}\ \alpha_2\ \mathbf{q_i\ A_2}$

$\mathbf{E_2 =}\ \alpha_2\ \mathbf{q_i}$**...2**

If 1 is divided by 2 we get

$$\frac{E_1}{E_2} = \frac{\alpha_1}{\alpha_2}$$

Hence

$$\frac{E_1}{\alpha_1} = \frac{E_2}{\alpha_2}$$

Hence

$$\frac{E_1}{\alpha_1} = \frac{E_2}{\alpha_2} = \frac{E_3}{\alpha_3} = \frac{E_4}{\alpha_4} = \text{-----------} = \frac{E_b}{\alpha_b}$$

Hence ratio of Emissive power to absoptivity is constant for all bodies at given temperature

$$\frac{E_1}{\alpha_1} = \frac{E_b}{\alpha_b}$$

But $\alpha_b=1$

Hence

$$\frac{E_1}{\alpha_1}=E_b \quad ; \quad \frac{E_1}{E_b}=¿ \qquad \alpha_1$$

$$\varepsilon_1=\alpha_1$$

Kirchoff’s law and is valid only when the body is in thermal equilibrium with the surroundings. However, while analyzing radiation problems in practice we assume that Kirchoff’s law holds good even if the body is not in thermal equilibrium with the surroundings, as the error involved is not very significant.

<u>Prove that Emissive Power of black body is π times the Intensity of radiation</u>

Consider an elemental area dA_1 whose total emissive power is E_1. This total radiant energy emitted by dA_1 can be intercepted by a hemisphere as shown in Fig 10.10.

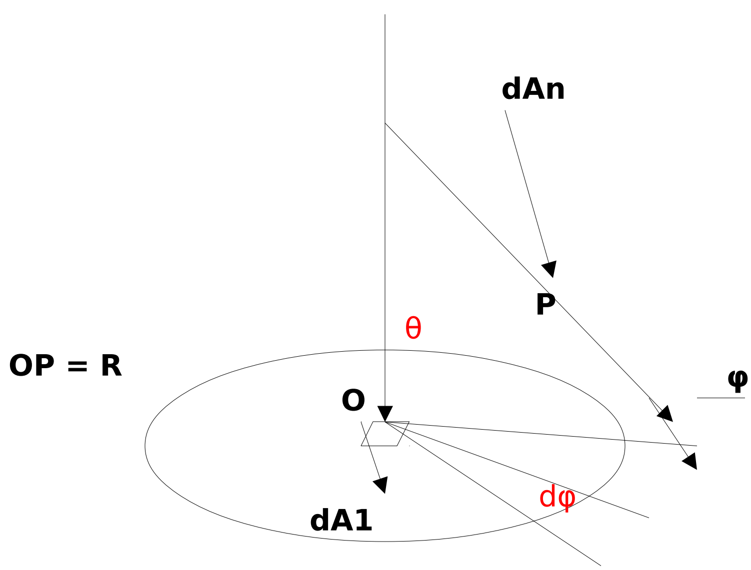


Fig Radiation from a differential area dA_1 into surrounding hemisphere centered at dA_1.

Intensity of radiation , I $= \dfrac{dQ_{1-2}}{dA_1 Cos\theta x \frac{d A_2}{R^2}}$

$$dQ_{1-2}=dA_1 Cos\theta x \frac{d A_2}{R^2}$$

$d A_2=¿$ x y where x = Rdθ ; y = rdφ But r= R sinθ hence y = R sinθ dφ

Hence $A_2 = (Rd\theta)(R\sin\theta\, d\varnothing)$; $d\,A_2 = R^2 \sin\theta\, d\theta\, d\varnothing$

$$dQ_{1-2} = I_b\, dA_1\, Cos\theta x \frac{R^2 \sin\theta\, d\theta\, d\varnothing}{R^2}$$

$$dQ_{1-2} = I_b\, dA_1\, Cos\theta\, Sin\theta\, d\theta\, d\varnothing$$

$$dQ_{1-2} = I_b\, dA_1 \frac{\sin 2\theta}{2} d\theta\, d\varnothing$$

$$Q_{1-2} = I_b dA_1 \int_{\theta=0}^{\theta=\frac{\pi}{2}} \frac{\sin 2\theta}{2} d\theta \int_{\varnothing=0}^{\varnothing=2\pi} d\varnothing$$

$$Q_{1-2} = I_b dA_1 2\pi \int_{\theta=0}^{\theta=\frac{\pi}{2}} \frac{\sin 2\theta}{2} d\theta$$

$$Q_{1-2} = I_b dA_1 \pi \left(\frac{-\cos 2\theta}{2} \right)_{\theta=0}^{\theta=\frac{\pi}{2}}$$

$$Q_{1-2} = -I_b dA_1 \frac{\pi}{2} (-1-1)$$

$$Q_{1-2} = E_b\, dA_1 \pi$$

But $Q_{1-2} = E_b dA_1$

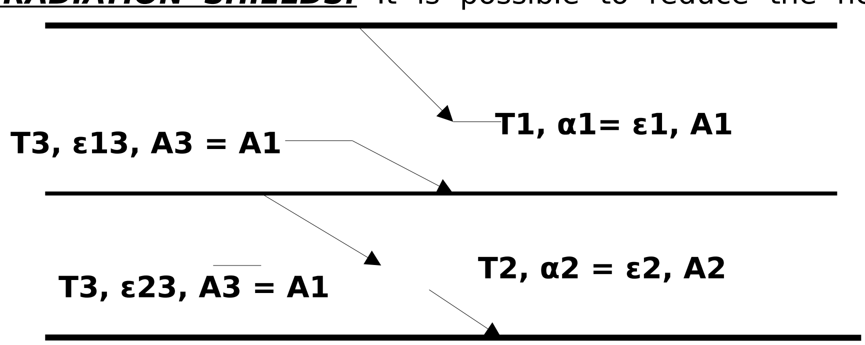
$$E_b dA_1 = I_b dA_1 \pi$$

$$E_b = \pi\, I_b$$

View Factor or shape factor

For any two surfaces, the orientation of them with respect to each other affects the fraction of the radiation energy leaving one surface and striking the other directly. The concept of "VIEW FACTOR" (also called as CONFIGURATION FACTOR/SHAPE FACTOR) has been utilised to formalise the effects of orientation in the radiation heat exchange between surfaces.

PLANE RADIATION SHIELDS: It is possible to reduce the net radiation heat exchange between two infinite parallel gray surfaces by introducing a



third surface in between them. If the third surface, known as the radiation shield is assumed to be very thin, then both sides of this surface can be assumed to be at the same temperature.

Radiation Heat Exchange Between Two Parallel Infinite Graysurfaces in presence of a radiation shield

Fig.shows a scheme for radiation heat exchange between two parallel infinite gray surfaces at two different temperatures T_1 and T_2 in presence of a radiation shield at a uniform temperature, T_3.

$$\frac{Q_{1-3}}{A_1}=\frac{\sigma\left(T_1^4-T_3^4\right)}{\frac{1}{\epsilon_1}+\frac{1}{\epsilon_{13}}-1}$$

$$\frac{Q_{3-1}}{A_1}=\frac{\sigma\left(T_3^4-T_2^4\right)}{\frac{1}{\epsilon_{32}}+\frac{1}{\epsilon_2}-1}$$

For steady state conditions, these two must be equal..Therefore we have

$$\frac{Q_{1-3}}{A_1}=\frac{Q_{3-1}}{A_1}$$

$$\frac{A_1\sigma\left(T_1^4-T_3^4\right)}{\frac{1}{\epsilon_1}+\frac{1}{\epsilon_{13}}-1}=\frac{A_{13}\sigma\left(T_3^4-T_2^4\right)}{\frac{1}{\epsilon_{32}}+\frac{1}{\epsilon_2}-1}$$

$$\left(T_1^4-T_3^4\right)=\frac{\frac{1}{\epsilon_1}+\frac{1}{\epsilon_{13}}-1}{\frac{1}{\epsilon_{32}}+\frac{1}{\epsilon_2}-1}\quad x\left(T_3^4-T_2^4\right)$$

$$T_1^4+\frac{\frac{1}{\epsilon_1}+\frac{1}{\epsilon_{13}}-1}{\frac{1}{\epsilon_{32}}+\frac{1}{\epsilon_2}-1}\quad T_2^4\qquad ¿\,T_3^4+\frac{\frac{1}{\epsilon_1}+\frac{1}{\epsilon_{13}}-1}{\frac{1}{\epsilon_{32}}+\frac{1}{\epsilon_2}-1}T_3^4$$

$$T_1^4+\frac{\frac{1}{\epsilon_1}+\frac{1}{\epsilon_{13}}-1}{\frac{1}{\epsilon_{32}}+\frac{1}{\epsilon_2}-1} \qquad T_2^4 \qquad \text{¿}\, T_3^4\left(1+\frac{\frac{1}{\epsilon_1}+\frac{1}{\epsilon_{13}}-1}{\frac{1}{\epsilon_{32}}+\frac{1}{\epsilon_2}-1}\right)$$

$$T_3=\left(\frac{T_1^4+\frac{\frac{1}{\epsilon_1}+\frac{1}{\epsilon_{13}}-1}{\frac{1}{\epsilon_{32}}+\frac{1}{\epsilon_2}-1}T_2^4}{1+\frac{\frac{1}{\epsilon_1}+\frac{1}{\epsilon_{13}}-1}{\frac{1}{\epsilon_{32}}+\frac{1}{\epsilon_2}-1}}\right)^{\frac{1}{4}}$$

Heat transfer with shield $Q=Q_{13}=Q_{32}$

$$\frac{Q}{A_1}=\frac{Q_{1-3}}{A_1}=\frac{A_1\sigma\left(T_1^4-T_3^4\right)}{\frac{1}{\epsilon_1}+\frac{1}{\epsilon_{13}}-1}$$

$$\frac{Q}{A_1}=\frac{Q_{1-3}}{A_1}=\frac{A_1\sigma\left(T_1^4-\frac{T_1^4+\frac{\frac{1}{\epsilon_1}+\frac{1}{\epsilon_{13}}-1}{\frac{1}{\epsilon_{32}}+\frac{1}{\epsilon_2}-1}T_2^4}{1+\frac{\frac{1}{\epsilon_1}+\frac{1}{\epsilon_{13}}-1}{\frac{1}{\epsilon_{32}}+\frac{1}{\epsilon_2}-1}}\right)}{\frac{1}{\epsilon_1}+\frac{1}{\epsilon_{13}}-1}$$

Special Case : If $\epsilon_1=\epsilon_2=\epsilon_{13}=\epsilon_{32}$

$$T_3=\left(\frac{T_1^4+T_2^4}{1+1}\right)^{\frac{1}{4}}$$

$$T_3=\left(\frac{T_1^4+T_2^4}{2}\right)^{\frac{1}{4}}$$

Heat transfer

$$\frac{Q_{withshield}}{A_1}=\frac{Q_{1-3}}{A_1}=\frac{\sigma\left(T_1^4-\frac{T_1^4+T_2^4}{2}\right)}{\frac{1}{\in}+\frac{1}{\in}-1}$$

$$\frac{Q}{A_1}=\frac{\sigma\left(\frac{T_1^4-T_2^4}{2}\right)}{\frac{2}{\in}-1}$$

Heat transfer with one shield

$$\frac{Q}{A_1}=\frac{\sigma\left(T_1^4-T_2^4\right)}{2\left(\frac{2}{\in}-1\right)}$$

Heat Transfer with out shield

$$\frac{Q}{A_1}=\frac{\sigma\left(T_1^4-T_2^4\right)}{\frac{1}{\in}+\frac{1}{\in}-1}$$

$$\frac{Q}{A_1}=\frac{\sigma\left(T_1^4-T_2^4\right)}{\frac{2}{\in}-1}$$

It can be seen from the above equation that when the emissivities of all surfaces are equal, the net radiation heat exchange between the surfaces in the presence of single radiation shield is 50% of the radiation heat exchange between the same two surfaces without the presence of a radiation shield. This statement can be generalised for N radiation shields as follows:

Heat transfer with n shield

$$\frac{Q}{A_1}=\frac{\sigma\left(T_1^4-T_2^4\right)}{\left(n+1\right)\left(\frac{2}{\in}-1\right)}$$

1. The temperature of a black surface is 0.2 m^2 in area is 540 ^oC. Calculate i) the total rate of energy emission ii) intensity of normal radiation iii) the wavelength of maximum monochromatic emissive power

Black surface ie ε=1; Area =0.2 m^2 ie A=0.2m^2 ; Temperature = 540^{o}C ie T=540+273 K = 813K

i) Q=? ii) I=? iii) $\lambda_{max}=?$

$i¿\, E_b=\sigma T^4$; $E_b=5.67\,x\,10^{-8}\,x\,813^4$; $E_b=2.4771\,x\,10^4\,W/m^2$

$Q=A\,E_b;\, Q=0.2\,x\,2.4771\,x\,10^4;\, Q=4954.22\,Watts$

$ii¿$ $E_b=\pi I_b$; $2.4771\,x\,10^4=\pi\,I_b$; $I_b=\dfrac{2.4771\,x\,10^4}{\pi}$;

$I_b=7884.5\,W/m^2\,solidAngle$

iii) $\lambda_{max}T=2898\,\mu mK$; $\lambda_{max}\,x\,813=2898$; $\lambda_{max}=\dfrac{2898}{813}\,\mu m$;

$\lambda_{max}=3{,}56\,\mu m$

2. A pipe carrying steam runs in a large room and exposed to air at 30 ^oC. The pipe surface temperature is 200 ^oC. Diameter of the pipe is 20 cm. If the total heat loss per meter length of the pipe is 1.9193 kW/m, determine the emissivity to the pipe surface .

$\phi=20\,cm=0.2\,m;\, T_w=200^0\,C=(200+273)\,K=473\,K$;

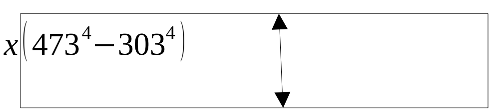
$T_\infty=30^0\,C=(30+273)\,K=303\,K$

$Heat\,loss\,per\,meter\,length=1.9193\,kW/m=1919.3\,W/m$; length L=1 m

$Heat\,loss¿\,the\,pipe\,Q=\in\sigma A\left(T_w^4-T_\infty^4\right)$;

$Q=\in\sigma A\left(T_w^4-T_\infty^4\right);$

$1919.3=\in x\,5.67\,x\,10^{-8}\,x\,(\pi x\,0.2\,x\,1)\,x\left(473^4-303^4\right)$

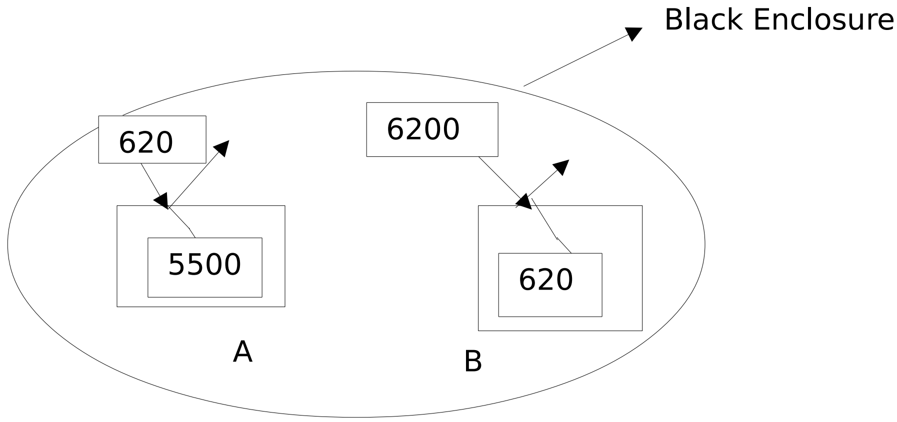


$\in=1.294$

$T_w=200^0\,C;$ $T_\infty=30^0\,C$

3. In an isothermal enclosure at uniform temperature two small surfaces A and B are placed. The irradiation to the surface by the enclosure is 6200 W/m^2. The absorption rates by the surfaces A and B are 5500 W/m^2 and 620 W/m^2 . When steady state established, calculate the following i) what are the heat

fluxes to each surface? What are their temperatures?. ii) Absorptivity of both surfaces iii) Emissive power of each surface iv) Emissivity of each surface .



For A

$$q_{iA}=6200\,W/m^2; q_{aA}=5500\,W/m^2$$

$$q_{reflected}=q_{iA}-q_{aA} \quad ; \quad q_{reflected}=6200-5500 \quad ; \quad q_{reflected}=700\,W/m^2$$

For B

$$q_{iB}=6200\,W/m^2; q_{aB}=620\,W/m^2$$

$$q_{reflected}=q_{iA}-q_{aA} \quad ; \quad q_{reflected}=6200-620 \quad ; \quad q_{reflected}=5580\,W/m^2$$

i) What are the heat fluxes to each surface

Heat flux on A = heat incident per unit area ; Hence Heat flux on A $q_A=6200\,W/m^2$

Heat flux on B = heat incident per unit area ; Hence Heat flux on B $q_B=6200\,W/m^2$

What are their temperature

For A

Under thermal equilibrium with enclosure and surface A

Energy absorbed = Energy emitted

$$q_{iA}A\alpha_A=E_A A \quad ; \quad q_{iA}\alpha_A=E_A;$$

$$q_{iA}\alpha_A=\epsilon_A\sigma T_A^4 \quad ; \quad q_{iA}\epsilon_A=\epsilon_A\sigma T_A^4 \quad q_{iA}=\sigma T_A^4$$

$$6200=5.67\,x\,10^{-8}\,x\,T_A^4 \quad ; \quad T_A=575.07\,K$$

For B

Under thermal equilibrium with enclosure and surface B

Energy absorbed = Energy emitted

$q_{iB} A \alpha_B = E_B A$; $q_{iB} \alpha_B = E_B$;

$q_{iB} \alpha_B = \epsilon_B \sigma T_B^4$; $q_{iB} \epsilon_B = \epsilon_B \sigma T_B^4$ $q_{iB} = \sigma T_B^4$

$6200 = 5.67 \, x \, 10^{-8} x \, T_B^4$; $T_B = 575.07 \, K$

ii) Absorbity on both the surfaces

$\alpha_A = \frac{q_{aA}}{q_i}$; $\alpha_A = \frac{5500}{6200}$; $\alpha_A = 0.887$

$\alpha_B = \frac{q_{aB}}{q_i}$; $\alpha_B = \frac{620}{6200}$; $\alpha_B = 0.1$

iii) Emissive Power of each surface

For A

$q_{iA} A \alpha_A = E_A A$; $q_{iA} \alpha_A = E_A$;

$6200 \, x \, 0.887 = E_A$

$E_A = 5500 \, W / m^2$

For B

$q_{iB} A \alpha_A = E_B A$; $q_{iB} \alpha_B = E_B$; $6200 \, x \, 0.1 = E_B$

$E_B = 620 \, W / m^2$

iv) Emissivity of each surface

$\epsilon_A = \alpha_A$; $\epsilon_A = 0.887$

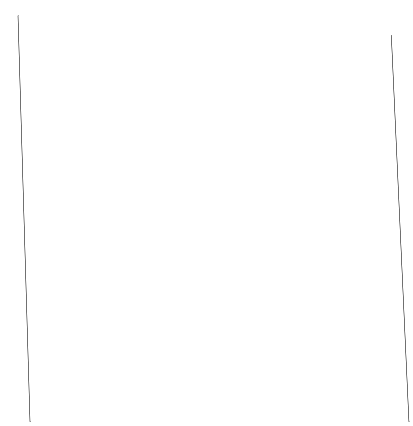
$\epsilon_B = \alpha_B$; $\epsilon_B = 0.1$

4. Calculate the net heat radiated exchange per m^2 for two large parallel plates maintained at 800 ºC and 300 ºC. The emissivities of two plates are 0.3 and 0.6 respectively

5. Calculate the net Radiant heat exchange per m^2 area for two large parallel planes at temperature of 427 ºC and 27 ºC respectively.Take ϵ for hot and cold planes to be 0.9 and 0.6 respectively. If a polished aluminum sheet is placed between them, find the percentage reduction in heat transfer, given ϵ for shield =0.04

6. Two large plates having emissivities of 0.3 and 0.6 maintained at a temperature of 900 ºC and 250 ºC. A radiation shield having and emissivity of 0.05 on both sides is placed between the two plates. Calculate i) heat transfer without shield ii) Heat transfer with the shield iii) percentage reduction in the heat transfer due to shield iv) temperature of the shield

Heat Transfer without shield A B



$$Q_1=\frac{\sigma A\left(T_1^4-T_2^4\right)}{\frac{1}{\epsilon_1}+\frac{1}{\epsilon_2}-1}$$

$$Q_1=\frac{5.67\,x\,10^{-8}\,x\,1\left(1173^4-523^4\right)}{\frac{1}{0.3}+\frac{1}{0.6}-1}$$

$Q_1=25.77\,x\,10^3\,watts$ /m^2

Heat Transfer with shield

$$Q_2=\frac{\sigma A\left(T_1^4-T_2^4\right)}{\left(\frac{1}{\epsilon_1}+\frac{1}{\epsilon_{13}}-1\right)+\left(\frac{1}{\epsilon_{32}}+\frac{1}{\epsilon_2}-1\right)}$$

$\epsilon_{13}=\epsilon_{32}=0.05$

$$Q_2=\frac{5.67\,x\,10^{-8}\,x\,1\left(1173^4-523^4\right)}{\left(\frac{1}{0.3}+\frac{1}{0.05}-1\right)+\left(\frac{1}{0.05}+\frac{1}{0.05}-1\right)}$$

$\epsilon_1=0.3$

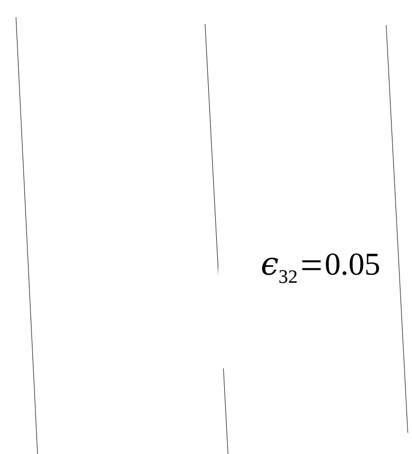
$\epsilon_2=0.6$

$Q_2=2397.27\,Watts/m^2$

Percentage reduction∈Heat Transfer due¿shield

$\epsilon_{13}=0.05$

$\frac{Q_1-Q_2}{Q_1}x\,100$ ie $\left(1-\frac{Q_2}{Q_1}\right)x\,100$



$$\left(1-\frac{2397.27}{25.77\,x\,10^3}\right)x\,100$$

=90.97%

Temperature of the shield

$\epsilon_{32}=0.05$

$$Q_{13}=Q_{13}$$

$\epsilon_{13}=0.($

$$\frac{\sigma A\left(T_1^4-T_3^4\right)}{\frac{1}{\epsilon_1}+\frac{1}{\epsilon_{13}}-1}=\frac{\sigma A\left(T_3^4-T_2^4\right)}{\frac{1}{\epsilon_{32}}+\frac{1}{\epsilon_2}-1}$$

$$\frac{\left(T_1^4-T_3^4\right)}{\frac{1}{\epsilon_1}+\frac{1}{\epsilon_{13}}-1}=\frac{\left(T_3^4-T_2^4\right)}{\frac{1}{\epsilon_{32}}+\frac{1}{\epsilon_2}-1}$$

$\epsilon_1=0.3$

$\epsilon_2=0.6$

$$\frac{\left(1173^4-T_3^4\right)}{\frac{1}{0.3}+\frac{1}{0.05}-1}=\frac{\left(T_3^4-523^4\right)}{\frac{1}{0.05}+\frac{1}{0.6}-1}$$

$$\frac{\left(1173^4-T_3^4\right)}{22.33}=\frac{\left(T_3^4-523^4\right)}{20.66}$$

$$1173^4-T_3^4=\frac{22.33}{20.66}\left(T_3^4-523^4\right)$$

$$1173^4-T_3^4=1.081\left(T_3^4-523^4\right)$$

$$1173^4-T_3^4=1.081\,T_3^4-1.081\,x\,523^4$$

$$1173^4+1.081\,x\,523^4=1.081\,T_3^4+T_3^4$$

$$1173^4+1.081\,x\,523^4=2.081\,T_3^4$$

$$T_3=\left(\frac{1173^4+1.081\,x\,523^4}{2.081}\right)^{\frac{1}{4}}$$

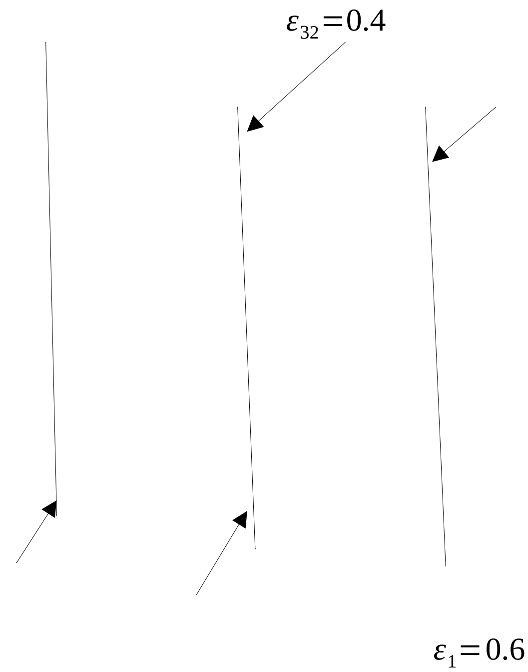
$$T_3=986.89\,K$$

7. Two large planes with emissivity of 0.6 at 900K and 300K. A radiation shield with one side polished and having emissivity of 0.05, while the emissivity of other side is 0.4 is proposed to be used. Which side of the shield should face the hotter plate, if the temperature of shield is to be kept minimum? Justify your answer

   Case 1: The face having lesser emissivity of radiation shield facing hotter plate ie $\epsilon_{13}=0.05$

And $\epsilon_{32}=0.4$

Temperature of the shield



$\varepsilon_2=0.6$

$Q_{13}=Q_{13}$

$$\frac{\sigma A\left(T_1^4-T_3^4\right)}{\frac{1}{\epsilon_1}+\frac{1}{\epsilon_{13}}-1}=\frac{\sigma A\left(T_3^4-T_2^4\right)}{\frac{1}{\epsilon_{32}}+\frac{1}{\epsilon_2}-1}$$

$$\frac{\left(T_1^4-T_3^4\right)}{\frac{1}{\epsilon_1}+\frac{1}{\epsilon_{13}}-1}=\frac{\left(T_3^4-T_2^4\right)}{\frac{1}{\epsilon_{32}}+\frac{1}{\epsilon_2}-1}$$

$$\frac{\left(900^4-T_3^4\right)}{\frac{1}{0.6}+\frac{1}{0.05}-1}=\frac{\left(T_3^4-300^4\right)}{\frac{1}{0.4}+\frac{1}{0.6}-1}$$

$\varepsilon_{13}=0.05$

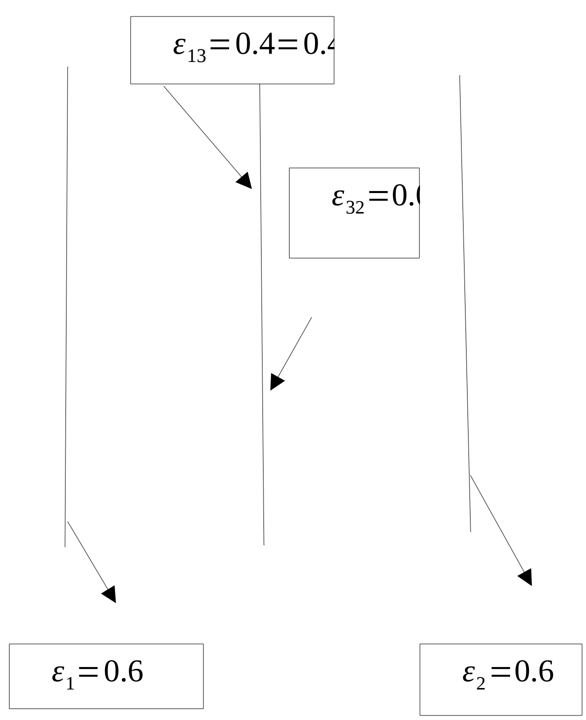
$$\frac{\left(900^4-T_3^4\right)}{20.67}=\frac{\left(T_3^4-300^4\right)}{3.17}$$

$T_3=554.10\,K$

Case 2: The face having lesser emissivity of radiation shield facing colder plate ie $\epsilon_{13}=0.4$

And $\epsilon_{32}=0.05$

Temperature of the shield



$Q_{13}=Q_{13}$

$$\frac{\sigma A\left(T_1^4-T_3^4\right)}{\frac{1}{\epsilon_1}+\frac{1}{\epsilon_{13}}-1}=\frac{\sigma A\left(T_3^4-T_2^4\right)}{\frac{1}{\epsilon_{32}}+\frac{1}{\epsilon_2}-1}$$

$$\frac{\left(T_1^4-T_3^4\right)}{\frac{1}{\epsilon_1}+\frac{1}{\epsilon_{13}}-1}=\frac{\left(T_3^4-T_2^4\right)}{\frac{1}{\epsilon_{32}}+\frac{1}{\epsilon_2}-1}$$

$$\frac{\left(900^4-T_3^4\right)}{\frac{1}{0.6}+\frac{1}{0.4}-1}=\frac{\left(T_3^4-300^4\right)}{\frac{1}{0.05}+\frac{1}{0.6}-1}$$

$$\frac{\left(900^4 - T_3^4\right)}{3.17} = \frac{\left(T_3^4 - 300^4\right)}{20.67}$$

$$T_3 = 868.83\,K$$

Hence face having lesser emissivity ie 0.05 should face higher temperature plate to keep the temperature of radiation shield to be minimum

8. Two parallel plates, each of 4m^2 area are large compared to gap of 5mm separating them. One plate has a temperature of 800K and surface emissivity of 0.6, while the other has a temperature of 300K and a surface emissivity of 0.9. Find the net energy exchange by radiation between them. If a polished metal sheet of surface emissivity 0.1 on both sides is now located centrally between two plates, what will be its study state temperature ? How the heat transfer would be altered. Neglect the Convection and its effects if any. Comment upon the significance of this exercise.

Hint since 4m^2 large compared to gap shape factor between plate =1 Hence Heat Transfer without shield as before But A=0.4m^2

$$Q_1 = \frac{\sigma A\left(T_1^4 - T_2^4\right)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

Heat transfer with shiled

$$Q_2 = \frac{\sigma A\left(T_1^4 - T_2^4\right)}{\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_{13}} - 1\right) + \left(\frac{1}{\epsilon_{32}} + \frac{1}{\epsilon_2} - 1\right)}$$

Percentage reduction∈Heat Transfer due¿shield

$$\frac{Q_1 - Q_2}{Q_1} x\,100 \qquad \text{ie} \qquad \left(1 - \frac{Q_2}{Q_1}\right) x\,100$$

Temperature of the shield

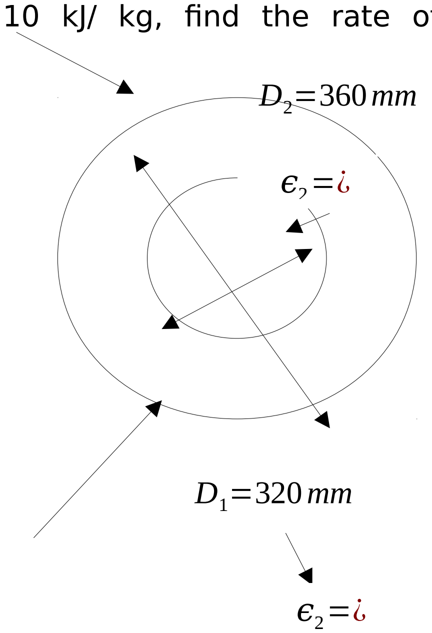
$$Q_{13} = Q_{13}$$

$$\frac{\sigma A\left(T_1^4 - T_3^4\right)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_{13}} - 1} = \frac{\sigma A\left(T_3^4 - T_2^4\right)}{\frac{1}{\epsilon_{32}} + \frac{1}{\epsilon_2} - 1}$$

9. Calculate the net Radiant heat exchange per m^2 area for two large parallel plates at temperature of 427 °C and 27 °C respectively. Take ϵ for hot and cold planes to be 0.9 and 0.6 respectively. If a polished aluminum sheet is placed between them, find the percentage reduction in heat transfer, given ϵ for shield =0.4

10. Two large parallel plates with emissivity of 0.5 each are maintained at different temperatures and are exchanging heat only by radiation. Two equally large radiation Shield with surface emissivity 0.05 are introduced in parallel to the plates . Find the percentage reduction in net radiative heat transfer

11. Liquid air boiling at – 153 °C is stored in a spherical container of diameter 320 mm the container is surrounded by concentric spherical shell diameter 360 mm in a room at 27 °C. The space between the two spheres is evacuated. The surface of the sphere are flashed with aluminum ($\epsilon = ¿$ 0.3) Taking the latent heat of vaporization of liquid a as 210 kJ/ kg, find the rate of evaporation of liquid air



$$Q = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1-\epsilon_1}{A_1\epsilon_1} + \frac{1}{A_1 F_{12}} + \frac{1-\epsilon_2}{A_2\epsilon_2}}$$

$D_2 = 360\,mm = 0.36\,m \qquad r_2 = 0.18\,m$

$D_1 = 320\,mm = 0.32\,m \qquad r_1 = 0.16\,m$

$A_1 = 4\pi r_1^2 \quad ; \quad A_1 = 4\pi\, 0.16^2 \quad ; \quad A_1 = 0.3216\,m^2$

$A_1 = 4\pi r_2^2 \quad ; \quad A_1 = 4\pi\, 0.18^2 \quad ; \quad A_2 = m^2$

$$Q = \frac{5.68\,x\,10^{-6}\left(120^4 - 300^4\right)}{\frac{1-0.3}{0.3216\,x\,0.3} + \frac{1}{0.3216\,x\,1} + \frac{1-0.3}{x\,0.3}}$$

$Q = -27.88\,watts$

$Negative\,sign\,indicates\,heat\,is\,transferred\,¿\,outside\,¿\,inside$

Rate of evaporation of liquid oxygen

$$\acute{m} = \frac{Q}{h_{fg}}$$

$$\acute{m} = \frac{27.88}{210\,x\,10^3}$$

$\acute{m} = 1.3276\,x\,10^{-4}\,kg/s$

$\acute{m} = 0.4779\, kg / hr$

12. The concentric spheres 20cms and 30cms in diameter are used to liquid O_2 (-153°C) in a room at 300K. The space between the spheres is evacuated. The surfaces of the spheres are highly polished as $\epsilon = ¿$ 0.04. Find the rate of air per hour

Boundary Layer

1. Define and explain hydrodynamic and thermal boundary layer in case of flow over a flat plate (06 4a June/July13, Dec18/Jan19)
2. Explain velocity and thermal boundary layer 06 June/July 14.Dec17/Jan18 ,June/July 16, Dec18/Jan19

3. With reference to fluid flow over a flat plate , discuss the concept of velocity boundary and thermal boundary layer with necessary sketches 05 Dec13/Jan14
4. For flow over flat plate, discuss the concepts of velocity boundary and thermal boundary layer with sketches. (04,5a,June/July2015)
5. Explain briefly with structures i) Boundary layer thickness ii) Thermal boundary layer thickness 08 June/July 2017

6 Obtain a relationship between drag coefficient c_m and heat transfer Coefficient h_m for the flow over a flat plate (06, 5c, Dec13/Jan 14)

7. Prove that $\frac{N_{ux}}{R_{ex} P_r} = \frac{C_{fx}}{2}$ with usual notations (08, 5b, Dec14/Jan15)

1. The exact expression for local Nusselt number for the laminar flow along a surface is given by

$$Nu_x = \frac{h_x x}{k} = 0.332 R_{ex}^{\frac{1}{2}} P_r^{\frac{1}{3}}$$

Show that the average heat transfer coefficient from x= 0 to x =L over the length L of the surface is given by $2h_L$ is the local heat transfer coefficient at x=L

2. An approximate expression for temperature profile in thermal boundary layer is given by

$$\frac{T_{(x,y)} - T_w}{T_\infty - T_w} = \frac{3}{2}\frac{y}{\delta_t(x)} - \frac{1}{2}\left[\frac{y}{\delta_t(x)}\right]^3 \text{, where } \delta_t(x) = 4.53\frac{x}{R_{ex}^{\frac{1}{2}} P_r^{\frac{1}{3}}}$$

Develop an expression for local heat transfer coefficient $h_{(x)}$

When a fluid flows over a body or inside a channel and if the temperatures of the fluid and the solid surface are different, heat transfer will take place between the solid surface and the fluid due to the macroscopic motion of the fluid relative to the surface. This mechanism of heat transfer is called as "*convective heat transfer*".If the fluid motion is due to an external force (by using a pump or a compressor) the heat transfer is referred to as "*forced convection*". If the fluid motion is due to a force generated in the fluid due to buoyancy effects resulting from density difference (density difference may be caused due to temperature difference in the fluid) then the mechanism of heat transfer is called as *“natural or free convection*

Velocity Boundary Layer:- Consider the flow of a fluid over a flat plate as shown in Fig.

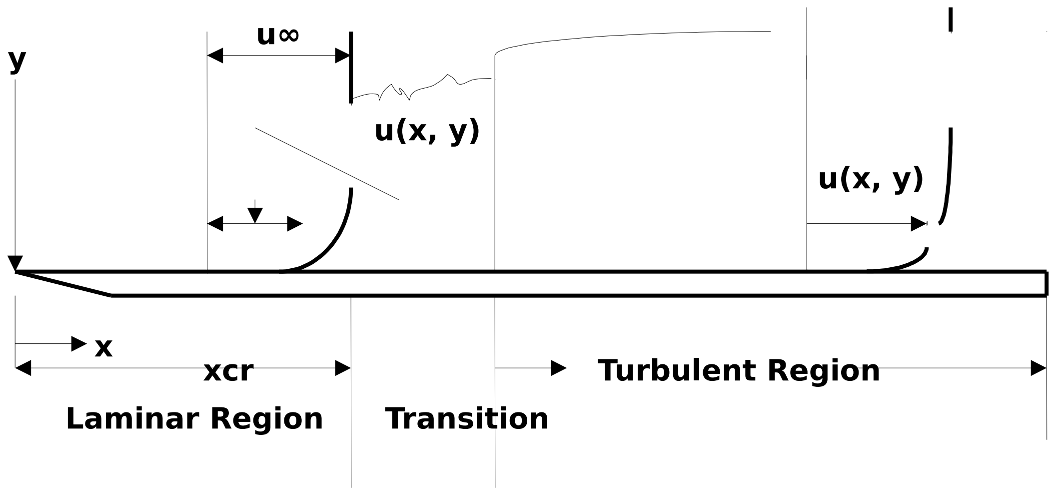


Fig. 5.2: Velocity boundary layer for flow over a flat plate

The fluid just before it approaches the leading edge of the plate has a velocity u_∞ which is parallel to the plate surface.As the fluid moves in x-direction along the plate,

A fluid flows over the plate as shown in fig, there is a region surrounding the plate surface where the fluid velocity changes from zero at the surface to the velocity u_∞ at the outer edge of the region. This region is called ***the velocity boundary layer***. The locus of the point where the velocity of the fluid is 99% of the stream velocity is called the boundary layer . In the region between the plate and boundary layer velocity varies from zero to 99% of stream velocity . and the region above the boundary layer the fluid have velocity equal to stream velocity. The variation of the x-component of velocity u(x,y) with respect to y at a particular location along the plate is shown in Fig

The distance measured normal to the surface from the plate surface to the point at which the fluid attains 99% of u_∞ is called “***velocity boundary layer thickness”*** and denoted by ***δ(x)***

Thus for flow over a flat plate, the flow field can be divided into two distinct regions, namely, (i) ***the boundary layer region*** in which the axial component of velocity u(x,y) varies rapidly with y with the result the velocity gradient (∂u /∂y) and hence the shear stress are very large and (ii) ***the potential flow region*** which is outside the boundary layer region, where the velocity gradients and shear stresses are negligible.

Drag coefficient and Drag force:- If the velocity distribution u(x,y) in the boundary layer at any 'x' is known then the viscous shear stress at the wall can be determined using Newton's law of viscosity. Thus if $\tau_w(x)$ is the wall-shear stress at any location x then

$$\tau_w(x) = \quad \mu\left(\frac{du}{dy}\right)_{y=0}$$

where μ is the absolute viscosity of the fluid.The drag coefficient is dimensionless wall shear stress.

The local drag coefficient, C_x at any 'x' is defined as

$$C_{fx}=\frac{\tau_w(x)}{\frac{1}{2}\rho U_\infty^2}$$

$$C_{fx}=\frac{\mu\left(\frac{du}{dy}\right)_{y=0}}{\frac{1}{2}\rho U_\infty^2}$$

$$C_{fx}=\frac{2v\left(\frac{du}{dy}\right)_{y=0}}{U_\infty^2}$$

Therefore if the velocity profile u(x,y) at any x is known then the local drag coefficient C_x at that location can be determined from Eq. 5.6.The average value of C_x for a total length L of the plate can be determined from the equation

$$\acute{C}_{fL}=\frac{1}{L} \quad \int_0^L C_{fx}\,dx$$

L

0

Substituting for C_x from Eq. 5.5 we have

$$C_{fx}=\frac{2v\left(\frac{du}{dy}\right)_{y=0}}{U_\infty^2}$$

$$\acute{C}_{fL}=\frac{\frac{1}{L}\int_0^L \tau_w(x)\,dx}{\frac{1}{2}\rho U_\infty^2}$$

$$\acute{C}_{fL}=\frac{\tau_w}{\frac{1}{2}\rho U_\infty^2}$$

L

Where τ_w is the average wall-shear stress for total length L of the plate.

The total drag force experienced by the fluid due to the presence of the plate can be written as

$$F_D = A_s\,\tau_w$$

Where A_s is the total area of contact between the fluid and the plate. If 'W' is the width of the plate then A_s = LW if the flow is taking place on one side of the plate and A_s = 2LW if the flow is on both sides of the plate.

Thermal boundary layer:-

Consider that a fluid at a uniform temperature T_∞ flows over a flat plate which is maintained at a uniform temperature T_w.Let T(x,y) is the temperature of the fluid at any location in the flow field.Let the dimensionless temperature of the fluid θ(x,y) be defined as

$$\theta(x,y)=\frac{T(x,y)-T_w}{T_\infty-T_w}$$

The fluid layer sticking to the plate surface will have the same temperature as the plate surface [$T(x,y)_{y=0} = T_w$] and therefore θ(x,y) = 0 at y = 0.Far away from the plate the fluid temperature is T_∞ and hence θ(x,y) → 1 as y → ∞. Therefore at each location x along the plate one can visualize a location y = $\delta_t(x)$ in the flow field at which θ(x,y) = 0.99. $\delta_t(x)$ is called ***"the thermal boundary layer thickness"*** as shown in Fig. 5.3. The locus of such points at which θ(x,y) = 0.99 is called the edge of the thermal boundary layer. The relative thickness of the thermal boundary layer $\delta_t(x)$

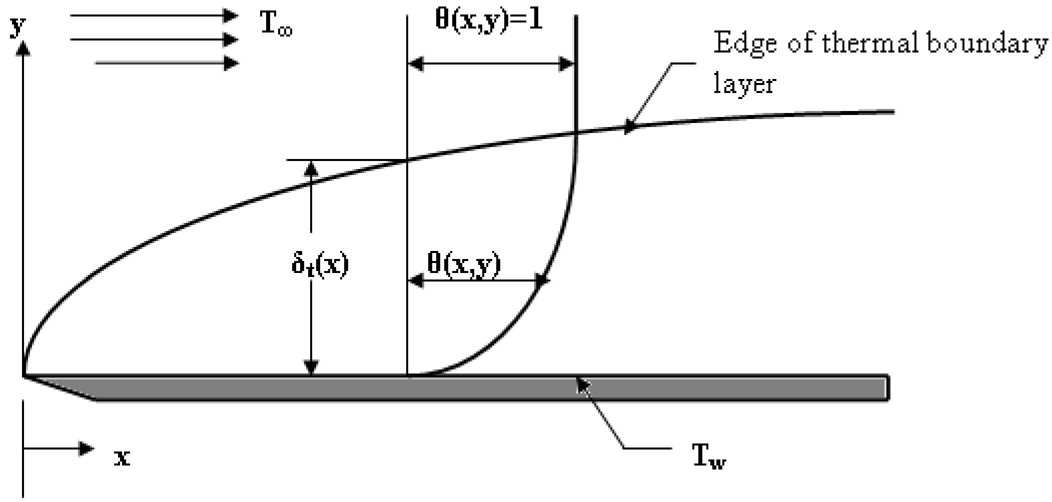


Fig. 5.4: Growth of thermal boundary layer for flow over a flat plate

The relative thickness of the thermal boundary layer $\delta_t(x)$ and the velocity boundary layer δ(x) depends on a dimensionless number called ***"Prandtl number"*** of the fluid.It is denoted by Pr The Prandtl number for fluids range from 0.01 for liquid metals to more than 100,000 for heavy oils. For fluids with Pr = 1   such as gases $\delta_t(x) = \delta(x)$, for fluids with Pr << 1such as liquid metals $\delta_t(x) >> \delta(x)$ and for fluids with Pr >> 1, like oils $\delta_t(x) << \delta(x)$.

Average heat transfer Coefficient

Consider laminar flow along flat plate and Nusselt number is given by

$$N_{ux} = 0.332 R_{ex}^{\frac{1}{2}} P_r^{\frac{1}{3}}$$

$$\frac{h_x x}{K} = 0.332 \left(\frac{U_\infty x}{\vartheta} \right)^{\frac{1}{2}} P_r^{\frac{1}{3}}$$

$$h_x = \frac{K}{x} 0.332 \left(\frac{U_\infty}{\vartheta} \right)^{\frac{1}{2}} x^{\frac{1}{2}} P_r^{\frac{1}{3}}$$

Average Heat transfer coefficient

$$h_m = \frac{1}{L} \int_0^L h_x \, dx$$

$$h_m = \frac{1}{L}\int_0^L K\,0.332\left(\frac{U_\infty}{\vartheta}\right)^{\frac{1}{2}} P_r^{\frac{1}{3}} \frac{x^{\frac{1}{2}}}{x}$$

$$h_m = 0.332\frac{1}{L} K\left(\frac{U_\infty}{\vartheta}\right)^{\frac{1}{2}} P_r^{\frac{1}{3}} \int_0^L \frac{x^{\frac{1}{2}}}{x}$$

$$h_m = 0.332\,K\,\frac{1}{L}\left(\frac{U_\infty}{\vartheta}\right)^{\frac{1}{2}} P_r^{\frac{1}{3}} \int_0^L x^{\frac{-1}{2}}$$

$$h_m = 0.332\,K\,\frac{1}{L}\left(\frac{U_\infty}{\vartheta}\right)^{\frac{1}{2}} P_r^{\frac{1}{3}} \left(\frac{x^{\frac{-1}{2}+1}}{\frac{-1}{2}+1}\right)_0^L$$

$$h_m = 0.332\,K\,\frac{1}{L}\left(\frac{U_\infty}{\vartheta}\right)^{\frac{1}{2}} P_r^{\frac{1}{3}}\, 2\,L^{\frac{1}{2}}$$

$$h_m = 0.664\,K\,\frac{1}{L}\left(\frac{U_\infty}{\vartheta}\right)^{\frac{1}{2}} P_r^{\frac{1}{3}}\, 2\,L^{\frac{1}{2}}$$

$$h_m = 0.664\frac{1}{L} K\left(\frac{U_\infty L}{\vartheta}\right)^{\frac{1}{2}} P_r^{\frac{1}{3}}$$

$$\frac{h_m L}{K} = 0.664\left(\frac{U_\infty L}{\vartheta}\right)^{\frac{1}{2}} P_r^{\frac{1}{3}}$$

$$\acute{N}_{uL} = 0.664\,R_{eL}^{\frac{1}{2}} P_r^{\frac{1}{3}}$$

$$\acute{N}_{uL} = 2*0.332\,R_{eL}^{\frac{1}{2}} P_r^{\frac{1}{3}}$$

$$\acute{N}_{uL} = 2\,N_{uL}$$

Hence Average Nusselt Number is equal to twice the local Nusselts Number at $x = L$

<u>Reynolds Analogy</u> Or $\left(\frac{C_x}{2} = S_{tx} P_r^{\frac{2}{3}}\right)$ $\frac{C_x}{2} = S_{tx} P_r^{\frac{2}{3}}$

Exact expression for local drag coefficient and local Nusselt number for laminar flow over plate

$$\frac{C_x}{2}=0.332\,R_{ex}^{\frac{-1}{2}}$$ ---------1

$$N_{ux}=0.332\,R_{ex}^{\frac{1}{2}}P_r^{\frac{1}{3}}$$ -------------2

By the definition of local Stanton Number $S_{tx}=\frac{h_x}{\rho\,C_p U_\infty}$

$$S_{tx}=\frac{h_x\frac{x}{K}}{\rho\,C_p U_\infty\frac{x}{K}\frac{\mu}{\mu}}$$

$$S_{tx}=\frac{h_x\frac{x}{K}}{\frac{\mu C_p}{K}\frac{\rho U_\infty x}{\mu}}$$

$$S_{tx}=\frac{N_{ux}}{P_r R_{ex}}$$ ------------3

Substituting 2 in3

$$S_{tx}=\frac{0.332\,R_{ex}^{\frac{1}{2}}P_r^{\frac{1}{3}}}{P_r R_{ex}}$$

$$S_{tx}=\frac{0.332\,R_{ex}^{\frac{-1}{2}}}{P_r^{\frac{2}{3}}}$$ --------------4

Substituting 1 in 4

$$S_{tx}=\frac{\frac{C_x}{2}}{P_r^{\frac{2}{3}}}$$

$$\frac{C_x}{2}=S_{tx}P_r^{\frac{2}{3}}$$

This expression is referred to as Reynolds Analogy which relates local drag coefficient and local Stanton number

In case of average values

$\frac{C_m}{2} = S_{tm} P_r^{\frac{2}{3}}$, where C_m and S_{tm} are the mean drag coefficient and mean Stanton Number respectively

Forced Convection VTU Questions

1. Using dimensional analysis obtain a relation between N_u, R_e and P_r for forced Convection heat transfer (5a, 10 June/July 2013)
2. Obtain an empirical expression in terms of dimensionless numbers for heat transfer coefficient in the case of forced convection heat transfer. (08,5a, June/July 2017)(Dec18/jan19)
3. Using dimensional analysis obtain a relation between N_u, R_e and P_r for forced Convection heat transfer (5a, 10 June/July 2013)
4. Obtain an empirical expression in terms of dimensionless numbers for heat transfer coefficient in the case of forced convection heat transfer. (08,5a, June/July 2017)(Dec18/jan19)
5. Using Bulkingham's $\pi - ¿$ theorem obtain the relationship between various dimensionless numbers $\left(N_u = \varnothing\left(P_r\right)\left(G_r\right)\right)$ for free convection heat transfer 8 Dec15/jan16, , June/July14,June/July 2018, June/July16

6. Using dimensional analysis show that for free convection heat transfer Nu=B $Gr^a Pr^b$ with usual notations 10 Dec16/Jan17

7. Explain the significance of Grashoff number 02 June/July 2017, Dec2018/Jan19

8. Explain the significance of following non-dimensional numbers i) Prandtl number ii) Grashoff number iii)Nasselt number iv) Stanton number v) Pecelt number 06 June/July 2015 , Dec14/Jan15, June/July 2018, June/July 2017

9. Explain the physical significance of the following dimensionless numbers i)Reynolds number ii) Prandtl number iii)Nusselt number iv) Stanton number (06, 8a, 15 scheme, June/July 18)(Dec15/Jan16) (june/July16)

Define stanton number and explain its physical significance (04, 5a, Dec14/Jan15

relation between N_u, R_e and P_r for forced Convection heat transfer

Heat transfer coefficient, h , depends on ρ, LV, μ, C_p, K

$$h = f\left(\rho, LV, \mu, C_p, K\right)$$

$$f\left(h, \rho, LV, \mu, C_p, K\right) = 0$$

No of variables n =7

No of fundamental dimension , m=4 ie M, L, T, θ

Hence number of π terms ¿ $n-m$

Hence number of π terms ¿ $7-4$ =3

Choose 3 repetitive variables ρ, L, V (fluid property, Geometry, dynamic)

$\pi_1 = \rho^{a1} L^{b1} \mu^{c1} K^{d1} \quad h$

$\pi_2 = \rho^{a2} L^{b2} \mu^{c2} \quad K^{d2} V$

$\pi_3 = \rho^{a3} L^{b3} \mu^{c3} \quad K^{d3} C_p$

$\rho = \frac{kg}{m^3}$ ¿ $\frac{M}{L^3} = M L^{-3}$; $L = L$; $V = \frac{m}{sec} = \frac{L}{T} = L T^{-1}; \mu = \frac{N-sec}{m^2}$ ¿ $\frac{ML T^{-2} T}{L^2}$
¿ $M L^{-1} T^{-1}$;

$h = \frac{W}{m^2 K} = \frac{Nm}{sec\, m^2 K} = \frac{ML T^{-2} L}{T L^2 \theta} = M T^{-2} \theta^{-1}$; $C_p = \frac{J}{kg-K} = \frac{Nm}{kgK} = \frac{ML T^{-2} L}{M\theta} = L^2 T^{-2} \theta^{-1}$;

$K = \frac{W}{mK} = \frac{Nm}{sec\, mK} = \frac{ML T^{-2} L}{T L\theta} = ML T^{-2} \theta^{-1}$

$\pi_1 = \rho^{a1} L^{b1} \mu^{c1} k^{d1} \quad h$

$M^0 L^0 T^0 \theta^0 = (M L^{-3})^{a1} L^{b1} (M L^{-1} T^{-1})^{C1} (ML T^{-2} \theta^{-1})^{d1} M T^{-2} \theta^{-1}$

Comparing the powers of θ in RHS and LHS

$0 = -d_1 - 1$; $d_1 = -1$

Comparing the powers of T in RHS and LHS

$0 = -C_1 - 2 d_1 - 2$; $0 = -C_1 - 2(-1) - 2$; $C_1 = 2 - 2$; $C_1 = 0$

Comparing the powers of M in RHS and LHS

$0 = a_1 + C_1 + d_1 + 1 ; 0 = a_1 + 0 + (-1) + 1$; $0 = a_1 - 0 - 1 + 1$; $a_1 = 0$

Comparing the powers of L in RHS and LHS

$0 = -3 a_1 + b_1 - C_1 + d_1 ;$ $0 = -3(0) + b_1 - 0 + (-1)$; $0 = 0 + b_1 - 0 - 1$; $b_1 = +1$

Hence $\pi_1 = \rho^0 L^1 \mu^0 \quad K^{-1} h$

$$\pi_1=\frac{hL}{K}=N_u$$

$$\pi_2=\rho^{a2}L^{b2}\mu^{c2} \quad K^{d2}V$$

$$M^0L^0T^0\theta^0=\left(ML^{-3}\right)^{a2}L^{b2}\left(ML^{-1}T^{-1}\right)^{C2}\left(MLT^{-2}\theta^{-1}\right)^{d2}LT^{-1}$$

Comparing the powers of θ in RHS and LHS

$$0=-d_2+0 \quad ; \quad d_2=0$$

Comparing the powers of T in RHS and LHS

$$0=-C_2-2d_2-1 \quad ;$$

$$0=-C_2-2(0)-1 \quad ;$$

$$C_2=0-1 \quad ;$$

$$C_2=-1$$

Comparing the powers of M in RHS and LHS

$$0=a_2+C_2+d_2+0;$$

$$0=a_2+(-1)+0+0 \quad ;$$

$$0=a_2-1+0+0 \quad ;$$

$$a_2=+1$$

Comparing the powers of L in RHS and LHS

$$0=-3a_2+b_2-C_2+d_2+1;$$

$$0=-3(1)+b_2-(-1)+0+1;$$

$$0=-3+b_2+1+0+1 \quad 0=-1+b_2$$

$$b_2=1$$

$$\pi_2=\rho^1L^1\mu^{-1} \quad K^0V$$

$$\pi_2=\frac{\rho LV}{\mu} \quad ; \quad \pi_2=R_e$$

$$\pi_3=\rho^{a3}L^{b3}\mu^{c3} \quad K^{d3}C_p$$

$$M^0 L^0 T^0 \theta^0 = (M L^{-3})^{a3} L^{b3} (M L^{-1} T^{-1})^{C3} (ML T^{-2} \theta^{-1})^{d3} L^2 T^{-2} \theta^{-1}$$

Comparing the powers of θ in RHS and LHS

$0 = -d_3 - 1$; $d_3 = -1$

Comparing the powers of T in RHS and LHS

$0 = -C_3 - 2d_3 - 2$;

$0 = -C_3 - 2(-1) - 1$;

$C_3 = 2 - 1$;

$C_3 = 1$

Comparing the powers of M in RHS and LHS

$0 = a_3 + C_3 + d_3 + 0;$

$0 = a_3 + 1 + (-1) + 0$;

$0 = a_3 + 1 - 1 + 0$;

$a_3 = 0$

Comparing the powers of L in RHS and LHS

$0 = -3a_3 + b_3 - C_3 + d_3 + 2$

$0 = -3(0) + b_3 - (1) + (-1) + 2$

$0 = 0 + b_3 - 1 - 1 + 2$

$b_3 = 0$

$\pi_3 = \rho^0 L^0 \mu^1 \quad K^{-1} C_p$

$\pi_3 = \frac{\mu C_p}{K}; \pi_3 = P_r$

$\pi_1 = f(\pi_2, \pi_3)$

$N_u = f(R_e, P_r)$

$N_u = C R_e^m P_r^n$

Natural Convection

Heat transfer coefficient, h , depends on $\rho, L, \beta g \Delta\theta, \mu, C_p, K$

$$h = f\left(\rho, L, \beta g \Delta\theta, \mu, C_p, K\right)$$

$$f\left(h, \rho, L, \beta g \Delta\theta, \mu, C_p, K\right) = 0$$

No of variables n =7

No of fundamental dimension , m=4 ie M, L, T, θ

Hence number of π terms ¿$n-m$

Hence number of π terms ¿$7-4$ =3

Choose 3 repetitive variables ρ, L, V (fluid property, Geometry, dynamic)

$$\pi_1 = \rho^{a1} L^{b1} \mu^{c1} K^{d1} \quad h$$

$$\pi_2 = \rho^{a2} L^{b2} \mu^{c2} \quad K^{d2} \beta g \Delta\theta$$

$$\pi_3 = \rho^{a3} L^{b3} \mu^{c3} \quad K^{d3} C_p$$

$\rho = \frac{kg}{m^3}$ ¿ $\frac{M}{L^3} = M L^{-3}$; $L = L$; $\beta g \Delta\theta = \frac{1}{K} \frac{m}{s^2} K = \frac{L}{T^2} = L T^{-2}; \mu = \frac{N-sec}{m^2}$

¿ $\frac{ML T^{-2} T}{L^2}$ ¿ $M L^{-1} T^{-1}$;

$h = \frac{W}{m^2 K} = \frac{Nm}{sec\, m^2 K} = \frac{ML T^{-2} L}{T L^2 \theta} = M T^{-2} \theta^{-1}$; $C_p = \frac{J}{kg-K} = \frac{Nm}{kgK} = \frac{ML T^{-2} L}{M\theta} = L^2 T^{-2} \theta^{-1}$;

$$K = \frac{W}{mK} = \frac{Nm}{sec\, mK} = \frac{ML T^{-2} L}{T L\theta} = ML T^{-2} \theta^{-1}$$

$$\pi_1 = \rho^{a1} L^{b1} \mu^{c1} k^{d1} \quad h$$

$$M^0 L^0 T^0 \theta^0 = \left(M L^{-3}\right)^{a1} L^{b1} \left(M L^{-1} T^{-1}\right)^{C1} \left(ML T^{-2} \theta^{-1}\right)^{d1} M T^{-2} \theta^{-1}$$

Comparing the powers of θ in RHS and LHS

$0 = -d_1 - 1$; $d_1 = -1$

Comparing the powers of T in RHS and LHS

$0=-C_1-2d_1-2$; $0=-C_1-2(-1)-2$; $C_1=2-2$; $C_1=0$

Comparing the powers of M in RHS and LHS

$0=a_1+C_1+d_1+1; 0=a_1+0+(-1)+1$; $0=a_1-0-1+1$; $a_1=0$

Comparing the powers of L in RHS and LHS

$0=-3a_1+b_1-C_1+d_1;$ $0=-3(0)+b_1-0+(-1)$; $0=0+b_1-0-1$; $b_1=+1$

Hence $\pi_1=\rho^0 L^1 \mu^0$ $K^{-1}h$

$$\pi_1=\frac{hL}{K}=N_u$$

$$\pi_2=\rho^{a2} L^{b2} \mu^{c2} \quad K^{d2} \beta g \Delta \theta$$

$$M^0 L^0 T^0 \theta^0=(ML^{-3})^{a2} L^{b2} (ML^{-1}T^{-1})^{C2} (MLT^{-2}\theta^{-1})^{d2} LT^{-2}$$

Comparing the powers of θ in RHS and LHS

$0=-d_2+0$; $d_2=0$

Comparing the powers of T in RHS and LHS

$0=-C_2-2d_2-2$;

$0=-C_2-2(0)-2$;

$C_2=0-2$;

$C_2=-2$

Comparing the powers of M in RHS and LHS

$0=a_2+C_2+d_2+0;$

$0=a_2+(-2)+0+0$;

$0=a_2-2+0+0$;

$a_2=+2$

Comparing the powers of L in RHS and LHS

$0=-3a_2+b_2-C_2+d_2+1;$

$0=-3(2)+b_2-(-2)+0+1;$

$0=-6+b_2+2+0+1 \quad 0=-3+b_2$

$b_2=+3$

$\pi_2=\rho^2 L^{+3} \mu^{-2} \quad K^0 \beta g \Delta \theta$

$\pi_2=\frac{\rho^2 L^3 \beta g \Delta \theta}{\mu^2} \quad ; \quad \pi_2=G_r$

$\pi_3=\rho^{a3} L^{b3} \mu^{c3} \quad K^{d3} C_p$

$M^0 L^0 T^0 \theta^0=\left(M L^{-3}\right)^{a3} L^{b3}\left(M L^{-1} T^{-1}\right)^{C3}\left(ML T^{-2} \theta^{-1}\right)^{d3} L^2 T^{-2} \theta^{-1}$

Comparing the powers of θ in RHS and LHS

$0=-d_3-1 \quad ; \quad d_3=-1$

Comparing the powers of T in RHS and LHS

$0=-C_3-2 d_3-2 \quad ;$

$0=-C_3-2(-1)-1 \quad ;$

$C_3=2-1 \quad ;$

$C_3=1$

Comparing the powers of M in RHS and LHS

$0=a_3+C_3+d_3+0 ;$

$0=a_3+1+(-1)+0 \quad ;$

$0=a_3+1-1+0 \quad ;$

$a_3=0$

Comparing the powers of L in RHS and LHS

$0=-3 a_3+b_3-C_3+d_3+2$

$0=-3(0)+b_3-(1)+(-1)+2$

$0=0+b_3-1-1+2$

$b_3=0$

$$\pi_3=\rho^0 L^0 \mu^1 \quad K^{-1} C_p$$

$$\pi_3=\frac{\mu C_p}{K}; \pi_3=P_r$$

$$\pi_1=f(\pi_2, \pi_3)$$

$$N_u=f(G_r, P_r)$$

$$N_u=C G_r^m P_r^n$$

Dimensionless Numbers involved in Convection

Prandtl Number:

It is defined as the ratio of Kinematic viscosity to thermal diffusivity

$$Pr=\frac{\vartheta}{\alpha}; Pr=\frac{\frac{\mu}{\rho}}{\frac{K}{\rho C_p}} \quad ; \quad Pr=\frac{\mu C_p}{K}$$

Pradtl Number is the parmeter which relates the thickness of hydraodynaamic and thermal boundary layer

If Pr=1 then $\delta_t(x)=\delta_h(x)$

If $Pr<1$ then $\delta_t(x)>\delta_h(x)$

If $Pr>1$ then $\delta_t(x)<\delta_h(x)$

Nusselt Number: is the ratio of convective heat transfer to conductive heat transfer across the boundary

$$N_u=\frac{hA\Delta T}{\frac{KA\Delta T}{L}} \quad ; \quad N_u=\frac{hL}{K}$$

Nusselt number close to 1 indicates Heat transfer of similar magnitude which is a charecterstic of slug flow or laminar flow (Range 1-10)

A large Nusselt Number corresponds to more active convection with turbulent flow (Range of 100 to 1000)

Reynolds Number: It is the ratio of Inertia force to viscous force within the fluid which is subjected to relative internal movement due to different fluid velocities

$$\rho AV L^2$$

$$R_e = \frac{Inertia\,force}{Viscous\,force} \quad ;$$

$$R_e = \frac{mass * Accerlation}{shear\,stress * Area}; R_e = \frac{ma}{\tau * A} \quad ; R_e = \frac{\rho U_\infty L}{\mu}$$

Reynold number is an important parameter to determine whether flow is laminar or turbulent which is very much essential in designing pipe

In flow over the plate if Reynolds number is less than $5x10^5$ flow is laminar

If Re $¿ 5\,x\,10^5$ then flow is turbulent

In flow through tube if Re $¿ 5\,x\,10^5$ flow is laminar if Re $¿ 2300\,flow\,is\,turbulent$

Grashoff Number: It is dimensionless number which approximates the ratio of the buoyancy force to viscous force acting on fluid. It involved in the study of situations invoving matural convection and is analogous to Reynold Number

$$G_r = \frac{\rho^2 L^3 \beta g\, \Delta\theta}{\mu^2}$$ where

$\rho\,is\,the\,density\,, L\,is\,the\,Charecterstic\,length\,, \beta\,is\,coefficient\,of\,thermal\,expansion\,, g\,is\,the\,acceraltion$

due to gravity , $\Delta\theta$ is the temperature difference between the surface and fluid over which it flows, μ is absolute viscosity

Rayleigh Number : is the product of Grshof Number and Prandtl Number . It determines whether the flow is laminar and turbulent in natural convection Heat transfer

$$R_a = G_r P_r \quad ; \quad R_a = \frac{\frac{\rho^2 L^3 \beta g\, \Delta\theta}{\mu^2} * \mu C_p}{K} \quad ; \quad R_a = \frac{\rho^2 L^3 \beta g\, \Delta\theta\, C_p}{\mu K}$$

$\rho\,is\,the\,density\,, L\,is\,the\,Charecterstic\,length\,, \beta\,is\,coefficient\,of\,thermal\,expansion\,, g\,is\,the\,acceraltion$

due to gravity , $\Delta\theta$ is the temperature difference between the surface and fluid over which it flows, μ is absolute viscosity , C_p is the specific heat of fluid, K is thermal conductivity of material .

<u>**Stanton Number**</u>: It is the ratio of heat transferred into the fluid to thermal capacity of the fluid

$$S_{tx} = ¿\,the\,fluid\,\frac{Heat\,transferred\,¿}{thermal\,capacity\,of\,fluid}$$

$$S_{tx}=\frac{hA\Delta T}{mC_p\Delta T} \quad ; \quad S_{tx}=\frac{hA\Delta T}{\rho A U_\infty C_p \Delta T} \quad ; \quad S_{tx}=\frac{h}{\rho U_\infty C_p}$$

$$S_{tx}=\frac{h\frac{x}{K}}{\rho U_\infty C_p \frac{x}{K}\frac{\vartheta}{\vartheta}}$$

$$S_{tx}=\frac{h\frac{x}{K}}{\frac{U_\infty x}{\vartheta}\frac{\rho\vartheta C_p}{K}}$$

$$S_{tx}=\frac{h\frac{x}{K}}{\frac{U_\infty x}{\vartheta}\frac{\mu C_p}{K}}$$

$$S_{tx}=\frac{N_{ux}}{R_{ex}P_r}$$

Peclet Number: It is defined as the ratio of the thermal energy convected to the fluid to the thermal energy conducted within the fluid_

$$P_e=\frac{Heat\,transfer\,by\,convection}{Heat\,transfer\,by\,conduction}$$ within the fluid

It is the product of Reynold Number and Prandtl Number

$$P_e=R_e P_r$$

$$P_e=\frac{\rho U_\infty L}{\mu}\frac{\mu C_p}{K} \quad ;$$

$$P_e=\frac{\rho C_p U_\infty L}{K}$$

1. The exact expression for local Nusselt number for the laminar flow along a surface is given by

$$Nu_x=\frac{h_x x}{k}=0.332 R_{ex}^{\frac{1}{2}} P_r^{\frac{1}{3}}$$

Show that the average heat transfer coefficient from x= 0 to x =L over the length L of the surface is given by $2h_L$ is the local heat transfer coefficient at x=L 05 Dec13/Jan14

2. An approximate expression for temperature profile in thermal boundary layer is given by

$$\frac{T_{(x,y)}-T_w}{T_\infty - T_w} = \frac{3}{2}\frac{y}{\delta_t(x)} - \frac{1}{2}\left[\frac{y}{\delta_t(x)}\right]^3 \text{, where } \delta_t(x) = 4.53\frac{x}{R_{ex}^{\frac{1}{2}} P_r^{\frac{1}{3}}}$$

3. Develop an expression for local heat transfer coefficient $h_{(x)}$ 4b June/July13 06

4. A fan provides a speed upto 50 m/ s is used in low speed wind tunnel with atmospheric air at 27 °C.If this wind Tunnel is used to study the boundary layer behavior over a flat plate upto are $R_e=10^8$. What should be the minimum length?. At what distance from the leading edge would transition occur, if critical Reynolds number $R_{ecr}=5x10^5$? 4b, 08 Dec17/Jan18
5. Calculate the approximate Reynold numbers and state if the flow is laminar or turbulent for the following
i) A 10 m long yatch sailing at 13 km per hour in sea water, ρ ¿ 1000 kg/m³ and $\mu=$¿ 1.3 x10⁻³kg/ms
ii) A compressor disc of radius 0.3 m rotating at 15000 RPM in air at 5 bar and 400 °C

$$¿\mu=\frac{1.46\,x\,10^{-6}T^{\frac{3}{2}}}{(110+T)}$$ kg/ms 4c, 08 Dec17/Jan18

6. A fan provides a speed upto 50 m/ s is used in low speed wind tunnel with atmospheric air at 27 °C.If this wind Tunnel is used to study the boundary layer behavior over a flat plate upto are $R_e=10^8$. What should be the minimum length?. At what distance from the leading edge would transition occur, if critical Reynolds number $R_{ecr}=5x10^5$? 4b, 08 Dec17/Jan18

7.

Forced Convection PROBLEMS

1. A plate of length 750 mm and width 250 mm has been placed longitudinally in a stream of crude oil which of which flows with a velocity of 5m/s. If the oil has a specific gravity of 0.8 and kinetic viscosity of 10^{-4} m² /s. Calculate i) boundary layer thickness at the middle of the plate ii) Shear stress at the middle of the plate iii) friction drag on one side of the plate 4b,06 Dec14/Jan15 06
Note: $\rho = 1000 * sp\,gravity = 1000*0.8 = 800\,kg/m^3$
Calculate i) boundary layer thickness at the middle of the plate ii) Shear stress at the middle of the plate
Since question has been asked at the middle
Take $x=\frac{0.750}{2}=0.325\,m\,for\,reynolds\,number,thickness\,of\,boundary\,layer\,cal\,culation$

For iii) sub question friction Drag force
Entire plate is to be considered
Here Reyonold number to be calculated by taking x =0.750
Then calculate $\acute{C}_{fL}=1.328\, R_{ex}^{-0.5}$

$Average\, shear\, stress\, \tau_{av}=\acute{C}_{fL}\frac{1}{2}\rho U_{\infty}^{2}$

$Total\, Drag\, Force=Average\, shear\, stress*Area$ where Area $¿\, w*L$
L=0.750m

2. Air at 20 ºC flows over thin plate with a velocity of 3 m/sec. The plate is 2 m long and 1 m wide. Estimate the boundary layer thickness at the trailing edge of the plate and the total drag force experienced by the plate. (4c, 06, June/July16)

3. Air at 30 ºC and atmospheric pressure is flowing over a flat plate at a velocity 3 m/s, plate is 30 cm wide and at a temperature of 60 ºC ,calculate at X= 0.3m
i) thickness of velocity and thermal boundary layer
ii) local and average friction coefficients
iii)local and average heat transfer coefficients
iv) total drag force on the plate 4b, 10, June/July2018
$T_{\infty}=30^{o}C\,;T_{s}=60^{o}C$; $U_{\infty}=3\,m/s$
At $x=0.3\,m$ i) $\delta_{h}=?$; $\delta_{t}=?$ ii) $C_{fx}=?$ $\acute{C}_{fL}=?$ iii) $h_{x=0.3}$=?, $h_{average}$=?

iii) Total Drag force=?

Solution

$$T_{f}=\frac{T_{s}+T_{\infty}}{2}=\frac{60+30}{2}=45^{o}C$$

$$\rho=\frac{1.128+1.093}{2}=1.1105\,kg/m^{3} \quad ; \quad \gamma=\frac{16.96+17.95}{2}\,x\,10^{-6}=17.455\,x\,10^{-6}\,m^{2}/s$$

$$P_{r}=\frac{0.669+.698}{2}=0.6835 \quad ; \quad k=\frac{0.02756+0.02826}{2}=0.02791\,W/mK$$

$$R_{ex}=\frac{U_{\infty}x}{\gamma}$$; At x=0.3m $$R_{ex}=\frac{3*0.3}{17.455*10^{-6}}=0.5156\,x\,10^{5}$$

$Since\, R_{ex}<5\,x\,10^{5}$ flow over the plate along width of 0.3 m is laminar

$i¿\,\delta_{hx}=5\,x\,R_{ex}^{-0.5}$ from data Hand Book Page number

At x=0.3m

$$\delta_{hx}=5*0.3*\left(0.51\,x\,10^{5}\right)^{-0.5} \quad ; \quad \delta_{hx}=6.605*10^{-3}\,m$$

Thickness of Hydrodynamic boundary layer thickness at x=0.3m is $6.605\,mm$

$$\delta_{Tx}=\delta_{hx}P_{r}^{-0.333} \quad ; \quad \delta_{Tx}=6.605*10^{-3}*0.6835^{-0.3333} \quad ; \quad \delta_{Tx}=7.498*10^{-3}\,m$$

Thermal Boundary layer thickness at x=0.3m is $7.498\,mm$

ii) Local Drag coefficient and average Drag coefficient

$C_{fx}=0.664\,R_{ex}^{-0.5}$; $C_{fx}=0.664*\left(0.5156\,x\,10^{5}\right)^{-0.5}$; $C_{fx}=2.924*10^{-3}$

$\acute{C}_{fL}=1.328\,R_{ex}^{-0.5}$; $\acute{C}_{fL}=1.328*\left(0.5156\,x\,10^{5}\right)^{-0.5}$; $\acute{C}_{fL}=5.84846*10^{-3}$

iii) Local Heat transfer coefficient at x=0.3m and average heat transfer coefficient

$Local\ Nusselt\ Number\ N_{ux}=0.332\,R_{ex}^{0.5}\,P_{r}^{0.333}$

$At\ x=0.3\,m$, $N_{ux}=0.332*¿$; $N_{ux}=66.4$ 1

$N_{ux}=\frac{h_x x}{K};$ $66.41=\frac{h_x*0.3}{0.02791}$; $h_x=6.178\,W/m^2K$

Local Heat Transfer coefficient at x=0.3m is $6.178\,W/m^2K$

Average heat transfer coefficient over width x=0.3 = 2 h_x since $\acute{N}_u=2\,N_{ux}$

Hence Average Heat transfer coefficient = $2*6.178=12.357\,W/m^2K$

iv)Total Drag force

$Average\ shear\ stress\ \tau_{av}=\acute{C}_{fL}\frac{1}{2}\rho U_{\infty}^{2}$

$\tau_{av}=5.84846*10^{-3}\frac{1}{2}$ * $1.1105*3^2$; $\tau_{av}=0.0292\,N/m^2$

$Total\ Drag\ Force=Average\ shear\ stress*Area$ where Area $¿\,w*L$

$F=0.0292*(0.3*1)$ Length is assumed as 1m

4. Air at 20 ᵒC and at a pressure of 1 bar is flowing over a flat plate at a velocity of 3m/s, if the plate is 280 mm wide and 56ᵒC. Calculate the following quantities x =280 mm, given that the properties of air at bulk mean temperature 38 ᵒC are $\rho=¿$ 1.1374 kg /m³ ,K =0.02732 W/mᵒC , $C_p=¿$ 1.005 kJ/ kg K, $\gamma=16.768\,x\,10^{-6}$, P_r =0.7. Determine i) boundary layer thickness ii) thickness of thermal boundary layer iii) local heat transfer Coefficient iv) Average convective heat transfer coefficient v) Rate of heat transfer by Convection vi) total drag force on the plate 4b,12 Dec15/jan16
5. Air at 27 ᵒC and 1 atmosphere pressure flows over a heated plate with a velocity of 2 m/s. The plate is at uniform temperature of 60 ᵒC. Calculate the heat transfer rate from first 0.2m of the plate (08, 5b, Dec17/Jan2018)
6. Air at a temperature of 20 ᵒC ,flows over a flat plate 3m/s. The plate is 50 cmx25 cm. Find the heat lost per hour if the air flow is parallel to 50 cm side of the plate. If 25cm side is kept parallel to the air flow, what will be the effect on heat transfer? Temperature of the plate is 100 ᵒC (5c,08, 5c, Dec14/Jan15)
7. Atmospheric air at 275 K and free stream velocity 20 m/s flows over a flat plate of length 1.5 m long maintained at 325 K. Calculate

i) The average heat transfer Coefficient over the region where the boundary layer is laminar

ii) Find the average heat transfer over the entire length 1.5 m of the plate.

iii) Calculate the total heat transfer rate from the plate to air over the length of 1.5 m and width 1 m, assume transition occurs at Reynold number $2x10^5$. Take air properties at mean Temperature of 300K

K=0.026W/m °C, P_r=0.708 γ =$16.8x10^{-6}$ m^2/s, $\mu=1.98\,x\,10^{-5} kg/ms$ (12, 5c, June/July14)

Length up to which flow is laminar

$Critical\,Reynold\,number\; R_{ec}=\frac{U_\infty L_c}{\gamma}$; At x=1.5m

$2*10^5=\frac{20*L_c}{16.8*10^{-6}};$ $L_c=0.168\,m$; ie upto 0.168 flow is laminar remaining length flow is turbulent

Average heat transfer coefficient over laminar region

$$Average\;Nusselt\;Number=2\,N_{ux}=2*0.332\,R_{ex}^{0.5}\,P_r^{0.333}$$

$$Average\;Nusselt\;Number=0.664\,R_{ex}^{0.5}\,P_r^{0.333}$$

$$N_u=0.664*\left(2\,x\,10^5\right)^{0.5}0.708^{0.3333} \;;\quad N_u=264.69$$

$$264.69=\frac{h*0.168}{0.026}; \;;\quad h'_{laminar}=40.96\,W/m^2K$$

Average heat Transfer coefficient for entire length (ie Laminar region + Turbulent Region)

$\acute{N}_{uL}=P_r^{0.333}\left(0.037\,R_{eL}^{0.8}-A\right)$ where $A=0.037\,R_{cr}^{0.8}-0.664\,R_{cr}^{0.5}$

$$A=0.037\left(2*10^5\right)^{0.8}-0.664\left(2\,x\,10^5\right)^{0.8} \;;\quad A=347.26$$

$$R_{eL}=\frac{U_\infty L}{\gamma};R_{eL}=\frac{20*1.5}{16.8*10^{-6}};\quad R_{eL}=1.786*10^5$$

$$\acute{N}_{uL}=0.708^{0.3333}\left(0.037\left(1.786\,x\,10^6\right)^{0.5}-347.26\right) \;;\quad \acute{N}_{uL}=3000$$

$$\acute{N}_{uL}=\frac{\acute{h}\,L}{K};\quad 66.41=\frac{\acute{h}*1.5}{0.026};\quad \acute{h}=52\,W/m^2K$$

Total Heat Transfer

$$Q=hA\left(T_s-T_\infty\right) \;;\quad Q=52*(1.5*1)\left[325-275\right] \;;\quad Q=3900\,Watts$$

8. Air at 20 °C flows past 800 mm long plate at velocity of 45 m/s. If the surface of the plate is maintained at 300°C, determine i) the heat transfer from the entire plate length to air taking into consideration both laminar and turbulent portion of the boundary layer ii) the percentage error if the boundary layer is assumed to be turbulent nature from the very leading edge of the plate . Assume unit width of the plate and critical Reynolds number to be $5x10^5$.(12, 5b, June/July16)

$$T_\infty = 20^o C ; T_s = 300^o C \quad ; \quad U_\infty = 45\, m/s$$

$$T_f = \frac{T_s + T_\infty}{2} = \frac{300+20}{2} = 160^o C$$

$$\gamma = 30.09\, x\, 10^{-6} m^2/s \quad ; \quad P_r = 0.682 \quad ; \quad k = 0.03640\, W/mK$$

$Critical\ Reynold\ number\ R_{ec}\ for\ exteranl\ flow\ is \quad 5\, x\, 10^5$

$$Critical\ Reynold\ number\ R_{ec} = \frac{U_\infty L_c}{\gamma} \qquad 5\, x\, 10^5 = \frac{45 * L_c}{30.09\, x\, 10^{-6}} ; L_c = 0.334\, m$$

Since length is 800 mm ie 0.8m greater than critical length flow is laminar turbulent
Hence from data hand book Page number

$\acute{N}_{uL} = P_r^{0.333}\left(0.037\, R_{eL}^{0.8} - 871\right)$ if $Critical\ Reynold\ number\ R_{ec}\ for\ exteranl\ flow\ is$ 5 $x\, 10^5$

$$R_{eL} = \frac{U_\infty L}{\gamma} ; R_{eL} = \frac{45 * 0.8}{30.09\, x\, 10^{-6}} ; \qquad R_{eL} = 1.196 * 10^6$$

$$\acute{N}_{uL} = 0.682^{0.3333}\left(0.037\left(1.196\, x\, 10^6\right)^{0.8} - 871\right) \quad ; \quad \acute{N}_{uL} = 1604.79$$

$$\acute{N}_{uL} = \frac{\acute{h} * L}{K} ; \qquad 1604.79 = \frac{\acute{h} * 0.8}{0.03640} ; \qquad \acute{h} = 73.01\, W/m^2 K$$

Total Heat Transfer

$$Q_1 = hA\left(T_s - T_\infty\right) \quad ; \quad Q_1 = 73.01 * (0.8 * 1)[300 - 20] \quad ; \quad Q_1 = 16356.11\, Watts$$

Case 2:
Assuming Boundary layer assumed to be turbulent over entire length
Page No 113 /equation 1.4.1

$$\acute{N}_{uL} = 0.037\, R_{eL}^{0.8} P_r^{0.333}$$

$$\acute{N}_{uL} = 0.037 * \left(1.196\, x\, 10^6\right)^{0.8} * 0.682^{0.3333} \quad ; \quad \acute{N}_{uL} = 2371.57$$

$$\acute{N}_{uL} = \frac{\acute{h} * L}{K} ; \qquad 2371.57 = \frac{\acute{h} * 0.8}{0.03640} ; \qquad \acute{h} = 107.90\, W/m^2 K$$

Total Heat Transfer

$$Q_2 = hA\left(T_s - T_\infty\right) \quad ; \quad Q_2 = 107.90 * (0.8 * 1)[300 - 20] \quad ; \quad Q_2 = 24171.09\, Watts$$

Percentage by considering the plate is turbulent nature from the leading edge

$$\frac{Q_1 - Q_2}{Q_1}\, x\, 100 \quad ; \quad \frac{16356.11 - 24171.09}{16356.11}\, x\, 100 \quad ; \quad -47$$

9. Atmosphere air at 275K and free stream velocity of 20 m/s flows over a long flat plate maintained at uniform temperature of 325K, calculate i) Average heat transfer coefficient over the region of the laminar boundary layer ii) average heat transfer coefficient over the entire length of 1.5 m iii) Total heat transfer coefficient over the entire length of 1.5 m (5b, 10, Dec18/Jan19)

10. Air flows over a flat plate at 30°C , 0.4m wide and 0.75m long with a velocity of 20 m/s . Determine the heat transfer from the surface of the plate assuming plate is maintained at 90 °C

Use $N_{uL}=0.664\,R_e^{0.5}\,P_r^{0.33}$ for laminar

$N_{uL}=\left(0.036\,R_e^{0.8}-0.836\right)P_r^{0.333}$ for turbulent (7b. 15 scheme June/July 2018 08),

June/July 2013

11. Air at velocity of 3 m/s and at 20 °C flows over a flat plate along its length. The length width and thickness of the plate are 100 cm, 50 cm and 2 cm respectively. The top surface of the plate is maintained at 100 °C. Calculate the heat lost by the plate and temperature of bottom surface of the plate for the steady state conditions. The thermal conductivity of the plate may be taken as 23 W/mK

(08, 5c, Dec17/Jan2018)

12. The velocity of water flowing through a tube of 2.2 cm dia is 2m/s. Steam condensing at 150°C on the surface of the tube heats the water from 15°C to 60°C over the length of tube. Find the heat transfer coefficient and the length of the tube neglecting the tube and steam side film resistance. Take the following properties of water at mean temperature : $\rho=850\,kg/m^3$

$C_p=2000\,J/kg^oC$ $\gamma=5.1\,x\,10^{-6}\,m^2/sec$ K=0.2W/ m^oC

INTERNAL FLOW

Critical Reynold number is 2300 which determines the flow is laminar or turbulent unless and until it is specified in the problem

Characteristic length is Diameter

$$R_{eD}=\frac{U_\infty D}{\gamma}$$

13. A tube 5m long is maintained at 100 °C by steam jacketing. A fluid flows through the tube at the rate of 175 kg/hr at 30 °C . The diameter of the tube is 2 cms. Find out the average heat transfer coefficient. Take the following properties of the fluid

$\rho=850\,kg/m^3$, C_p=2000J/kg°C, γ =5.1x10^{-6} m^2/s, K=0.2W/m°C, (10,5b, June/July17)

$$R_{eD}=\frac{U_\infty D}{\gamma};$$

If mass flow rate is given please note down to calculate the velocity of fluid as follows

$m=\rho A_f U_\infty$ where m is the mass flow rate in kg/s , A_f is the area of flow ,

$U_\infty = the\,velocity of\,fluid\,inside\,the\,tube$

$m=175\,kg/hr$ $\quad m=\frac{175}{3600}$ kg/s $\quad ;A_f=\frac{\pi ¿ 0.02^2}{4}$

Hence, $\frac{175}{3600}=\frac{850*\pi ¿ 0.02^2}{4}*U_\infty$; $U_\infty=0.1820\,m/s$

$R_{eD}=\frac{0.1820*0.02}{5.1*10^{-6}}$; $R_{eD}=713.88$

Since $R_{eD}<2300$ flow is laminar

From Page 114 equation 1.2.1 for circular tube constant wall temperature $N_u=3.66$

$N_u=\frac{\acute{h}*D}{K}$; $\quad 3.66=\frac{\acute{h}*0.02}{0.2}$; $h=36.6\,W/m^2K$

If rate of heat transfer asked

$Q=h\,A_{HT}\left(\left(T_s-T_\infty\right)\right)$

14. Lubricating oil at a temperature of 60 °C enters a 1 cm diameter tube with a velocity of 3.5 m/ s. The tube surface is maintained at 30 °C. Calculate the tube length required to cool the oil to 45 °C. Assume that the oil has the following average properties for the temperature range of this problem $\rho=¿$ 865kg/m^3 , K=0.14 W/mK , Cp=1.78kJ/kgK, and $\gamma=¿$ 9x10^{-6} m^2/s (08 ,Dec18/Jan19, 7b, 15 scheme)
15. Water flows at a velocity of 12 m/ s in a straight tube of 60 mm diameter. The tube surface temperature is maintained at 70 °C and the flowing water is heated from the inlet temperature of 15 °C to an outlet temperature of 45 °C. Taking the principal properties of water at the mean bulk temperature of 30 °C as $\rho=995.7\,kg/m^3$, C_p=4.174kJ/kgK ,K=0.61718W/mK , γ =0.805x10^{-6} m^2/s, and P_r=5.42 . calculate i)heat transfer Coefficient from the tube surface to the water.ii) the heat transferred iii) the length of the tube (10,5b, Dec16/Jan17)

$$R_{eD}=\frac{U_\infty D}{\gamma};R_{eD}=\frac{12*0.06}{0.0805*10^{-6}} \quad ; \quad R_{eD}=8.944*10^5$$

$R_{eD}>2300$, Hence the flow is turbulent

From Data Hand Book Page number equation

$N_{uD}=0.023 R_{eD}^{0.8} P_r^n$ wher n=0.3 for cooling of fluid n=0.4 for heating of fluid

In the problem water is heated from 15^0C to 45^0C Hence n=0.4

$$N_{uD}=0.023 R_{eD}^{0.8} P_r^{0.4} \quad ; \quad N_{uD}=0.023\left(8.944*10^5\right)^{0.8}*5.42^{0.3333} \quad ; \quad N_{uD}=609.47$$

$$N_{uD}=\frac{\acute{h}*D}{K} \quad ; \quad 609.47=\frac{\acute{h}*0.06}{0.61718} \quad ; \quad h=26841\,W/m^2K$$

$$Q=h\,A_{HT}\,LMTD$$

$$LMTD=\frac{\theta_i-\theta_0}{\ln\frac{\theta_i}{\theta_0}}$$; where $\theta_i=T_s-T_{wi}=70-15=55^oC$; $\theta_0=T_s-T_{wo}=70-45=25^oC$

$$LMTD=\frac{\theta_i-\theta_0}{\ln\frac{\theta_i}{\theta_0}}; LMTD=\frac{55-25}{\ln\frac{55}{25}} \quad ; \quad LMTD=38.04^0C$$

$$A_{HT}=\pi DL=\pi*0.06*L$$

$$Also\ Rate\ of\ Heat\ transfer=Heat\ gained\ by\ water\ ie\ Q=m_w C_{pw}\left(T_{wo}-T_{wi}\right)$$

$$m_w=\rho A_f U_\infty=\rho\frac{\pi D^2}{4}U_\infty; m_w=995.7\left(\frac{\pi\,0.06^2}{4}\right)12$$ kg/s

$$Q=995.7\left(\frac{\pi\,0.06^2}{4}\right)12*4.174*\left(45-15\right) \quad ; \quad Q=4.23*10^6\,Watts$$

$$4.23*10^6=26841*\left(\pi*0.06*L\right)*38.04 \quad ; \quad L=21.97\,m$$

16. Air at 2 atm and 200°C is heated as it flows at a velocity of 12m/s through a tube with a diameter of 3cm. A constant heat flux condition is maintained at the wall and the wall temperature is 20°C above the air temperature all along the lenth of the tube . Calculate
    i) The heat transfer per unit length of tube
    ii) The increase in bulk temperature of air over a 4m length of the tube.

Take the following properties of air Pr=0.681 , $\mu=¿$ 2.57 $x\,10^{-5}\,kgm/s$ K=0.0396W/mK and $C_p=1.025\,kJ/kgK$ (Dec13/Jan14)

17. Hot air at atmospheric pressure and 80 °C enters an 8m long uninsulated square duct of cross section 0.2 m x 0.2m that passes through the attic of a house at a rate of 0.15 m^3/s. The duct is observed to be nearly isothermal at 60 °C. Determine the exit temperature of the air and the rate of heat loss from the duct to attic space (10,5c,June/July2015)

18. Consider air at atmospheric pressure and 100 ^oC enters a 2m long tube of 4 cm diameter with a velocity of 9 m/s. A 1 KW electric heater is wound on the outer surface of the tube, find i) Exit temperature of air ii) mass flow rate of air iii) wall temperature. Assume that the rate of heat absorption by air per unit area is uniform throughout length of the tube(10, 5b, June/July2018)
19. A refrigerated truck is moving on a highway 90 km/hr in desert area, where the ambient air temperature is 50 ^oC. The body of the truck is a rectangular box measuring 10m(length) x 4m (width)x3m(Height) . Assume that the boundary layer on the four walls is turbulent. The heat transfer takes only from the four surfaces and the wall surfaces of the truck is maintained at 10^oC. Neglecting the heat transfer from the front and back and assuming flow to be parallel to 10m long side. Calculate i) the heat lost from four surfaces ii) The power required to overcome the resistance acting on the four surfaces. The properties of air (at t_f =30 ^oC) take $\rho = 1.165\, kg/m^3$, C_p=1.005kJ/kgK, K=0.02673W/m ^oC, γ =16x10^{-6}m^2/s , P_r=0.701 (10 , 5b Dec15/Jan16) (5b, 10, Dec15/Jan16)
20. A hot plate 1 mx0.5 m at 130^oC is kept vertically in still air at 20 ^oC. Find i) heat transfer coefficient ii) Initial rate of cooling the plate in ^oC/min.iii) Time required for cooling plate 180^oC to 80 ^oC if the heat transfer is due to convection only

    Take mass of the plate as 20 kg, Cp= 400 J/kgK, Assume 0.5 mm size is vertical and convection takes place from both sides of the plate (10, 4c, Dec18/Jan19)

1. The exact expression for local Nusselt number for the laminar flow along a surface is given by

$$Nu_x = \frac{h_x x}{k} = 0.332 R_{ex}^{\frac{1}{2}} P_r^{\frac{1}{3}}$$

Show that the average heat transfer coefficient from x= 0 to x =L over the length L of the surface is given by $2h_L$ is the local heat transfer coefficient at x=L 05 Dec13/Jan14

8. An approximate expression for temperature profile in thermal boundary layer is given by

$$\frac{T_{(x,y)} - T_w}{T_\infty - T_w} = \frac{3}{2}\frac{y}{\delta_t(x)} - \frac{1}{2}\left[\frac{y}{\delta_t(x)}\right]^3$$, where $\delta_t(x) = 4.53 \frac{x}{R_{ex}^{\frac{1}{2}} P_r^{\frac{1}{3}}}$

9. Develop an expression for local heat transfer coefficient $h_{(x)}$ 4b June/July13 06

10. A fan provides a speed upto 50 m/ s is used in low speed wind tunnel with atmospheric air at 27 °C.If this wind Tunnel is used to study the boundary layer behavior over a flat plate upto are $R_e=10^8$. What should be the minimum length?. At what distance from the leading edge would transition occur, if critical Reynolds number $R_{ecr}=5x10^5$? 4b, 08 Dec17/Jan18
11. Calculate the approximate Reynold numbers and state if the flow is laminar or turbulent for the following

iv) A 10 m long yatch sailing at 13 km per hour in sea water, ρ ¿ 1000 kg/m³ and $\mu =$¿ 1.3 x10^{-3}kg/ms

v) A compressor disc of radius 0.3 m rotating at 15000 RPM in air at 5 bar and 400 °C

$$¿\,\mu = \frac{1.46\,x\,10^{-6}\,T^{\frac{3}{2}}}{(110+T)}$$ kg/ms 4c, 08 Dec17/Jan18

12. A fan provides a speed upto 50 m/ s is used in low speed wind tunnel with atmospheric air at 27 °C.If this wind Tunnel is used to study the boundary layer behavior over a flat plate upto are $R_e=10^8$. What should be the minimum length?. At what distance from the leading edge would transition occur, if critical Reynolds number $R_{ecr}=5x10^5$? 4b, 08 Dec17/Jan18

13.

Forced Convection PROBLEMS

1. A plate of length 750 mm and width 250 mm has been placed longitudinally in a stream of crude oil which of which flows with a velocity of 5m/s. If the oil has a specific gravity of 0.8 and kinetic viscosity of 10^{-4} m^2 /s. Calculate i) boundary layer thickness at the middle of the plate ii) Shear stress at the middle of the plate iii) friction drag on one side of the plate 4b,06 Dec14/Jan15 06

   Note: $\rho = 1000 * sp\,gravity = 1000 * 0.8 = 800\ kg/m^3$

   Calculate i) boundary layer thickness at the middle of the plate ii) Shear stress at the middle of the plate

   Since question has been asked at the middle

   Take $x = \frac{0.750}{2} = 0.325\,m\,for\ reynolds\ number, thickness\ of\ boundary\ layer\ calculation$

   For iii) sub question friction Drag force

   Entire plate is to be considered

   Here Reyonold number to be calculated by taking x =0.750

   Then calculate $\acute{C}_{fL} = 1.328\, R_{ex}^{-0.5}$

$$Average\ shear\ stress\ \tau_{av} = \acute{C}_{fL} \frac{1}{2} \rho U_\infty^2$$

$Total\ Drag\ Force = Average\ shear\ stress * Area$ where Area $¿\, w*L$

L=0.750m

2. Air at 20 °C flows over thin plate with a velocity of 3 m/sec. The plate is 2 m long and 1 m wide. Estimate the boundary layer thickness at the trailing edge of the plate and the total drag force experienced by the plate. (4c, 06, June/July16)

3. Air at 30 °C and atmospheric pressure is flowing over a flat plate at a velocity 3 m/s, plate is 30 cm wide and at a temperature of 60 °C ,calculate at X= 0.3m

   i) thickness of velocity and thermal boundary layer

   ii) local and average friction coefficients

   iii)local and average heat transfer coefficients

   iv) total drag force on the plate 4b, 10, June/July2018

   $T_\infty = 30^o C; T_s = 60^o C$; $U_\infty = 3\,m/s$

   At $x = 0.3\,m$ i) $\delta_h = ?$; $\delta_t = ?$ ii) $C_{fx} = ?$ $\acute{C}_{fL} = ?$ iii) $h_{x=0.3}$=?, $h_{average}$=?

vi) Total Drag force=?

Solution

$$T_f = \frac{T_s + T_\infty}{2} = \frac{60+30}{2} = 45^o C$$

$$\rho = \frac{1.128 + 1.093}{2} = 1.1105\, kg/m^3 \quad ; \quad \gamma = \frac{16.96 + 17.95}{2}\, x\, 10^{-6} = 17.455\, x\, 10^{-6}\, m^2/s$$

$$P_r = \frac{0.669 + .698}{2} = 0.6835 \quad ; \quad k = \frac{0.02756 + 0.02826}{2} = 0.02791\, W/mK$$

$R_{ex} = \frac{U_\infty x}{\gamma}$; At x=0.3m $R_{ex} = \frac{3 * 0.3}{17.455 * 10^{-6}} = 0.5156\, x\, 10^5$

$Since\ R_{ex} < 5\, x\, 10^5$ flow over the plate along width of 0.3 m is laminar

$i¿\, \delta_{hx} = 5\, x\, R_{ex}^{-0.5}$ from data Hand Book Page number

At x=0.3m

$\delta_{hx} = 5 * 0.3 * (0.51\, x\, 10^5)^{-0.5}$; $\delta_{hx} = 6.605 * 10^{-3}\, m$

Thickness of Hydrodynamic boundary layer thickness at x=0.3m is $6.605\, mm$

$\delta_{Tx} = \delta_{hx} P_r^{-0.333}$; $\delta_{Tx} = 6.605 * 10^{-3} * 0.6835^{-0.3333}$; $\delta_{Tx} = 7.498 * 10^{-3}\, m$

Thermal Boundary layer thickness at x=0.3m is $7.498\, mm$

ii) Local Drag coefficient and average Drag coefficient

$C_{fx} = 0.664\, R_{ex}^{-0.5}$; $C_{fx} = 0.664 * (0.5156\, x\, 10^5)^{-0.5}$; $C_{fx} = 2.924 * 10^{-3}$

$\acute{C}_{fL} = 1.328\, R_{ex}^{-0.5}$; $\acute{C}_{fL} = 1.328 * (0.5156\, x\, 10^5)^{-0.5}; \acute{C}_{fL} = 5.84846 * 10^{-3}$

iii) Local Heat transfer coefficient at x=0.3m and average heat transfer coefficient

$Local\ Nusselt\ Number\ N_{ux} = 0.332\, R_{ex}^{0.5} P_r^{0.333}$

$At\ x = 0.3\, m$, $N_{ux} = 0.332 * ¿$; $N_{ux} = 66.4$ 1

$N_{ux} = \frac{h_x x}{K};$ $66.41 = \frac{h_x * 0.3}{0.02791}$; $h_x = 6.178\, W/m^2 K$

Local Heat Transfer coefficient at x=0.3m is $6.178\, W/m^2 K$

Average heat transfer coefficient over width x=0.3 = 2 h_x since $\acute{N}_u = 2\, N_{ux}$

Hence Average Heat transfer coefficient = $2 * 6.178 = 12.357\, W/m^2 K$

iv)Total Drag force

$Average\ shear\ stress\ \tau_{av} = \acute{C}_{fL} \frac{1}{2} \rho U_\infty^2$

$\tau_{av} = 5.84846 * 10^{-3} \frac{1}{2}$ * $1.1105 * 3^2$; $\tau_{av} = 0.0292\, N/m^2$

$Total\ Drag\ Force = Average\ shear\ stress * Area$ where Area $¿\, w * L$

$F = 0.0292 * (0.3 * 1)$ Length is assumed as 1m

4. Air at 20 ºC and at a pressure of 1 bar is flowing over a flat plate at a velocity of 3m/s, if the plate is 280 mm wide and 56ºC. Calculate the following quantities x =280 mm, given that the properties of air at bulk mean temperature 38 ºC are $\rho = ¿$ 1.1374 kg /m³ ,K =0.02732 W/mºC , $C_p = ¿$

1.005 kJ/ kg K, $\gamma = 16.768\, x 10^{-6}$, P_r =0.7. Determine i) boundary layer thickness ii) thickness of thermal boundary layer iii) local heat transfer Coefficient iv) Average convective heat transfer coefficient v) Rate of heat transfer by Convection vi) total drag force on the plate 4b,12 Dec15/jan16

5. Air at 27 °C and 1 atmosphere pressure flows over a heated plate with a velocity of 2 m/s. The plate is at uniform temperature of 60 °C. Calculate the heat transfer rate from first 0.2m of the plate (08, 5b, Dec17/Jan2018)
6. Air at a temperature of 20 °C ,flows over a flat plate 3m/s. The plate is 50 cmx25 cm. Find the heat lost per hour if the air flow is parallel to 50 cm side of the plate. If 25cm side is kept parallel to the air flow, what will be the effect on heat transfer? Temperature of the plate is 100 °C (5c,08, 5c, Dec14/Jan15)
7. Atmospheric air at 275 K and free stream velocity 20 m/s flows over a flat plate of length 1.5 m long maintained at 325 K. Calculate

iv) The average heat transfer Coefficient over the region where the boundary layer is laminar

v) Find the average heat transfer over the entire length 1.5 m of the plate.

vi) Calculate the total heat transfer rate from the plate to air over the length of 1.5 m and width 1 m, assume transition occurs at Reynold number $2x10^5$. Take air properties at mean Temperature of 300K

K=0.026W/m °C, P_r=0.708 γ =16.8x10^{-6} m^2/s, $\mu = 1.98\, x 10^{-5} kg/ms$ (12, 5c, June/July14)

Length up to which flow is laminar

$$Critical\, Reynold\, number\, R_{ec} = \frac{U_\infty L_c}{\gamma}$$; At x=1.5m

$$2*10^5 = \frac{20*L_c}{16.8*10^{-6}};\quad L_c = 0.168\, m$$; ie upto 0.168 flow is laminar remaining length flow is turbulent

Average heat transfer coefficient over laminar region

$$Average\, Nusselt\, Number = 2\, N_{ux} = 2*0.332\, R_{ex}^{0.5}\, P_r^{0.333}$$

$$Average\, Nusselt\, Number = 0.664\, R_{ex}^{0.5}\, P_r^{0.333}$$

$$N_u = 0.664*\left(2\, x\, 10^5\right)^{0.5} 0.708^{0.3333} \quad ; \quad N_u = 264.69$$

$$264.69 = \frac{h*0.168}{0.026}; \quad ; \quad h'_{laminar} = 40.96\, W/m^2 K$$

Average heat Transfer coefficient for entire length (ie Laminar region + Turbulent Region)

$$N'_{uL} = P_r^{0.333}\left(0.037\, R_{eL}^{0.8} - A\right) \quad \text{where} \quad A = 0.037\, R_{cr}^{0.8} - 0.664\, R_{cr}^{0.5}$$

$$A = 0.037\left(2*10^5\right)^{0.8} - 0.664\left(2\, x\, 10^5\right)^{0.8} \quad ; \quad A = 347.26$$

$$R_{eL} = \frac{U_\infty L}{\gamma}; R_{eL} = \frac{20*1.5}{16.8*10^{-6}}; \quad R_{eL} = 1.786*10^5$$

$$\acute{N}_{uL}=0.708^{0.3333}\left(0.037\left(1.786\,x\,10^{6}\right)^{0.5}-347.26\right) \quad ; \qquad \acute{N}_{uL}=3000$$

$$\acute{N}_{uL}=\frac{\acute{h}\,L}{K}; \qquad 66.41=\frac{\acute{h}*1.5}{0.026}; \qquad \acute{h}=52\,W/m^{2}\,K$$

Total Heat Transfer

$$Q=hA\left(T_{s}-T_{\infty}\right) \quad ; \quad Q=52*(1.5*1)\left[325-275\right] \quad ; \quad Q=3900\,Watts$$

8. Air at 20 °C flows past 800 mm long plate at velocity of 45 m/s. If the surface of the plate is maintained at 300°C, determine i) the heat transfer from the entire plate length to air taking into consideration both laminar and turbulent portion of the boundary layer ii) the percentage error if the boundary layer is assumed to be turbulent nature from the very leading edge of the plate . Assume unit width of the plate and critical Reynolds number to be $5x10^5$.(12, 5b, June/July16)

$$T_{\infty}=20^{o}\,C\,;T_{s}=300^{o}C \quad ; \qquad U_{\infty}=45\,m/s$$

$$T_{f}=\frac{T_{s}+T_{\infty}}{2}=\frac{300+20}{2}=160^{o}\,C$$

$$\gamma=30.09\,x\,10^{-6}\,m^{2}/s \quad ; \quad P_{r}=0.682 \quad ; \quad k=0.03640\,W/mK$$

$$Critical\,Reynold\,number\,R_{ec}\,for\,exteranl\,flow\,is \qquad 5\,x\,10^{5}$$

$$Critical\,Reynold\,number\,R_{ec}=\frac{U_{\infty}\,L_{c}}{\gamma} \qquad\qquad 5\,x\,10^{5}=\frac{45*L_{c}}{30.09\,x\,10^{-6}};L_{c}=0.334\,m$$

Since length is 800 mm ie 0.8m greater than critical length flow is laminar turbulent
Hence from data hand book Page number

$$\acute{N}_{uL}=P_{r}^{0.333}\left(0.037\,R_{eL}^{0.8}-871\right) \quad \text{if} \quad Critical\,Reynold\,number\,R_{ec}\,for\,exteranl\,flow\,is \quad 5 \quad x\,10^{5}$$

$$R_{eL}=\frac{U_{\infty}\,L}{\gamma};R_{eL}=\frac{45*0.8}{30.09\,x\,10^{-6}}; \qquad R_{eL}=1.196*10^{6}$$

$$\acute{N}_{uL}=0.682^{0.3333}\left(0.037\left(1.196\,x\,10^{6}\right)^{0.8}-871\right) \quad ; \qquad \acute{N}_{uL}=1604.79$$

$$\acute{N}_{uL}=\frac{\acute{h}*L}{K}; \qquad 1604.79=\frac{\acute{h}*0.8}{0.03640}; \qquad \acute{h}=73.01\,W/m^{2}\,K$$

Total Heat Transfer

$$Q_{1}=hA\left(T_{s}-T_{\infty}\right) \quad ; \quad Q_{1}=73.01*(0.8*1)\left[300-20\right] \quad ; \quad Q_{1}=16356.11\,Watts$$

Case 2:

Assuming Boundary layer assumed to be turbulent over entire length

Page No 113 /equation 1.4.1

$$\acute{N}_{uL}=0.037\,R_{eL}^{0.8}\,P_{r}^{0.333}$$

$$\acute{N}_{uL}=0.037*\left(1.196\,x\,10^{6}\right)^{0.8}*0.682^{0.3333} \quad ; \quad \acute{N}_{uL}=2371.57$$

$$\acute{N}_{uL}=\frac{\acute{h}*L}{K}; \quad 2371.57=\frac{\acute{h}*0.8}{0.03640}; \quad \acute{h}=107.90\,W/m^2\,K$$

Total Heat Transfer

$$Q_2=hA\left(T_s-T_\infty\right) \quad ; \quad Q_2=107.90*(0.8*1)[300-20] \quad ; \quad Q_2=24171.09\,Watts$$

Percentage by considering the plate is turbulent nature from the leading edge

$$\frac{Q_1-Q_2}{Q_1}x\,100 \quad ; \quad \frac{16356.11-24171.09}{16356.11}x\,100 \quad ; \quad -47$$

9. Atmosphere air at 275K and free stream velocity of 20 m/s flows over a long flat plate maintained at uniform temperature of 325K, calculate i) Average heat transfer coefficient over the region of the laminar boundary layer ii) average heat transfer coefficient over the entire length of 1.5 m iii) Total heat transfer coefficient over the entire length of 1.5 m (5b, 10, Dec18/Jan19)
10. Air flows over a flat plate at 30°C , 0.4m wide and 0.75m long with a velocity of 20 m/s . Determine the heat transfer from the surface of the plate assuming plate is maintained at 90 °C

    Use $N_{uL}=0.664\,R_e^{0.5}\,P_r^{0.33}$ for laminar

    $N_{uL}=\left(0.036\,R_e^{0.8}-0.836\right)P_r^{0.333}$ for turbulent (7b. 15 scheme June/July 2018

08),

    June/July 2013
11. Air at velocity of 3 m/s and at 20 °C flows over a flat plate along its length. The length width and thickness of the plate are 100 cm, 50 cm and 2 cm respectively. The top surface of the plate is maintained at 100 °C. Calculate the heat lost by the plate and temperature of bottom surface of the plate for the steady state conditions. The thermal conductivity of the plate may be taken as 23 W/mK

    (08, 5c, Dec17/Jan2018)
12. The velocity of water flowing through a tube of 2.2 cm dia is 2m/s. Steam condensing at 150°C on the surface of the tube heats the water from 15°C to 60°C over the length of tube. Find the heat transfer coefficient and the length of the tube neglecting the tube and steam side film resistance. Take the following properties of water at mean temperature : $\rho=850\,kg/m^3$

    $C_p=2000\,J/kg^oC$ $\gamma=5.1\,x\,10^{-6}\,m^2/sec$ K=0.2W/ m^oC

INTERNAL FLOW

Critical Reynold number is 2300 which determines the flow is laminar or turbulent unless and until it is specified in the problem

Characteristic length is Diameter

$$R_{eD} = \frac{U_\infty D}{\gamma}$$

13. A tube 5m long is maintained at 100 °C by steam jacketing. A fluid flows through the tube at the rate of 175 kg/hr at 30 °C . The diameter of the tube is 2 cms. Find out the average heat transfer coefficient. Take the following properties of the fluid

$\rho = 850\, kg/m^3$, C_p=2000J/kg°C, γ =5.1x10^{-6} m^2/s, K=0.2W/m°C, (10,5b, June/July17)

$$R_{eD} = \frac{U_\infty D}{\gamma};$$

If mass flow rate is given please note down to calculate the velocity of fluid as follows

$m = \rho A_f U_\infty$ where m is the mass flow rate in kg/s , A_f is the area of flow ,

$U_\infty = the\ velocity\ of\ fluid\ inside\ the\ tube$

$m = 175\, kg/hr$ $\quad m = \frac{175}{3600}$ kg/s $\quad ; A_f = \frac{\pi ¿ 0.02^2}{4}$

Hence, $\frac{175}{3600}=\frac{850*\pi¿0.02^2}{4}*U_\infty$; $U_\infty=0.1820\,m/s$

$R_{eD}=\frac{0.1820*0.02}{5.1*10^{-6}}$; $R_{eD}=713.88$

Since $R_{eD}<2300$ flow is laminar

From Page 114 equation 1.2.1 for circular tube constant wall temperature $N_u=3.66$

$N_u=\frac{\acute{h}*D}{K}$; $3.66=\frac{\acute{h}*0.02}{0.2}$; $h=36.6\,W/m^2K$

If rate of heat transfer asked

$Q=h\,A_{HT}((T_s-T_\infty))$

14. Lubricating oil at a temperature of 60 ºC enters a 1 cm diameter tube with a velocity of 3.5 m/ s. The tube surface is maintained at 30 ºC. Calculate the tube length required to cool the oil to 45 ºC. Assume that the oil has the following average properties for the temperature range of this problem $\rho=¿$ 865kg/m^3 , K=0.14 W/mK , Cp=1.78kJ/kgK, and $\gamma=¿$ 9x10^{-6} m^2/s (08 ,Dec18/Jan19, 7b, 15 scheme)
15. Water flows at a velocity of 12 m/ s in a straight tube of 60 mm diameter. The tube surface temperature is maintained at 70 ºC and the flowing water is heated from the inlet temperature of 15 ºC to an outlet temperature of 45 ºC. Taking the principal properties of water at the mean bulk temperature of 30 ºC as $\rho=995.7\,kg/m^3$, C$_p$=4.174kJ/kgK ,K=0.61718W/mK , γ =0.805x10^{-6} m^2/s, and P$_r$=5.42 . calculate i)heat transfer Coefficient from the tube surface to the water.ii) the heat transferred iii) the length of the tube (10,5b, Dec16/Jan17)

$R_{eD}=\frac{U_\infty D}{\gamma};R_{eD}=\frac{12*0.06}{0.0805*10^{-6}}$; $R_{eD}=8.944*10^5$

$R_{eD}>2300,$ Hence the flow is turbulent

From Data Hand Book Page number equation

$N_{uD}=0.023R_{eD}^{0.8}P_r^n$ wher n=0.3 for cooling of fluid n=0.4 for heating of fluid

In the problem water is heated from 15^{0}C to 45^{0}C Hence n=0.4

$N_{uD}=0.023R_{eD}^{0.8}P_r^{0.4}$; $N_{uD}=0.023(8.944*10^5)^{0.8}*5.42^{0.3333}$; $N_{uD}=609.47$

$N_{uD}=\frac{\acute{h}*D}{K}$; $609.47=\frac{\acute{h}*0.06}{0.61718}$; $h=26841\,W/m^2K$

$Q=h\,A_{HT}\,LMTD$

$$LMTD = \frac{\theta_i - \theta_0}{\ln \frac{\theta_i}{\theta_0}}$$; where $\theta_i = T_s - T_{wi} = 70 - 15 = 55^o C$; $\theta_0 = T_s - T_{wo} = 70 - 45 = 25^o C$

$$LMTD = \frac{\theta_i - \theta_0}{\ln \frac{\theta_i}{\theta_0}}; LMTD = \frac{55 - 25}{\ln \frac{55}{25}} \quad ; \quad LMTD = 38.04^0 C$$

$$A_{HT} = \pi DL = \pi * 0.06 * L$$

$$Also\ Rate\ of\ Heat\ transfer = Heat\ gained\ by\ water\ ie\ Q = m_w C_{pw} (T_{wo} - T_{wi})$$

$$m_w = \rho A_f U_\infty = \rho \frac{\pi D^2}{4} U_\infty; m_w = 995.7 \left(\frac{\pi\, 0.06^2}{4} \right) 12$$ kg/s

$$Q = 995.7 \left(\frac{\pi\, 0.06^2}{4} \right) 12 * 4.174 * (45 - 15) \quad ; \quad Q = 4.23 * 10^6\ Watts$$

$$4.23 * 10^6 = 26841 * (\pi * 0.06 * L) * 38.04 \quad ; \quad L = 21.97\ m$$

16. Air at 2 atm and 200°C is heated as it flows at a velocity of 12m/s through a tube with a diameter of 3cm. A constant heat flux condition is maintained at the wall and the wall temperature is 20°C above the air temperature all along the lenth of the tube . Calculate
    iii) The heat transfer per unit length of tube
    iv) The increase in bulk temperature of air over a 4m length of the tube.

Take the following properties of air Pr=0.681 , $\mu =$ ¿ 2.57 $x\, 10^{-5}\, kgm/s$ K=0.0396W/mK and $C_p = 1.025\, kJ/kgK$ (Dec13/Jan14)

17. Hot air at atmospheric pressure and 80 °C enters an 8m long uninsulated square duct of cross section 0.2 m x 0.2m that passes through the attic of a house at a rate of 0.15 m³/s. The duct is observed to be nearly isothermal at 60 °C. Determine the exit temperature of the air and the rate of heat loss from the duct to attic space (10,5c,June/July2015)
18. Consider air at atmospheric pressure and 100 °C enters a 2m long tube of 4 cm diameter with a velocity of 9 m/s. A 1 KW electric heater is wound on the outer surface of the tube, find i) Exit temperature of air ii) mass flow rate of air iii) wall temperature. Assume that the rate of heat absorption by air per unit area is uniform throughout length of the tube(10, 5b, June/July2018)
19. A refrigerated truck is moving on a highway 90 km/hr in desert area, where the ambient air temperature is 50 °C. The body of the truck is a rectangular box measuring 10m(length) x 4m (width)x3m(Height) . Assume that the boundary layer on the four walls is turbulent. The heat transfer takes only from the four surfaces and the wall surfaces of the truck is maintained at 10°C. Neglecting the heat transfer from the front and back and assuming flow to be parallel to 10m long side. Calculate i) the heat lost from four surfaces ii) The power required to overcome the resistance acting on the four surfaces.

The properties of air (at t_f =30 °C) take $\rho = 1.165\, kg/m^3$, C_p=1.005kJ/kgK, K=0.02673W/m °C, γ =16x10^{-6}m^2/s , P_r=0.701 (10 , 5b Dec15/Jan16) (5b, 10, Dec15/Jan16)

20. A hot plate 1 mx0.5 m at 130°C is kept vertically in still air at 20 °C. Find i) heat transfer coefficient ii) Initial rate of cooling the plate in °C/min.iii) Time required for cooling plate 180°C to 80 °C if the heat transfer is due to convection only
Take mass of the plate as 20 kg, Cp= 400 J/kgK, Assume 0.5 mm size is vertical and convection takes place from both sides of the plate (10, 4c, Dec18/Jan19)

Free Convection

Problems

- **i) On Vertical Plate Characteristic length = Hieght**
- **ii) Vertical Cylinder Characteristic length = Hieght**
- **iii) Horizontal Cylinder Characteristic length = Diameter**

For Vertical surface or vertical Cylinder

Laminar if $G_r P_r < 10^9$

Turbulent if $G_r P_r > 10^9$

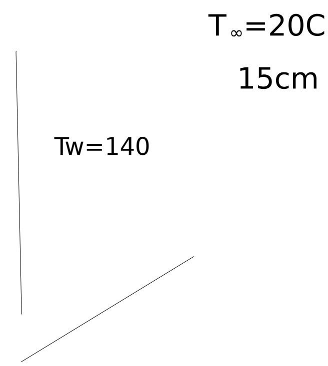
1. A hot plate 1mx0.5m at 130°C is kept vertically in still air at 20°C.Find i) Heat transfer Coefficient ii) heat lost to surroundings 4c, 06 June/July 2015

2. A vertical plate 15 cm high and 10 cm wide is maintained at 140°C. Calculate the maximum heat dissipation rate from both the sides of the

10c

plates to air at 20 °C. The radiation heat transfer coefficient is 9.0 W/m^{2}K. For Air at 80°C,take γ =21.09x10^{-6} P_r=0.692, k_f= 0.03W/mK, 4c,10 Dec13/Jan14

$$G_r=\frac{\beta g\,\Delta\theta\,L^3}{\gamma^2}$$



$$\beta=\frac{1}{T_f+273}$$

$$T_f=\frac{T_w+T_\infty}{2} \quad ; \quad T_f=\frac{140+20}{2}=80^0C$$

$$\beta=\frac{1}{80+273}=\frac{1}{353}$$; L=hiegth=15cm=0.15m

$$G_r=\frac{1\,x\,9.81\,(140-20)\,0.15^3}{353\,x\,(21.09\,x\,10^{-6})^2} \quad ; \quad G_r=2.530\,x\,10^7$$

$G_r\,P_r=2.53\,x\,10^7\,x\,0.692$; $G_r\,P_r=1.751\,x\,10^7$; $G_r\,P_r<10^9$ hence laminar

For turbulent ¿ Data Book 6th edition − equation 1.2 Page 135

$N_u=0.59\,(G_r\,P_r)^{0.25}$ for $10^4<G_r\,P_r<10^9$

$N_u=0.59\,(1.751\,x\,10^7)^{0.25}$; $N_u=38.16$

$N_u=\frac{hL}{K}$; $634.34=\frac{h\,x\,0.15}{0.03}$;

$h=7.633\,w/m^2K$

$h_{conv}=7.633\,w/m^2K$

$h_{rad}=9\,w/m^2K$

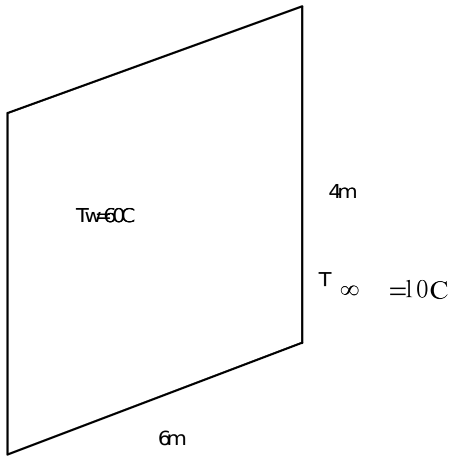
Total $h=7.633+9=16.633\,w/m^2K$

$A=(0.15\,x\,0.1)\,2$; $A=0.3\,m^2$ Since both the surfaces are exposed to air

$Q=hA\,(T_w-T_\infty)$; $Q=16.633*0.3*(140-20)$; $Q=59.97$ Watts

3. A vertical plate 4 m high and 6 m wide is maintained at 60 °C and exposed to atmospheric air at 10 °C. Calculate the heat transfer from both sides of the plate. For air at 35°C take K=0.027 W/mK $\gamma=$¿ 16.5 x10^{-6} m^2/ s P_r =0.7 (10 Dec 2016/Jan17)

Solution:



$$G_r=\frac{\beta g \Delta \theta L^3}{\gamma^2}$$

$$\beta=\frac{1}{T_f+273}$$

$$T_f=\frac{T_w+T_\infty}{2} \quad ; \quad T_f=\frac{60+10}{2}=35^0 C$$

$$\beta=\frac{1}{35+273}=\frac{1}{308} \quad ; \quad \text{L=hiegth=4m}$$

$$G_r=\frac{1\,x\,9.81(60-10)4^3}{308\,x\left(21.09\,x\,10^{-6}\right)^2} \quad ; \quad G_r=3.743\,x\,10^{11}$$

$$G_r P_r=3.743\,x\,10^{11}\,x\,0.7 \quad ; \quad G_r P_r=2.62\,x\,10^{11} \quad ; \quad G_r P_r>10^9 \quad \text{hence turbulent}$$

For turbulent ¿ *Data Book* 6 *th edition* − *equation* 2 .1 Page 135

$$N_u=0.10\left(G_r P_r\right)^{0.333}$$

$$N_u=0.10\left(2.62\,x\,10^{11}\right)^{0.333} \quad ; \quad N_u=634.34$$

$N_u = \frac{hL}{K}$; $634.34 = \frac{h\,x\,4}{0.027}$;

$h = 4.28\,w/m^2\,K$

$A = (6\,x\,4)\,2$; $A = 48\,m^2$ Since both the surfaces are exposed to air

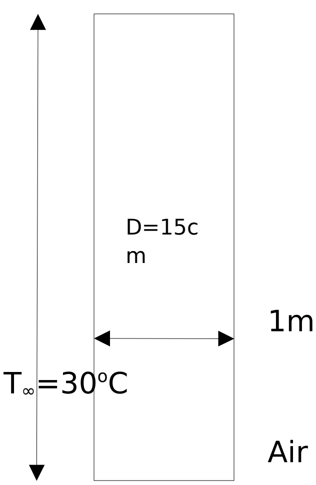
$Q = hA\,(T_w - T_\infty)$; $Q = 4.28 * 48 * (60-10)$; $Q = 10.276 * 10^3$ Watts

$Q = 10$ .276 kW

4. Calculate the convection heat loss from a radiator 0.5 m wide and 1 m high maintained at a temperature of 84 ºC in a room at 20 ºC. Treat the radiator as a vertical plate (08 ,Dec18/Jan19, 8b, 15 scheme) Hint : Vertical Plate L= height = 1m

5. A vertical pipe 15 cm OD, 1 m long has a surface temperature of 90ºC. If the surrounding air is 30 ºC. What is the rate of heat loss by free convection? 4c,08 June/July13

$G_r = \frac{\beta g \Delta\theta L^3}{\gamma^2}$



$T_w = 90ºC$

$\beta = \frac{1}{T_f + 273}$

$T_f = \frac{T_w + T_\infty}{2}$; $T_f = \frac{90+30}{2} = 60^0\,C$

At $60^0\,C$,

K=0.02896W/mK ; $\gamma = ¿$ 18.97 x10^{-6} m^2/ s P_r =0.696

$\beta = \frac{1}{60+273} = \frac{1}{333}$; L=hiegth=1m

$G_r = \frac{1\,x\,9.81\,(90-30)\,1^3}{333\,x\,(18.97\,x\,10^{-6})^2}$; $G_r = 4.91\,x\,10^9$

$G_r\,P_r = 4.91\,x\,10^9\,x\,0.696$; $G_r\,P_r = 3.418\,x\,10^9$; $G_r\,P_r > 10^9$ hence Turbulent

$For\,turbulent\,¿\,Data\,Book\,6\,th\,edition-equation\,2$ .1 Page 135

$N_u=0.10\left(G_r P_r\right)^{0.333}$

$N_u=0.10\left(3.418\,x\,10^9\right)^{0.333}$; $N_u=149.54$

$N_u=\frac{hL}{K}$; $149.54=\frac{h\,x\,1}{0.02896}$;

$h=4.33\,w/m^2\,K$

$A=\pi DL=\pi*0.15*1$; $A=0.4712\,m^2$ Since both the surfaces are exposed to air

$Q=hA\left(T_w-T_\infty\right)$; $Q=4.33*0.4712*\left(90-30\right)$; $Q=122.45$ Watts

6. The water in a tank 20 ºC is heated by passing the steam through a pipe of 50 cm long and 5cm dia. If the pipe surface temperature is maintained at 80 ºC i) find the heat loss from the pipe per hour if the pipe kept horizontal ii) if the pipe is kept vertical , then also find out the heat loss from the pipe per hour 10 June/July 2017

Fluid is water

$\beta=\frac{1}{T_f+273}$

$T_f=\frac{T_w+T_\infty}{2}$; $T_f=\frac{80+20}{2}=50^0 C$

At $50^0 C$, water properties

$\gamma=\frac{0.657+0.478}{2}\,x\,10^{-6}=0.5675\,x\,10^{-6}\,m^2/s$

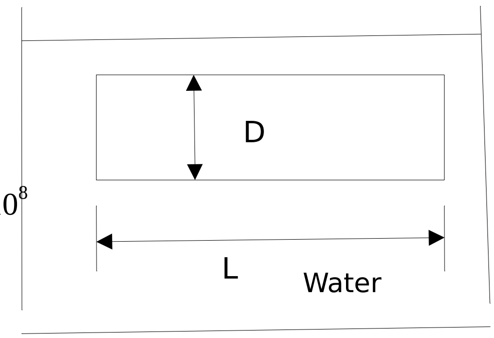
$K=\frac{0.6280+0.6513}{2}=0.63965\,W/mK$

$P_r=\frac{4.340+3.020}{2}=3.68$

$\beta\,for\,water\,at$ $50^0 C=0.48\,x\,10^{-3}$ From data Hand book Page No 29 for water

$Case\,1: Horizontal\,Cylinder$

$G_r = \frac{\beta g \Delta\theta D^3}{\gamma^2}$;When the pipe is Horizontal Characteristic length is Diameter



$G_r = \frac{\beta g \Delta\theta D^3}{\gamma^2}$

$G_r = \frac{0.48\,x\,10^{-3}\,x\,9.81(80-20)\,0.05^3}{(0.5675\,x\,10^{-6})^2}$; $G_r = 1.09\,x\,10^8$

$G_r P_r = 1.09\,x\,10^8\,x\,3.68$; $G_r P_r = 4.0354\,x\,10^8$

$Horizontal\,cylinder$

$¿\,Data\,Book\,6\,th\,edition - equation\,3$ .1 Page 137

$N_u = C(G_r P_r)^m$ and $C = 0.125, m = 0.333\,for\,10^7 < G_r P_r > 10^{12}$

$N_u = 0.125(G_r P_r)^{0.333}$

$N_u = 0.125(4.0354\,x\,10^8)^{0.333}$; $N_u = 91.76$

$N_u = \frac{hD}{K}$; $91.76 = \frac{h\,x\,0.05}{0.63965}$; $h = 1173.93\,w/m^2\,K$

$A = \pi DL = \pi * 0.05 * 0.5$; $A = 0.0785\,m^2$

$Q = hA(T_w - T_\infty)$; $Q = 1173.93 * 0.0785 * (80-20)$; $Q = 5532.02$ Watts

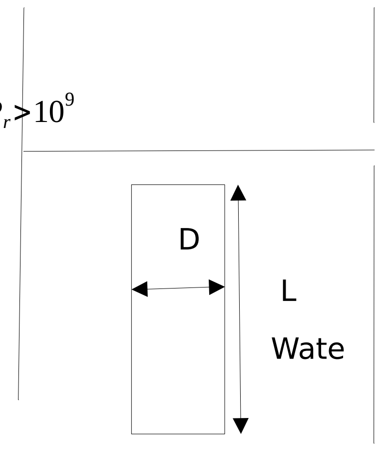
Case 2: Vertical cylinder

$G_r = \frac{\beta g \Delta\theta D^3}{\gamma^2}$;When the pipe is Horizontal Characteristic length is Diameter

$G_r = \frac{\beta g \Delta\theta L^3}{\gamma^2}$

$$G_r = \frac{0.48\,x\,10^{-3}\,x\,9.81(80-20)\,0.5^3}{(0.5675\,x\,10^{-6})^2} \quad ; \qquad G_r = 1.0965\,x\,10^{11}$$

$$G_r P_r = 1.09\,x\,10^{11}\,x\,3.68 \quad ; \qquad G_r P_r = 4.0354\,x\,10^{11} \quad ; \quad G_r P_r > 10^9$$



Vertical cylinder

¿ *Data Book 6th edition − equation* 2 .1 Page 135

$$N_u = 0.1(G_r P_r)^{0.333}$$

$$N_u = 0.1(G_r P_r)^{0.333}$$

$$N_u = 0.125(4.0354\,x\,10^{11})^{0.333} \quad ; \qquad N_u = 732.42$$

$$N_u = \frac{h\,L}{K} \quad ; \qquad 732.42 = \frac{h\,x\,0.5}{0.63965} \quad ;$$

$$h = 936.95\,w/m^2 K$$

$$A = \pi DL = \pi * 0.05 * 0.5 \quad ; \qquad A = 0.0785\,m^2$$

$$Q = hA(T_w - T_\infty) \quad ; \qquad Q = 936.95 * 0.0785 * (80-20) \quad ; \qquad Q = 4415.29\,watts$$

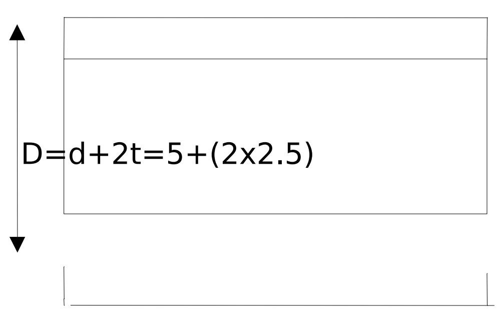
7. A steam pipe 5 cm in diameter is lagged with insulating material of 2.5cm thick.The surface temperature is 80 °C and emissivity of the insulating material surface is 0.93.Find the total heat loss from 10m length of pipe considering the heat loss by natural convection and radiation. The temperature of the air surrounding 20 °C . Also find overall heat transfer coefficient 4b,08 June/July 2015, (10 , 8b, June/July 18 15 scheme)

Please not that outer Diameter of Cylinder = Inner Dia +2 thickness

$$D = 5 + (2\,x\,2.5) = 10\,cm = 0.1\,m$$

$$\beta = \frac{1}{T_f + 273}$$

$$T_f = \frac{T_w + T_\infty}{2} \quad ; \qquad T_f = \frac{80+20}{2} = 50^0 C$$



$$\beta = \frac{1}{50+273} = \frac{1}{323}$$

Properties of air at $50^0 C$ Page 33 $T_w = 80^oC$

$T_\infty = 20^0C$

$\gamma = 17.95 \, x \, 10^{-6}$ $P_r = 0.698;$ $K = 0.02896 \, W/mK$

Considering is Steam pipe is Horizontal

$$G_r = \frac{\beta g \Delta\theta D^3}{\gamma^2}$$

$$G_r = \frac{1 \, x \, 9.81 (80 - 20) 0.1^3}{323 \, x \left(17.95 \, x \, 10^{-6}\right)^2} \quad ; \qquad G_r = 5.655 \, x \, 10^6$$

$$G_r P_r = 5.655 \, x \, 10^6 \, x \, 0.698 \quad ; \qquad G_r P_r = 3.947 \, x \, 10^6$$

Horizontal cylinder

¿ $Data \, Book \, 6th \, edition - equation \, 3$ .1 Page 137

$N_u = C \left(G_r P_r\right)^m$ and $C = 0.48, m = 0.25 \, for \, 10^4 < G_r P_r > 10^7$

$$N_u = 0.48 \left(3.947 \, x \, 10^6\right)^{0.25}$$

$$N_u = 0.48 \left(4.0354 \, x \, 10^8\right)^{0.25} \quad ; \qquad N_u = 21.4$$

$$N_u = \frac{hD}{K} \quad ; \qquad 21.4 = \frac{h \, x \, 0.1}{0.02896} \quad ;$$

$$h = 6.046 \, w/m^2 K$$

$$A = \pi DL = \pi * 0.1 * 1 \quad ; \qquad A = 0.3141 m^2$$

$Q = hA \left(T_w - T_\infty\right)$; $Q = 6.046 * 0.3141 * (80 - 20)$; $Q = 1139.72$ Watts

Heat transfer by radiation

$$Q_{rad} = \epsilon \sigma A \left(T_w^4 - T_\infty^4\right)$$

$$T_w = 80^o C + 273 = 353 \, K$$

$$T_\infty = 20^o C + 273 = 293 \, K$$

$$Q_{rad} = 0.93 \, x \, 5.65 \, x \, 10^{-8} \, x \, 0.3141 \left(353^4 - 293^4\right)$$

$$Q_{rad} = 1351.34 \, Watts$$

Total Heat Transfer $Q = Q_{Conv} + Q_{rad}$

$$Q = 1139.72 + 1351.34 = 2491.06\, Watts$$

Overall Heat Transfer Coefficient $¿ \frac{Q}{A\left(T_w - T_\infty\right)}$

$$U = \frac{2491.06}{0.3141\, x \left(80 - 20\right)} = 132.18\, W/m^2 K$$

8. Two horizontal steam pipes having 100 mm and 300 mm are so laid in a boiler house that the mutual heat transfer may be neglected. The surface temperature of each of the steam pipe is 475 ^oC. If the temperature of the ambient air is 35 ^oC, calculate the ratio of heat transfer coefficients and heat losses per metre length of the pipes(04, 4c, Dec14/Jan15

MODULE 5

Heat Exchanger

Heat Exchanger is the device which facilitate heat transfer between two or more fluids at different temperatures

Classification of Heat exchanger:

1. Nature of Heat Exchange process
   a) Direct   b) Indirect type ----- i) Regenerator ii) Recuperator

In Regenerator hot and cold fluid pass alternatively through a space containing solid particles (matrix) which provides alternatively provide sink and source for heat flow examples IC Engines gas turbines

In Recuperator heat transfer takes place between hot and cold fluid through a dividing wall between the hot and cold fluid flow example : Automobile Radiator , economizer, eavaportor, condenser etc

2. Relative direction of fluid motion:
   a) Parallel flow Heat exchanger in which cold and hot fluid flows in the same direction
   b) Counter flow Heat exchanger in which cold and hot fluid flows in the opposite direction
   c) Cross flow Heat exchanger in which hot and cold fluids cross each other in right angles
3. Design and constructional features:
   a) Concentric tube Heat exchanger b) Shell and tube c) Multiple shell and tube passesd) Compact Heat exchanger
4. Physical state of fluids :
   i) Condenser in which hot fluid is condensed which releases latent heat of vaporization and heats cold fluid
   ii) Evaporator in which cold fluid evaporates as the hot fluid transfer the heat to cold fluid

Log Mean Temperature of Heat Exchanger (LMTD):

It is defined as the temperature difference which, if constant , would give the same heat transfer as actually occurs under variable conditions of temperature difference

$$\text{LMTD} = \frac{\left[\theta_i - \theta_o\right]}{\ln \frac{\theta_i}{\theta_o}}$$

$\theta_i \wedge \theta_o$ are the temperature difference between hot and cold fluid at inlet and outlet of Heat exchanger

LMTD for Parallel flow Heat Exchanger

$\theta_i = T_{hi} - T_{ci} \wedge \theta_o = T_{ho} - T_{co}$

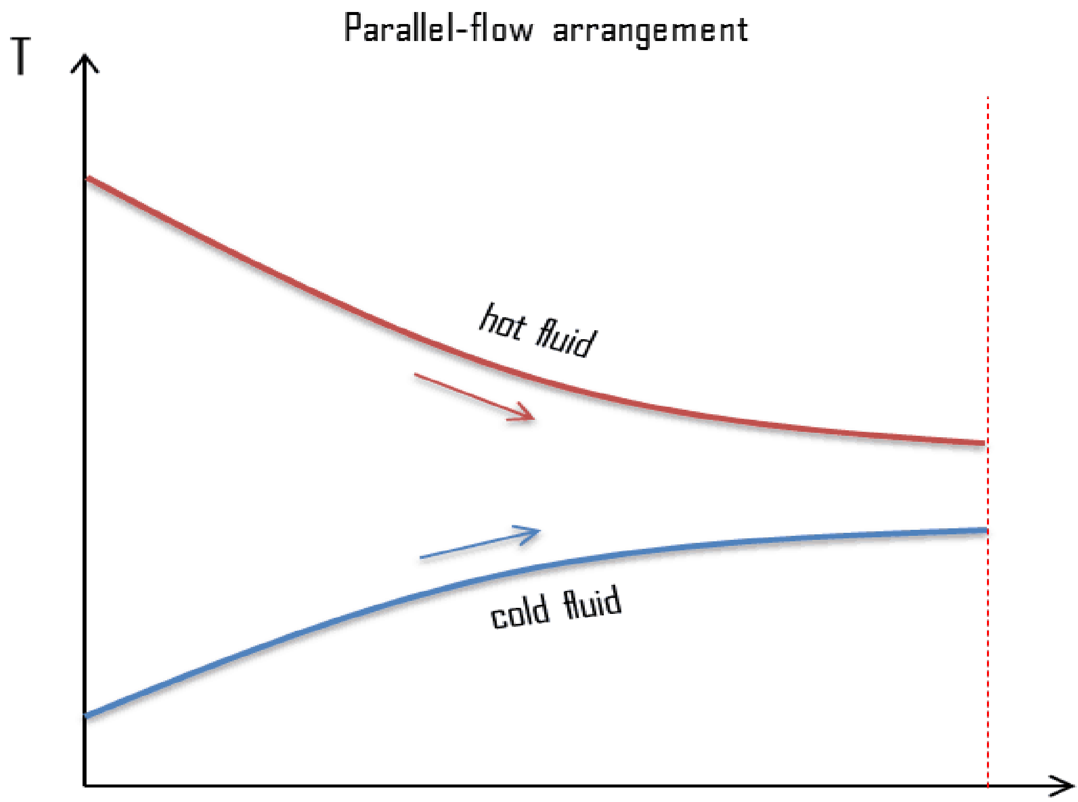
For Counter flow Heat Exchanger

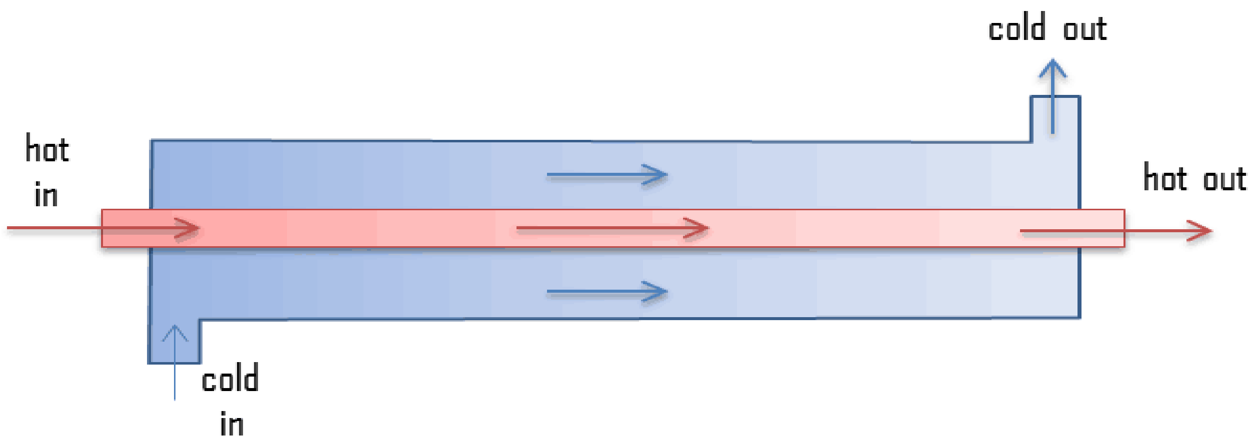
$\theta_i = T_{hi} - T_{c0} \wedge \theta_o = T_{ho} - T_{ci}$

Assumption in Derivation of LMTD and Effectiveness equation :

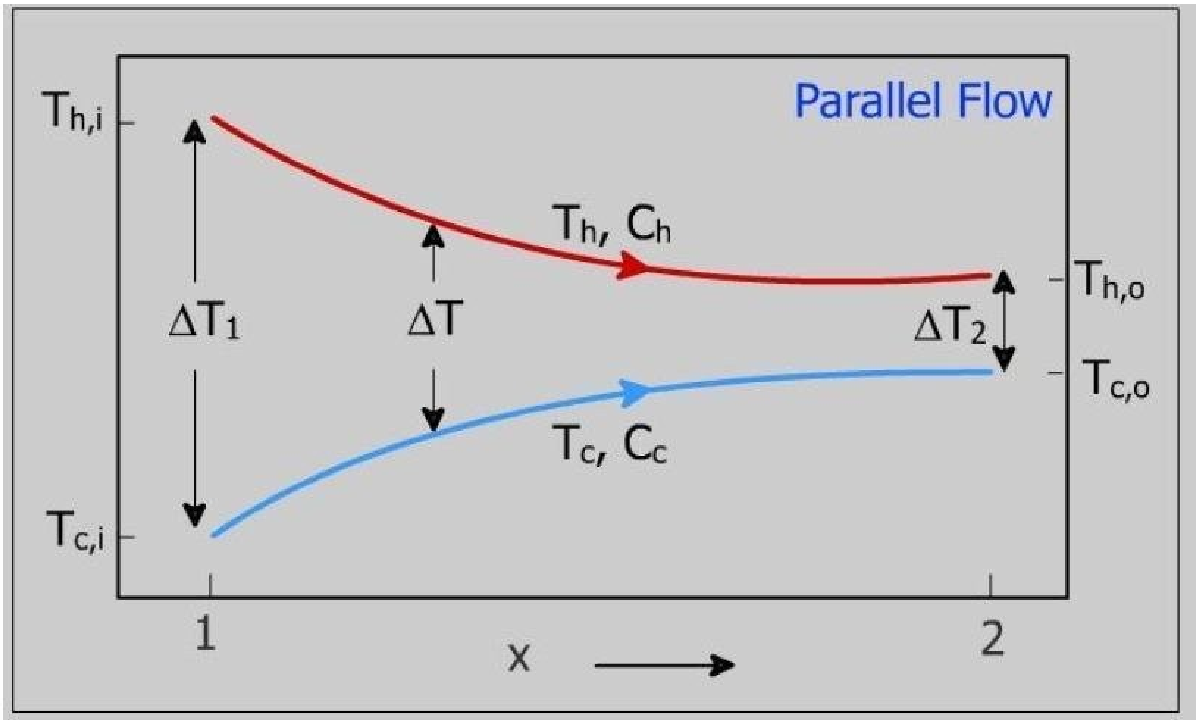
1. Overall Heat transfer coefficient the same
2. The flow conditions for hot and cold fluid is steady
3. Specific Heat and mass flow rate for hot and fluid are constant
4. Heat Exchanger is completely is insulated ie there is no heat loss to surroundings
5. There is no phase change in either of hot and cold fluid during heat transfer

LMTD for Parallel flow Heat HE (Derivation)





Consider a small element in heat exchanger of area dA at x from one end as shown in fig



Let θ is the temperature difference between hot and cold fluid, $T_h \wedge T_c$ are the hot and cold fluid at x respectively

Hence, $\theta = T_h - T_c$

Differentiating above equation

$$d\theta = dT_h - dT_c$$

Rate of heat transfer between and cold fluid through small element in consideration

$$dQ = m_c C_{pc}(+dT_c) = m_h C_{ph}(-dT_h) = UdA\theta$$

Where $m_h \wedge ¿$ m_c are the mass flow rate of hot and cold fluid in heat exchanger $C_{pc} \wedge C_{pc}$ are the specific heat of hot and cold fluid respectively , U is the overall heat transfer coefficient

$$dT_c = \frac{UdA\theta}{m_c C_{pc}} \quad ; \quad dT_h = \frac{-UdA\theta}{m_h C_{ph}}$$

$$d\theta = \frac{-UdA\theta}{m_h C_{ph}} - \frac{UdA\theta}{m_c C_{pc}}$$

$$d\theta = -UdA\theta\left(\frac{1}{m_h C_{ph}} + \frac{1}{m_c C_{pc}}\right)$$

Rate of heat transfer between cold fluid

$$Q = m_c C_{pc}(T_{c0} - T_{ci}) \quad = \quad m_h C_{ph}(T_{hi} - T_{h0})$$

Where $T_{hi} \wedge T_{h0}$ are inlet and outlet temperature respectively, $T_{ci} \wedge \text{¿} \quad T_{c0}$ are inlet and outlet temperature respectively

$$m_c C_{pc} = \frac{Q}{(T_{c0} - T_{ci})} \quad ; \quad m_h C_{ph} = \frac{Q}{(T_{hi} - T_{h0})}$$

$$d\theta = -UdA\theta\left(\frac{(T_{hi} - T_{h0})}{Q} + \frac{(T_{c0} - T_{ci})}{Q}\right)$$

$$\frac{d\theta}{\theta} = \frac{-UdA}{Q}(T_{hi} - T_{h0} + T_{c0} - T_{ci})$$

$$\frac{d\theta}{\theta} = \frac{-UdA}{Q}\left[(T_{hi} - T_{ci}) - (T_{h0} - T_{c0})\right]$$

$$\frac{d\theta}{\theta} = \frac{-UdA}{Q}\left[\theta_i - \theta_o\right]$$

Integrating above equation from inlet to outlet

$$\frac{d\theta}{\theta} = \text{¿} - \frac{U}{Q}\left[\theta_i - \theta_o\right]\int_0^A dA$$

$$\int_{\theta_i}^{\theta_o} \text{¿}$$

$$\ln\frac{\theta_o}{\theta_i} \qquad \text{¿} - \frac{U}{Q}\left[\theta_i - \theta_o\right]A$$

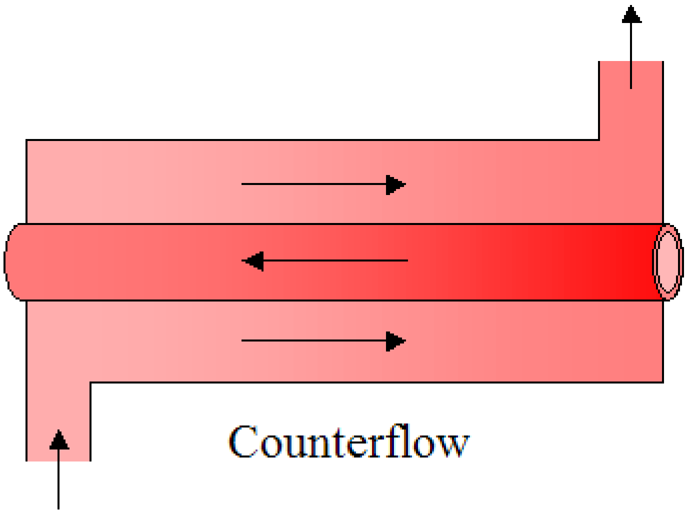
$$\ln\frac{\theta_i}{\theta_o} \qquad \text{¿} + \frac{U}{Q}\left[\theta_i - \theta_o\right]A$$

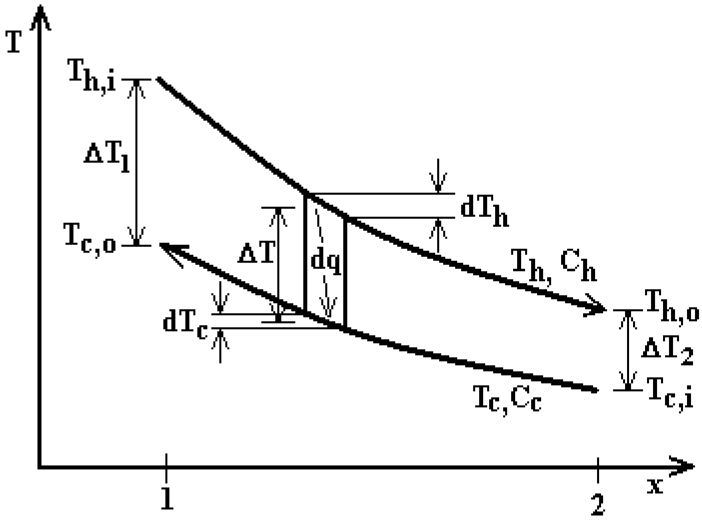
$$Q \qquad \text{¿}\, UA\frac{\left[\theta_i - \theta_o\right]}{\ln\frac{\theta_i}{\theta_o}}$$

$$Q \qquad \text{¿}\, UA\, LMTD$$

Where LMTD = $\dfrac{\left[\theta_i - \theta_o\right]}{\ln\frac{\theta_i}{\theta_o}}$

LMTD for Counter flow Heat Exchanger





Consider a small element in heat exchanger of area dA at x from one end as shown in fig

Let θ is the temperature difference between hot and cold fluid, $T_h \wedge T_c$ are the hot and cold fluid at x respectively, Hence

$$\theta = T_h - T_c$$

$$d\theta = dT_h - dT_c$$

Rate of heat transfer between and cold fluid through small element in consideration

$$dQ = m_c C_{pc}(-dT_c) = m_h C_{ph}(-dT_h) = UdA\theta$$

Where $m_h \wedge ¿$ m_c are the mass flow rate of hot and cold fluid in heat exchanger $C_{pc} \wedge C_{pc}$ are the specific heat of hot and cold fluid respectively , U is the overall heat transfer coefficient

$$dT_c = \frac{-UdA\theta}{m_c C_{pc}} \quad ; \quad dT_h = \frac{-UdA\theta}{m_h C_{ph}}$$

$$d\theta = \frac{-UdA\theta}{m_h C_{ph}} - \left(\frac{-UdA\theta}{m_c C_{pc}} \right)$$

$$d\theta = \frac{-UdA\theta}{m_h C_{ph}} + \left(\frac{UdA\theta}{m_c C_{pc}} \right)$$

$$d\theta = -\left(\frac{UdA\theta}{m_h C_{ph}} - \frac{UdA\theta}{m_c C_{pc}} \right)$$

$$d\theta = -UdA\theta \left(\frac{1}{m_h C_{ph}} - \frac{1}{m_c C_{pc}} \right)$$

Rate of heat transfer between the cold fluid and hot fluid

$$Q = m_c C_{pc} \left(T_{c0} - T_{ci} \right) \quad = \quad m_h C_{ph} \left(T_{hi} - T_{h0} \right)$$

Where $T_{hi} \wedge T_{h0}$ are inlet and outlet temperature respectively, $T_{ci} \wedge ¿$ T_{c0} are inlet and outlet temperature respectively

$$m_c C_{pc} = \frac{Q}{\left(T_{c0} - T_{ci} \right)} \quad ; \quad m_h C_{ph} = \frac{Q}{\left(T_{hi} - T_{h0} \right)}$$

$$d\theta = -UdA\theta \left(\frac{\left(T_{hi} - T_{h0} \right)}{Q} - \frac{\left(T_{c0} - T_{ci} \right)}{Q} \right)$$

$$\frac{d\theta}{\theta} = \frac{-UdA}{Q} \left(T_{hi} - T_{h0} - T_{c0} + T_{ci} \right)$$

$$\frac{d\theta}{\theta} = \frac{-UdA}{Q} \left[\left(T_{hi} - T_{co} \right) - \left(T_{h0} - T_{ci} \right) \right]$$

For Counter flow HE $\theta_i = T_{hi} - T_{co}$ and $\theta_o = T_{h0} - T_{ci}$

$$\frac{d\theta}{\theta} = \frac{-UdA}{Q} \left[\theta_i - \theta_o \right]$$

$$\frac{d\theta}{\theta} = ¿ - \frac{U}{Q} \left[\theta_i - \theta_o \right] \int_0^A dA$$

$$\int_{\theta_i}^{\theta_o} ¿$$

$$\ln \frac{\theta_o}{\theta_i} \qquad ¿ - \frac{U}{Q} \left[\theta_i - \theta_o \right] A$$

$$\ln\frac{\theta_i}{\theta_o} \quad \text{¿} + \frac{U}{Q}\left[\theta_i - \theta_o\right] A$$

$$Q \quad \text{¿}\, UA \frac{\left[\theta_i - \theta_o\right]}{\ln\frac{\theta_i}{\theta_o}}$$

$$Q \quad \text{¿}\, UA\, LMTD$$

Where $LMTD = \frac{\left[\theta_i - \theta_o\right]}{\ln\frac{\theta_i}{\theta_o}}$

Correction factor for Multiple pass Heat Exchanger:

Above LMTD equation is applicable for only single pass Heat Exchanger. The analytical treatment for Multiple pass and Cross flow Heat Exchanger is difficult. Such cases may be solved easily by using Correction factor as flollows

$Q \quad \text{¿}\, UAF\left(LMTD\right)_{Counter\ flow}$ where F is the correction factor

Correction factor : can be determined from charts in Heat Transfer Data Hand Book for various tube and shell passes of Shell Tube HE and Cross flow Heat Exchanger by using Capacity ratio and Temperature ratio

Capacity ratio: It is the ratio of min heat capacity to maximum heat capacity of two fluids which exchanges heat from hot fluid to cold fluid

$$C = \frac{C_{min}}{C_{max}}$$

Heat capacity is the product of mass flow rate and specific heat ie mC

Heat capacity of hot fluid $C_h = \text{¿} \quad m_h C_{ph}$

Heat capacity of hot fluid $C_c = \text{¿} \quad m_c C_{pc}$

$$If\ C_c > C_h then\ C_c = C_{max}; \wedge C_h = C_{min}$$

$$If\ C_c < C_h then\ C_h = C_{max}; \wedge C_c = C_{min}$$

Temperature Ratio

Is defined as the ratio of temperature difference between rise in cold fluid to difference between the inlet temperatures of hot and cold fluids in Heat Exchanger

$$P = \frac{T_{hi} - T_{ci}}{T_{hi} - T_{ci}}$$

Overall Heat transfer coefficient:

In Double tube heat exchanger Equivalent coefficient which replaces coefficient of conduction through separating wall and convection heat transfer coefficient , Equivalence heat transfer coefficient is called overall heat transfer coefficient

For clean surface of separating wall

$$Total\,resistance = \frac{1}{h_i 2\pi r_i L} + \frac{\ln \frac{r_o}{r_i}}{2\pi LK} + \frac{1}{h_o 2\pi r_o L}$$

$$Q = \frac{(T_i - T_o)}{\frac{1}{h_i 2\pi r_i L} + \frac{\ln \frac{r_o}{r_i}}{2\pi LK} + \frac{1}{h_o 2\pi r_o L}}$$

Also $Q = U_o 2\pi r_o L (T_i - T_o)$ in terms of Overall heat transfer coefficient

Equating both we get

$$U_o = \frac{1}{\frac{1}{h_i}\frac{r_o}{r_i} + \frac{r_o}{K}\ln\frac{r_o}{r_i} + \frac{1}{h_o}}$$

Fouling Factor: In a Heat Exchanger, in normal operation, the tube surface is covered by deposition of dirt soot and scale etc. The phenomenon of scale and dirt deposit is called fouling . The heat transfer coefficient due to deposit is called fouling factor. Fouling factor due inside tube is called Inner Fouling factor , due to outside deposition is called outside fouling factor. Fouling factor lower the performance of Heat exchanger

Fouling factor can be determined by finding overall heat transfer coefficient for cleaned surface and un cleaned surface

$$R_f = \frac{I}{U_{Uncleaned}} - \frac{I}{U_{cleaned}}$$

Effects of fouling

Energy losses due to thermal inefficiencies

Energy losses due to thermal inefficiencies

Costs associates for periodic cleaning the surface of Heat exchanger

Hence overall Heat transfer coefficient with dirty tube surface

$$U_o = \frac{1}{\frac{1}{h_i}\frac{r_o}{r_i} + \frac{r_o}{r_i} R_{fi} + \frac{r_o}{K} \ln \frac{r_o}{r_i} + R_{fi} + \frac{1}{h_o}}$$

$$\varepsilon = \frac{C_c\left(T_{co} - T_{ci}\right)}{C_{min}\left(T_{hi} - T_{ci}\right)} = \frac{C_h\left(T_{hi} - T_{ho}\right)}{C_{min}\left(T_{hi} - T_{ci}\right)}$$

Effectiveness ε :

is defined as the actual Heat transfer rate to maximum possible heat transfer

$$\varepsilon = \frac{C_c\left(T_{co} - T_{ci}\right)}{C_{min}\left(T_{hi} - T_{ci}\right)} = \frac{C_h\left(T_{hi} - T_{ho}\right)}{C_{min}\left(T_{hi} - T_{ci}\right)}$$

Greater the effectiveness of heat exchanger greater the performance of heat exchanger

Number of transfer Units(NTU):

It is defined as the ratio of heat capacity of heat exchanger to heat capacity of flow

$$NTU = \frac{UA}{C_{min}}$$

For specified value of $\frac{U}{C_{min}}$ the NTU is measure of Area or physical size of Heat exchanger. The higher the value of NTU, the larger the physical size and also higher the effectiveness of Heat Exchanger.

For counter flow Heat Exchanger, for the specified NTU and capacity ratio, the effectiveness of heat exchanger is effectiveness of Heat exchanger more compared to other flow arrangements in Heat exchanger

Effectiveness of Parallel flow Heat Exchanger

Consider a small element in heat exchanger of area dA at x from one end as shown in fig

Let θ is the temperature difference between hot and cold fluid, $T_h \wedge T_c$ are the hot and cold fluid at x respectively, Hence

$$\theta = T_h - T_c$$

$$d\theta = dT_h - dT_c$$

Rate of heat transfer between and cold fluid through small element in consideration

$$dQ = m_c C_{pc}\left(+dT_c\right) = m_h C_{ph}\left(-dT_h\right) = UdA\theta$$

Where $m_h \wedge ¿$ m_c are the mass flow rate of hot and cold fluid in heat exchanger $C_{pc} \wedge C_{pc}$ are the specific heat of hot and cold fluid respectively , U is the overall heat transfer coefficient

$$dT_c = \frac{UdA\theta}{m_c C_{pc}} \quad ; \quad dT_h = \frac{-UdA\theta}{m_h C_{ph}}$$

$$d\theta = \frac{-UdA\theta}{m_h C_{ph}} - \frac{UdA\theta}{m_c C_{pc}}$$

$$d\theta = -UdA\theta\left(\frac{1}{m_h C_{ph}} + \frac{1}{m_c C_{pc}}\right)$$

$$\frac{d\theta}{\theta} = -UdA\left(\frac{1}{m_h C_{ph}} + \frac{1}{m_c C_{pc}}\right) \qquad dA$$

$$\int_{¿}^{T_{hi-¿T_{ci}}} T_{ho-¿T_{c0}} \frac{d\theta}{\theta} ¿ = -U\left(\frac{1}{m_h C_{ph}} + \frac{1}{m_c C_{pc}}\right)\int_0^A dA$$

where $hi \wedge ¿$ $T_¿$ T_{ho} are the temperature of hot fluid at inlet and outlet respectively and $ci \wedge ¿$ $T_¿$ T_{co} are the temperature of hot fluid at inlet and outlet respectively

$$T_{ho-¿T_{c0}} = -U\left(\frac{1}{m_h C_{ph}} + \frac{1}{m_c C_{pc}}\right)A$$

$$T^{¿}_{hi-¿T_{ci}}$$

$$(ln\theta)_¿$$

$$\ln ¿ \frac{T_{hi-¿T_{ci}}}{T_{ho-¿T_{c0}}} ¿ \; ¿ - UA\left(\frac{1}{C_h} + \frac{1}{C_c}\right)$$

where $C_h = m_h C_{ph}$ and $C_c = m_c C_{pc}$

$$T_{hi-¿T_{ci}} = e^{-UA\left(\frac{1}{C_h}+\frac{1}{C_c}\right)}$$

$$\frac{T_{ho-¿T_{c0}}}{¿}$$

$$¿$$

$$Q = m_c C_{pc}\left(T_{co} - T_{ci}\right) = m_h C_{ph}\left(T_{hi} - T_{ho}\right)$$

$$\varepsilon = \frac{C_c\left(T_{co} - T_{ci}\right)}{C_{min}\left(T_{hi} - T_{ci}\right)} = \frac{C_h\left(T_{hi} - T_{ho}\right)}{C_{min}\left(T_{hi} - T_{ci}\right)}$$

$$\frac{\varepsilon C_{min}\left(T_{hi} - T_{ci}\right)}{C_c} = \left(T_{co} - T_{ci}\right)$$

$$T_{co}=T_{ci}+\frac{\varepsilon C_{min}\left(T_{hi}-T_{ci}\right)}{C_c}$$

$$\frac{\varepsilon C_{min}\left(T_{hi}-T_{ci}\right)}{C_h}=\left(T_{hi}-T_{ho}\right)$$

$$T_{ho}=T_{hi}-\frac{\varepsilon C_{min}\left(T_{hi}-T_{ci}\right)}{C_h}$$

$$T_{hi-¿T_{ci}}=e^{-UA\left(\frac{1}{C_h}+\frac{1}{C_c}\right)}$$

$$\frac{T_{ho-¿T_{c0}}}{¿}$$

$$¿$$

$$T_{hi-¿T_{ci}}=e^{-UA\left(\frac{1}{C_h}+\frac{1}{C_c}\right)}$$

$$\frac{\left(T_{hi}-\frac{\varepsilon C_{min}\left(T_{hi}-T_{ci}\right)}{C_h}\right)-\left(T_{ci}+\frac{\varepsilon C_{min}\left(T_{hi}-T_{ci}\right)}{C_c}\right)}{¿}$$

$$T_{hi-¿T_{ci}}=e^{-UA\left(\frac{1}{C_h}+\frac{1}{C_c}\right)}$$

$$\frac{\left(T_{hi}-T_{ci}\right)-\varepsilon C_{min}\left(T_{hi}-T_{ci}\right)\left(\frac{1}{C_h}+\frac{1}{C_c}\right)}{¿}$$

$$T_{hi-¿T_{ci}}=e^{-UA\left(\frac{1}{C_h}+\frac{1}{C_c}\right)}$$

$$\frac{\left(T_{hi}-T_{ci}\right)\left(1-\varepsilon C_{min}\left(\frac{1}{C_h}+\frac{1}{C_c}\right)\right)}{¿}$$

$$1-\varepsilon C_{min}\left(\frac{1}{C_h}+\frac{1}{C_c}\right)=e^{-UA\left(\frac{1}{C_h}+\frac{1}{C_c}\right)}$$

$$1-e^{-UA\left(\frac{1}{C_h}+\frac{1}{C_c}\right)}=\varepsilon C_{min}\left(\frac{1}{C_h}+\frac{1}{C_c}\right)$$

$$\varepsilon=\frac{1-e^{-UA\left(\frac{1}{C_h}+\frac{1}{C_c}\right)}}{C_{min}\left(\frac{1}{C_h}+\frac{1}{C_c}\right)}$$

$If\, C_c>C_h then\, C_c=C_{max}; \wedge C_h=C_{min}$ and hence

$$\varepsilon=\frac{1-e^{-UA\left(\frac{1}{C_{min}}+\frac{1}{C_{max}}\right)}}{C_{min}\left(\frac{1}{C_{min}}+\frac{1}{C_{max}}\right)}$$

$$\varepsilon=\frac{1-e^{\frac{-UA}{C_{min}}\left(1+\frac{C_{min}}{C_{max}}\right)}}{1+\frac{C_{min}}{C_{max}}}$$

$$\varepsilon=\frac{1-e^{-NTU(1+C)}}{1+C}$$

$If\,C_c<C_h then\,C_h=C_{max};\wedge C_c=C_{min}$ and hence

$$\varepsilon=\frac{1-e^{-UA\left(\frac{1}{C_{max}}+\frac{1}{C_{min}}\right)}}{C_{min}\left(\frac{1}{C_{max}}+\frac{1}{C_{min}}\right)}$$

$$\varepsilon=\frac{1-e^{\frac{-UA}{C_{min}}\left(\frac{C_{min}}{C_{max}}+1\right)}}{\frac{C_{min}}{C_{max}}+1}$$

$$\varepsilon=\frac{1-e^{-NTU(C+1)}}{C+1}$$

Effectiveness of Counter flow Heat exchanger

Consider a small element in heat exchanger of area dA at x from one end as shown in fig

Let θ is the temperature difference between hot and cold fluid, $T_h\wedge T_c$ are the hot and cold fluid at x respectively, Hence

Consider a small element in heat exchanger

$$\theta=T_h-T_c$$

$$d\theta=dT_h-dT_c$$

Rate of heat transfer between and cold fluid through small element in consideration

$$dQ=m_cC_{pc}(-dT_c)=m_hC_{ph}(-dT_h)=UdA\theta$$

Where $m_h ∧ ¿$ m_c are the mass flow rate of hot and cold fluid in heat exchanger $C_{pc} ∧ C_{pc}$ are the specific heat of hot and cold fluid respectively , U is the overall heat transfer coefficient

$$dT_c = \frac{-UdA\theta}{m_c C_{pc}} \quad ; \quad dT_h = \frac{-UdA\theta}{m_h C_{ph}}$$

$$d\theta = \frac{-UdA\theta}{m_h C_{ph}} + \frac{UdA\theta}{m_c C_{pc}}$$

$$d\theta = -UdA\theta \left(\frac{1}{m_h C_{ph}} - \frac{1}{m_c C_{pc}} \right)$$

$$\frac{d\theta}{\theta} = -UdA \left(\frac{1}{m_h C_{ph}} - \frac{1}{m_c C_{pc}} \right) \quad dA$$

$$T_{hi-¿T_{c0}} \; T_{ho-¿T_{ci}} \frac{d\theta}{\theta} \; \int_{¿} ¿ = -U \left(\frac{1}{m_h C_{ph}} - \frac{1}{m_c C_{pc}} \right) \int_0^A dA$$

$hi ∧ ¿$ $where\, T_{¿}$ T_{ho} are the temperature of hot fluid at inlet and outlet respectively

and $ci ∧ ¿$ $T_{¿}$ T_{co} are the temperature of hot fluid at inlet and outlet respectively

$$T_{ho-¿T_{ci}} = -U \left(\frac{1}{m_h C_{ph}} - \frac{1}{m_c C_{pc}} \right) A$$

$$T^{¿}_{hi-¿T_{c0}}$$

$$(ln\theta)_{¿}$$

$$\ln ¿ \frac{T_{hi-¿T_{c0}} \; T_{ho-¿T_{ci}}}{¿} \; ¿ - UA \left(\frac{1}{C_h} - \frac{1}{C_c} \right)$$

$$T_{hi-¿T_{c0}} = e^{-UA\left(\frac{1}{C_h} - \frac{1}{C_c}\right)} \; \frac{T_{ho-¿T_{ci}}}{¿} \; ¿$$

$$Q = m_c C_{pc} (T_{co} - T_{ci}) = m_h C_{ph} (T_{hi} - T_{ho})$$

$$\varepsilon = \frac{C_c (T_{co} - T_{ci})}{C_{min} (T_{hi} - T_{ci})} = \frac{C_h (T_{hi} - T_{ho})}{C_{min} (T_{hi} - T_{ci})}$$

$$\frac{\varepsilon C_{min}\left(T_{hi}-T_{ci}\right)}{C_c}=\left(T_{co}-T_{ci}\right)$$

$$T_{co}=T_{ci}+\frac{\varepsilon C_{min}\left(T_{hi}-T_{ci}\right)}{C_c}$$

$$\frac{\varepsilon C_{min}\left(T_{hi}-T_{ci}\right)}{C_h}=\left(T_{hi}-T_{ho}\right)$$

$$T_{ho}=T_{hi}-\frac{\varepsilon C_{min}\left(T_{hi}-T_{ci}\right)}{C_h}$$

$$T_{hi-\text{¿}T_{c0}}=e^{-UA\left(\frac{1}{C_h}-\frac{1}{C_c}\right)}$$

$$\frac{T_{ho-\text{¿}T_{ci}}}{\text{¿}\,\text{¿}}$$

$$\frac{\left(T_{hi}-\frac{\varepsilon C_{min}\left(T_{hi}-T_{ci}\right)}{C_h}\right)-T_{ci}}{T_{hi}-\left(T_{ci}+\frac{\varepsilon C_{min}\left(T_{hi}-T_{ci}\right)}{C_c}\right)}=e^{-UA\left(\frac{1}{C_h}-\frac{1}{C_c}\right)}$$

$$\frac{\left(T_{hi}-T_{ci}\right)-\frac{\varepsilon C_{min}\left(T_{hi}-T_{ci}\right)}{C_h}}{\left(T_{hi}-T_{ci}\right)-\left(\frac{\varepsilon C_{min}\left(T_{hi}-T_{ci}\right)}{C_c}\right)}=e^{-UA\left(\frac{1}{C_h}-\frac{1}{C_c}\right)}$$

$$\frac{\left(T_{hi}-T_{ci}\right)\left(1-\frac{\varepsilon C_{min}}{C_h}\right)}{\left(T_{hi}-T_{ci}\right)\left(1-\frac{\varepsilon C_{min}}{C_C}\right)}=e^{-UA\left(\frac{1}{C_h}-\frac{1}{C_c}\right)}$$

$$\frac{\left(1-\frac{\varepsilon C_{min}}{C_h}\right)}{\left(1-\frac{\varepsilon C_{min}}{C_C}\right)}=e^{-UA\left(\frac{1}{C_h}-\frac{1}{C_c}\right)}$$

$$\left(1-\frac{\varepsilon C_{min}}{C_h}\right)=\left(1-\frac{\varepsilon C_{min}}{C_C}\right)e^{-UA\left(\frac{1}{C_h}-\frac{1}{C_c}\right)}$$

$$1-\frac{\varepsilon C_{min}}{C_h}=e^{-UA\left(\frac{1}{C_h}-\frac{1}{C_c}\right)}-\frac{\varepsilon C_{min}}{C_C}e^{-UA\left(\frac{1}{C_h}-\frac{1}{C_c}\right)}$$

$$1-e^{-UA\left(\frac{1}{C_h}-\frac{1}{C_c}\right)}=\frac{\varepsilon C_{min}}{C_h}-\frac{\varepsilon C_{min}}{C_C}e^{-UA\left(\frac{1}{C_h}-\frac{1}{C_c}\right)}$$

$$1-e^{-UA\left(\frac{1}{C_h}-\frac{1}{C_c}\right)}=\varepsilon C_{min}\left(\frac{1}{C_h}-\frac{1}{C_C}e^{-UA\left(\frac{1}{C_h}-\frac{1}{C_c}\right)}\right)$$

$$\frac{1-e^{-UA\left(\frac{1}{C_h}-\frac{1}{C_c}\right)}}{C_{min}\left(\frac{1}{C_h}-\frac{1}{C_C}e^{-UA\left(\frac{1}{C_h}-\frac{1}{C_c}\right)}\right)}=\varepsilon$$

$If\, C_c<C_h then\, C_h=C_{max};\wedge C_c=C_{min}$ and hence

$$\varepsilon=\frac{1-e^{-UA\left(\frac{1}{C_{max}}-\frac{1}{C_{min}}\right)}}{C_{min}\left(\frac{1}{C_{max}}-\frac{1}{C_{min}}e^{-UA\left(\frac{1}{C_{max}}-\frac{1}{C_{min}}\right)}\right)}$$

$$\varepsilon=\frac{1-e^{-UA\left(\frac{1}{C_{max}}-\frac{1}{C_{min}}\right)}}{C_{min}\left(\frac{1}{C_{max}}-\frac{1}{C_{min}}e^{-UA\left(\frac{1}{C_{max}}-\frac{1}{C_{min}}\right)}\right)}$$

$$\varepsilon=\frac{1-e^{\frac{-UA}{C_{min}}\left(\frac{C_{min}}{C_{max}}-1\right)}}{\left(\frac{C_{min}}{C_{max}}-e^{\frac{-UA}{C_{min}}\left(\frac{C_{min}}{C_{max}}-1\right)}\right)}$$

$$\varepsilon=\frac{1-e^{-NTU(C-1)}}{C-e^{-NTU(C-1)}}$$

$$\varepsilon=\frac{\frac{1-e^{-NTU(C-1)}}{e^{-NTU(C-1)}}}{\frac{C-e^{-NTU(C-1)}}{e^{-NTU(C-1)}}}$$

$$\varepsilon=\frac{\frac{1-e^{-NTU(C-1)}}{e^{-NTU(C-1)}}}{\frac{C-e^{-NTU(C-1)}}{e^{-NTU(C-1)}}}$$

$$\varepsilon=\frac{e^{-NTU(1-C)}-1}{C\,e^{-NTU(1-C)}-1}$$

$$\varepsilon = \frac{1-e^{-NTU(1-C)}}{1-Ce^{-NTU(1-C)}}$$

For Condensers and evaporators C=0 as $C_{max} = \infty$

$$\varepsilon = 1-e^{-NTU(1-C)}$$

Regenerators C=0

$$\varepsilon = \frac{1-e^{-NTU}}{1-Ce^{-NTU}}$$

NTU Method:

By Using Non Dimensional number ie NTU and C(Capacity ratio) effectiveness can be determined from equation of effectiveness in terms of NTU and C .From definition of Effectiveness Outlet temperatures can be found out . This is called NTU method. This method is more useful when outlet temperatures fluids are Unknown

NTU Method Calculation is given below steps

1. Calculate heat capacities of hot and cold fluid using $C_c = m_c C_{pc}$ and $C_h = m_c C_{ph}$
   $If\ C_c < C_h then\ C_h = C_{max}; \wedge C_c = C_{min}$
   $If\ C_h < C_C then\ C_h = C_{min}; \wedge C_c = C_{max}$
   $¿ calcualte\ Capcity\ ratio \quad C = \frac{C_{min}}{C_{max}}$
2. Calculate NTU $¿ \frac{UA}{C_{min}}$
3. Using C and NTU determine effectiveness of heat exchanger either using equation in terms C and NTU or plot NTU verses C
4. Determine the unknown temperatures of hot and cold fluids using equation of heat exchanger in terms of temperatures
5. Calculate Q using $Q = m_c C_{pc}(T_{c0} - T_{ci})$ = $m_h C_{ph}(T_{hi} - T_{h0})$

Boiling

Boiling Types: When evaporation occurs at a solid-liquid interface, it is called as *"boiling".* The boiling process occurs when the temperature of the surface T_w exceeds the saturation temperature T_{sat} corresponding to the liquid pressure. Heat is transferred from the solid surface to the liquid, and the appropriate form of Newton's law of cooling is

$$q_w = h\,[T_w - T_{sat}] = h\,\Delta T_e$$

Where $\Delta T_e = [T_w - T_{sat}]$ and is termed as the *“excess temperature”*.

Applications

1. Steam production (Power Plants, Space heating etc)
2. Heat absorption in refrigeration and Air conditioning systems
3. Distillation and Refining of liquids
4. Drying food materials
5. Cooling the machines liked nuclear reactors and rocket motors

The boiling process is characterized by the formation of vapour bubbles which grow and subsequently detach from the surface. Vapour bubble growth and dynamics depend, in a complicated manner, on the excess temperature ΔT_e, the nature of the surface, and the thermo-physical properties of the fluid, such as its surface tension. In turn the dynamics of vapour bubble growth affect fluid motion near the surface and therefore strongly influence the heat transfer coefficient.

Classification Boiling Process

1. Pool Boiling 2. Forced Convection boiling 3. Sub-cooled Boiling (Local Boiling)

4. Saturated Boiling

Pool Boiling

If the liquid is quiescent and if its motion near the surface is due to free convection and due to mixing induced by bubble growth and detachment, then such a boiling process is called *“pool boiling”.*

Forced Convection Boiling

In *“forced convection boiling”*, the fluid motion is induced by an external means as well as by free convection and bubble induced mixing

Boiling may also be classified as *“sub-cooled boiling(Local Boiling) ”* and *“saturated boiling”*.

Sub-cooled boiling(Local Boiling)

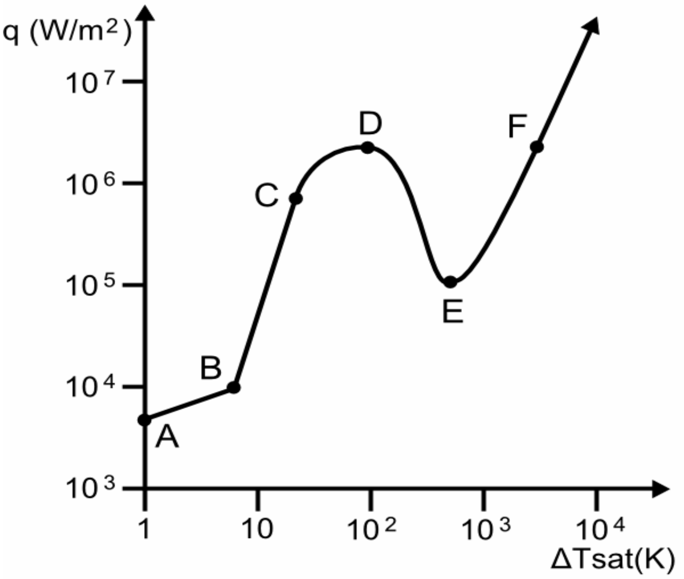
In sub-cooled boiling, the temperature of the liquid is below the saturation temperature and the bubbles formed at the surface may condense in the liquid.

Saturated Boiling

In saturated boiling, the temperature of the liquid slightly exceeds the saturation temperature. Bubbles formed at the surface are then propelled through the liquid by buoyancy forces, eventually escaping from a free surface.

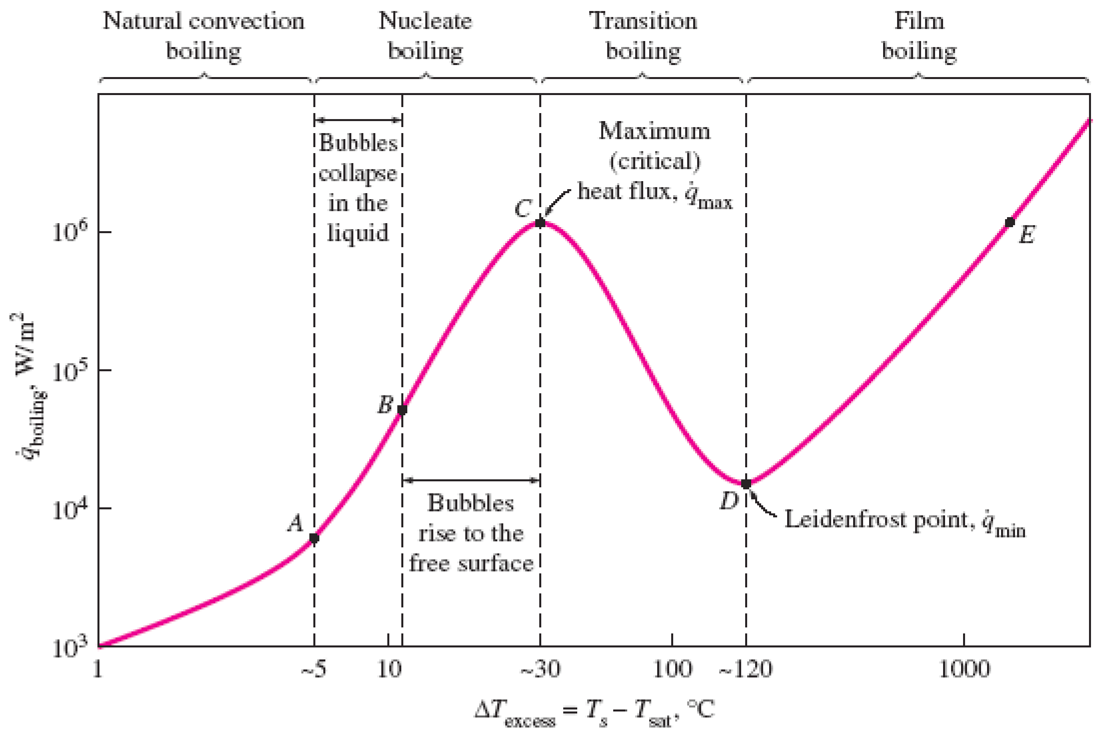
Pool Boiling Regimes: The first investigator who established experimentally the different regimes of pool boiling was Nukiyama. He immersed an electric resistance wire into a body of saturated water and initiated boiling on the surface of the wire by passing electric current through it. He determined the heat flux as well as the temperature from the measurements of current and voltage.

Figure shows characteristics of pool boiling for water at atmospheric pressure. This boiling curve illustrates the variation of heat flux or the heat transfer coefficient as a function of excess temperature ΔT_e. This curve pertains to water at 1 atm pressure.From Eq. (8.27) it can seen that



Pool boiling regimes:
A-B: Pure convection with liquid rising to surface for evaporation
B-C: Nucleate boiling with bubbles condensing in liquid
C-D: Nucleate boiling with bubbles rising to surface
D: Peak temperature
D-E: Partial nucleate boiling and unstable film boiling
E: Film boiling is stabilized
E-F: Radiation becomes a dominant mechanism for heat transfer

q_w depends on the heat transfer coefficient h and the excess temperature ΔT_e.



Free Convection Regime(point 0 to point A):- Free convection is said to exist if $\Delta T_e \leq 5\ ^0$ C. Heat transfer from the heated surface causes the liquid in the vicinity of the surface to be superheated . The superheated liquid raises to the free surface by natural convection where vapor produced by the evaporation

Nucleate Boiling Regime(Between points Aand B):- Nucleate boiling exists in the range $5\ ^0$ C $\leq \Delta T_e \leq 30\ ^0$ C. In this range, two different flow regimes may be distinguished. In the region A-B, very few bubbles are formed . These bubbles grow and get detached and rises to free surface In this regime most of the heat exchange is through direct transfer from the surface to liquid in motion at the surface, and not through vapour bubbles rising from the surface.

As ΔT_e is increased beyond 10 ^{0}C (Region B-C), the rate of bubble formation and number of location where bubble formed increases and the bubble generation rate is so high that continuous columns of vapour appear. As a result very high heat fluxes are obtainable in this region. In practical applications, the nucleate boiling regime is most desirable, because large heat fluxes are obtainable with small temperature differences. In the nucleate boiling regime, the heat increases rapidly with increasing excess temperature ΔT_e until the peak heat flux is reached. The location of this peak heat flux is called the *burnout point,* or *departure from nucleate boiling* (DNB), or the *critical heat flux* (CHF). The reason for calling the critical heat flux the burnout point is apparent from the Fig. 8.4. Such high values of ΔT_e may cause the burning up or melting away of the heating element.

Film Boiling Regime:- It can be seen from Fig. that after the peak heat flux is reached, any further increase in ΔT_e results in a reduction in heat flux. The reason for this curious phenomenon is the blanketing of the heating surface with a vapour film which restricts liquid flow to the surface and has a low thermal conductivity. This regime is called the *film boiling regime.*

The film boiling regime can be separated into three distinct regions namely (i) the *unstable film boiling region,* C-D (ii) the *stable film boiling region* and (iii) *radiation dominating region.*

In the unstable film boiling region, the vapour film is unstable, collapsing and reforming under the influence of convective currents and the surface tension. Here the heat flux decreases as the surface temperature increases, because the average wetted area of the heater surface decreases.

In the stable film boiling region, the heat flux drops to a minimum, because a continuous vapour film covers the heater surface.In the radiation dominating region, the heat flux begins to increase as the excess temperature increases, because the temperature at the heater surface is sufficiently high for thermal radiation effects to augment heat transfer through the vapour film.

Condensation:

Condensation occurs whenever a vapour comes into contact with a surface at a temperature lower than the saturation temperature of the vapour corresponding to its vapour pressure.

There are 2 types of condensation

i) Filmwise Condensation ii) Dropwise Condensation

If the liquid thus formed due to condensation wets the solid surface and the condensate flows on the surface in the form of a film and this type of condensation is called "***film-wise condensation***".

If the liquid thus formed due to condensation wets the solid surface and , the condensate collects in the form of droplets, which either grow in size or combine with neighboring droplets and eventually roll on the surface under the influence of gravity. This type of condensation is called “***drop-wise condensation***”.

During the condensation if traces of oil are present on highly polished surface, the film of condensation is broken into droplets and condensation takes place in the form of drops

Difference between Filmwise condensation and Dropwise condensation

1	During condensation a thin continuous film of liquid is formed. Film of liquid formed falls down due to gravity	A film is broken into droplets of liquid and fall down the surface in random fashion
2	It wets the surface	Does not wet the surface
3	Surface does not directly exposed to surface since film of condensate is formed between the surface and vapor	Larger area of condensing surface is exposed to vapor
4	Heat transfer rate is less as film of liquid formed gives thermal resistance for heat flow	Heat transfer rate is 5 to 10 times greater compared to filmwise condensation
5		Dropwise condensation is preferred to filmwise condesation

Laminar Film Condensation on vertical plate

Assumptions

1. The plate is maintained at a uniform temperature T_w is less than saturation temperature of the vapor
2. The film of liquid formed flows under the action of gravity only.
3. The condensate flow is laminar and fluid properties are constant
4. Heat transfer across the the condensate layer only due to pure conduction
5. Viscous shear and gravitational forces are assumed to be act on the fluid . Normal viscous forces and inertia forces are neglected
6. The shear stress at the liquid –vapor interface is negligible
7. Condensing vapor is entirely clean and free from gases , air and condensing impurities
8. The liquid film is in good thermal contact with the cooling surface

Consider a small element of thickness dx and breadth $(\delta - y)$ and unit width as shown in figure

1. A metal clad heating element of 10 mm diameter and of emissivity 0.92 is submerged in water bath horizontally. If the surface temperature of the plate is 260 ºC under study boiling conditions, calculate the power

dissipation per unit length of the heater. Assume that the water is exposed to atmospheric pressure and is at uniform temperature (7c,10,Dec14/Jan16)

1. Discuss modes of condensation (7a, 04, Dec17/Jan18)
2. Explain the influence of non- condensable gases in condensation process (7a,04,June/July15)
3. Explain filmwise and dropwise condensation (7a,04,June/July14)
4. Differentiate between the mechanism of filmwise and dropwise condensation. Explain Why dropwise condensation is preferred over filmwise condensation (7b,06,June/July15)

1. A vertical Square Plate 30 cm x 30 cm is exposed to steam at atmospheric pressure. The plate temperature is 98 °C. Calculate the heat transfer and mass of steam condensed per hour (7c, 07,June/July18)

Solution

At atmospheric pressure

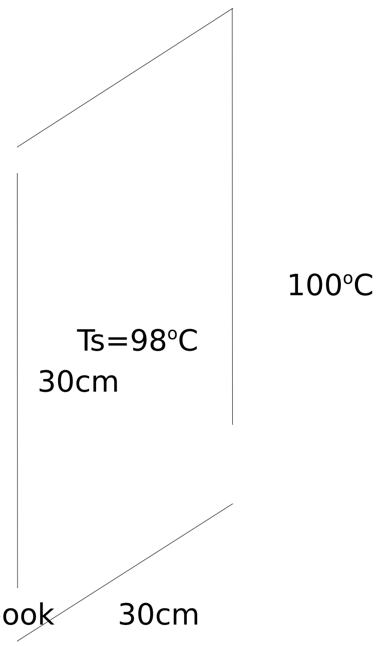
T_{sat}=100°C

T_v=T_{sat}=100°C

$At\ saturaturation\ temperature = 100^0 C, h_{fg} = 2256.9\, kJ/kg$ ie $h_{fg} = 2256.9\, x\, 10^3\, J/kg$

Plate temperature = 98°C ie T_s=98°C

$T_f = \frac{T_V + T_s}{2}$ ie $T_f = \frac{100+98}{2} = 99^o C$



At $99^o C$ Refer water properties

$$\rho = 974 + \frac{961-974}{100-80}(99-80) = 961.65\, kg/m^3$$

$$\gamma = \left[0.364 + \frac{0.293-0.364}{100-80}(99-80)\right] x\, 10^{-6} = 0.36045\, x\, 10^{-6}$$

$\mu_l = \rho\, \gamma$ ie $\mu_l = 961.65\, x\, 0.36045\, x\, 10^{-6} = 3.466\, x\, 10^{-4}$

$$k = 0.6687 + \frac{0.6804-0.6687}{100-80}(99-80) = \frac{0.6715\, W}{mK}$$

Heat transfer coefficient for vertical plate from data hand book

$$h = 0.943\left[\frac{k^3 \rho^2 g\, h_{fg}}{\mu_l L (T_V - T_s)}\right]^{0.25}$$

$L = vertical\ hieght\ for\ vertical\ plate\ ie\ L = 0.3\, m$

$$h = 0.943\left[\frac{0.6715^3\, x\, 961.65^2\, x\, 9.81\, x\, 2256.9\, x\, 10^3}{3.466\, x\, 10^{-4}\, x\, 0.3(100-98)}\right]^{0.25}$$

$h = 12390.96\, W/m^2 K$

Rate of heat transfer

$$Q=hA\left(T_V-T_s\right)$$

$$Q=12390.96\,x\left(0.3\,x\,0.3\right)x\left(100-98\right)$$

Q=2230.37 Watts

Mass of steam condensed

$$\acute{m}=\frac{Q}{h_{fg}} \qquad \acute{m}=\frac{2256.9\,x\,10^3}{2230.37} \text{ kg/s}$$

2. A vertical Square Plate 300 m x 300m is exposed to steam at atmospheric pressure. The plate temperature is 98°C. Calculate the heat transfer and the mass of steam condenser per hour (10b, 08,Dec18/Jan19,15 scheme)
3. Steam at 0.065bar condenses on a vertical plate of 0.6 m square. If the surface temperature of the plate is maintained at 15 °C , estimate the rate of condensation, T_s = 37.7 °C , h_{fg}=2412 x 10^3 J/ kg

   The properties of water at mean temperature 26.4 °C are listed below

   $\rho=$¿ 1000 kg/m³, K=0.913W/mK, μ =864 x 10^{-6} kg/ms (7c, 10, june/July2017)

4. A vertical plate 500 mm high and maintained at 30 °C is exposed to saturated steam at atmospheric pressure. Calculate the following i) rate of heat transfer ii) condensate rate / hour / m width of the plate for film condensation

   Properties of water at mean film temperature are $\rho=$¿ 980.3 kg/m³, K=66.4x10^{-2}W/m°C, μ =434 x 10^{-6} kg/ms and h_{fg}=2257 k J/kg. Assume vapour density is small compared to that of condensate (7c,07, Dec18/Jan19)

   (hint: Area A=0.5x1=0.5m², L=height 500mm =0.5m)

5. Dry saturated steam at atmospheric pressure condenses on a vertical tube of diameter 5 cm and length 1.5 m. If the surface is maintained at 80 °C, determine heat transfer rate and the mass of steam condenser per hr (7c,08,June/July2013)

At atmospheric pressure

T_{sat}=100°C

T_v=T_{sat}=100°C

$At\ saturaturation\ temperature=100^0\,C,\ h_{fg}=2256.9\,kJ/kg$ ie $h_{fg}=2256.9\,x\,10^3\,J/kg$

Plate temperature = 98°C ie T_s=98°C

D=5cm

$$T_f=\frac{T_V+T_s}{2} \quad \text{ie} \quad T_f=\frac{100+80}{2}=90^o\,C$$

At $90^o\,C$ Refer water properties

$\rho=974+\frac{961-974}{100-80}(90-80)=961.65\,kg/m^3$ $T_v=100°C$

L=1.5m

$\gamma=\left[0.364+\frac{0.293-0.364}{100-80}(90-80)\right]x\,10^{-6}=0.36045\,x\,10^{-6}$

$\mu_l=\rho\,\gamma$ ie $\mu_l=961.65\,x\,0.36045\,x\,10^{-6}=3.466\,x\,10^{-4}$

$k=0.6687+\frac{0.6804-0.6687}{100-80}(90-80)=\frac{0.6715\,W}{mK}$ $T_s=80°C$

Heat transfer coefficient for vertical plate from data hand book

$$h=0.943\left[\frac{k^3\rho^2 g\,h_{fg}}{\mu_l L(T_V-T_s)}\right]^{0.25}$$

$L=vertical\,hieght\,for\,vertical\,plate\,ie\,L=0.3\,m$

$$h=0.943\left[\frac{0.6715^3\,x\,961.65^2\,x\,9.81\,x\,2256.9\,x\,10^3}{3.466\,x\,10^{-4}\,x\,0.3(100-98)}\right]^{0.25}$$

$h=12390.96\,W/m^2K$

Rate of heat transfer

$Q=hA(T_V-T_s)$

$Q=12390.96\,x(0.3\,x\,0.3)\,x(100-98)$

Q=2230.37 Watts

Mass of steam condensed

$\acute{m}=\frac{Q}{h_{fg}}$ $\acute{m}=\frac{2256.9\,x\,10^3}{2230.37}$ kg/s

6. Dry saturated steam at a pressure of 2.0 bar condenses on the surface of vertical tube of height 1 m. The tube surface is kept at 117 °C. Estimate the thickness of the condensate film and heat transfer coefficient at a distance of 0.2 m from the upper end of the tube. Assume the condensate film to be laminar. Also calculate the average heat transfer coefficient over the entire length of the tube (10c , 08, June/July18,15 scheme)

7. A vertical tube (Taking experimental value) of 60 mm OD and 1.2 mtr long is exposed to steam at atmospheric pressure. The outer surface of the tube is maintained at a temperature of 50°C by circulating cold water through the tubes. Calculate i) rate of heat transfer to the coolant ii) the rate of condensation of steam. Assuming the condensation film is laminar and TPP of water at 75 °C are $\rho_L=¿$ 975kg/m^3 $\mu_L=3$ 75 x 10^{-6} Ns/m^2 K= 0.67W/m°C. The properties of saturated vapour tsat=100 °C $\rho v=¿$ 0.596 kg/m^3, h_{fg}=2257 k J/kg. (7c, 08,Dec15/jan16)

D=60mm

$$h = 0.943\left[\frac{k^3 \rho^2 g h_{fg}}{\mu_l L (T_V - T_s)}\right]^{0.25}$$

$L = vertical\, hieght\, for\, vertical\, Cylinder\, ie\, L = 1.2m$

$h_{fg} =$ ¿ 2257 k J/kg ¿ $2257\, x\, 10^3\, J/kg$

$T_V = T_{sat} = 100^o C$

$100^o C$

$$h = 0.943\left[\frac{0.67^3\, x\, 975^2\, x\, 9.81\, x\, 2257\, x\, 10^3}{375\, x\, 10^{-6}\, x\, 1.2(100-50)}\right]^{0.25}$$

DDDD L=1.2m

$h = 3862.1\, W/m^2 K$

Rate of heat transfer

$Q = hA(T_V - T_s)$

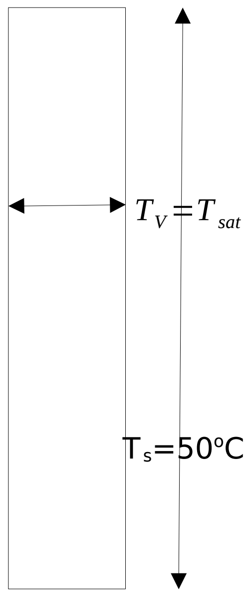
$Q = 3862.1\, x\,(\pi x\, 0.06\, x\, 1.2)\, x\,(100-98)$

Q= $43.679\, x\, 10^3$ Watts

Mass of steam condensed

$\acute{m} = \frac{Q}{h_{fg}}$ $\acute{m} = \frac{43.679\, x\, 10^3}{2257\, x\, 10^3}$ kg/s

$\acute{m} = 0.01935\, kg/s$



8. Saturated steam at 90°C and 70 kPa is condensed on outer surface of a 1.5 m long 2.5 m diameter vertical tube maintained at uniform temperature of 70 °C .Assuming film wise condensation, calculate the heat transfer rate on the tube surfaces (7b, 08, Dec17/Jan18)

(T_{sat}=90°C, $T_f = \frac{T_V + T_s}{2}$ ie $T_f = \frac{90+70}{2} = 80^0 C$ Take properties of water at 80°C for k, ρ, μ

Take $h_{fg}\, for\, either$ 90°C or 70 kPa=0.7 bar from steam table) L=vertical height of cylinder=2.5m

9. A tube of 13 mm in outer diameter and 1.5 m long is used to condense the steam at 40 kPa (T_{sat} =76 °C) .Calculate the heat transfer coefficient for this tube in a) horizontal position b) vertical position.Ttake average to wall temperature as 52 °C (7c, 08, Dec17/Jan18)

10. A tube of 15 mm outside diameter and 1.5 m long is used for condensing steam at 40 kPa. calculate the average heat transfer Coefficient when the tube is i) horizontal and ii) vertical and its surface temperature is mentioned at 52 ºC (7c,08,June/July15)

At 40kPa =0.4bar from steam table

T_{sat}=76ºC
T_v=T_{sat}=100ºC

$At\ saturaturation\ temperature=0.4\ \bar{h}_{fg}=2319.2\ kJ/kg$ ie $h_{fg}=2319.2\ x\ 10^{3}\ J/kg$

Surface temperature = 52ºC ie T_s=52ºC

$T_f=\frac{T_V+T_s}{2}$ ie $T_f=\frac{76+52}{2}=64^{o}C$

At $64^{o}C$ Refer water properties

$\rho=985+\frac{974-985}{80-60}(64-60)=982.8\ kg/m^{3}$

$\gamma=\left[0.478+\frac{0.364-0.478}{80-60}(64-60)\right]x\ 10^{-6}=0.4552\ x\ 10^{-6}\ m^{2}/s$

$\mu_l=\rho\ \gamma$ ie $\mu_l=982.8\ x\ 0.4552\ x\ 10^{-6}=4.473\ x\ 10^{-4}$

$k=0.6513+\frac{0.6687-0.6513}{80-60}(64-60)=0.65478\ W/mK$

Case 1: Horizontal Position

Heat transfer coefficient for Horizontal cylinder from data hand book

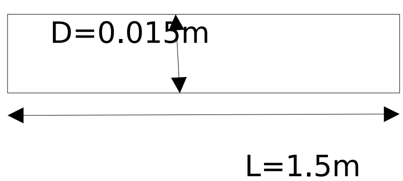
$$h=0.728\left[\frac{k^{3}\rho^{2}g\,h_{fg}}{\mu_l D(T_V-T_s)}\right]^{0.25}$$

$D=15\ mm=0.015\ m$

$$h=0.728\left[\frac{0.65478^{3}x\ 982.8^{2}x\ 9.81\ x\ 2319.2\ x\ 10^{3}}{4.473\ x\ 10^{-4}x\ 0.015(76-52)}\right]^{0.25}$$

$T_V=76^{0}C;T_s=52^{0}C$

$h=10183.47\ W/m^{2}K$



Rate of heat transfer

$Q=hA(T_V-T_s)$

$Q=10183.47\ x\ (\pi x\ 0.015\ x\ 1.5)\ x\ (76-52)$

$Q=$ ¿ 17275.84Watts

Mass of steam condensed

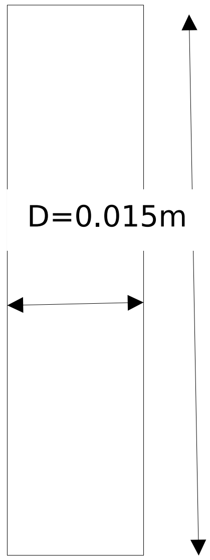
$$\acute{m}=\frac{Q'}{h_{fg}}$$

$$\acute{m}=\frac{17275.84}{2319.2\,x\,10^{3}}\ \text{kg/s}$$

$$\acute{m}=7.449\,x\,10^{-3}\,kg/s$$

Case 2 Vertical Position

L=Hieght of the cylinder=1.5m



$$h=0.943\left[\frac{k^3\rho^2\,g\,h_{fg}}{\mu_l\,L\left(T_V-T_s\right)}\right]^{0.25}$$

$$L=1.5\,m$$

$$h=0.943\left[\frac{0.65478^3\,x\,982.8^2\,x\,9.81\,x\,2319.2\,x\,10^3}{4.473\,x\,10^{-4}\,x\,1.5\left(76-52\right)}\right]^{0.25}$$

$$h=4171.3\,W/m^2K$$

L=1.5m

Rate of heat transfer

$$Q=hA\left(T_V-T_s\right)$$

$$Q=4171.3\,x\left(\pi x\,0.015\,x\,1.5\right)x\left(76-52\right)$$

$Q=$¿ 7077.6 Watts

Mass of steam condensed

$$\acute{m}=\frac{Q'}{h_{fg}}$$

$$\acute{m}=\frac{7077.6}{2319.2\,x\,10^{3}}\ \text{kg/s}$$

$$\acute{m}=3.052\,x\,10^{-3}\,kg/s$$

11. A 12 cm outside diameter and 2 m long tube is used in a big condenser to condense the steam at 0.4 bar. Estimate the unit surface conductance i) in vertical position ii) in horizontal position. Also find the amount of condensate formed per hour in both the cases. The saturation temperature of the steam 74.5°C . Average wall temperature =50°C.

The properties of water film at average temperature of $\frac{74.5+50}{2}$ =62.7°C are given below

$\rho=$¿ 982.2 kg/m^3, h_{fg}=2480 k J/kg ,K=0.65W/mK, and . μ =0.47 x 10^{-3}kg/ms

12. Air free saturated steam at 85 °C and pressure of 57.8 KPa condenses on the outer surface of 225 horizontal tubes of 1.27 cm outside diameter arranged in 15x15 array. Tube surfaces are maintained at uniform

temperature of 75 °C .Calculate the total condensation rate /metre length of the tube bundle.
(7c,08,June/July16)

T_{sat}=85°C
T_v=T_{sat}=85°C

$At\ saturaturation\ temperature = 85\,C\ h_{fg} = 2293\,kJ/kg$ ie $h_{fg} = 2293\,x\,10^3\,J/kg$
Surface temperature =75°C ie T_s=75°C

$T_f = \frac{T_V + T_s}{2}$ ie $T_f = \frac{85+75}{2} = 80^o\,C$

At $80^o\,C$ Refer water properties

$\rho = 974\,kg/m^3$

$\gamma = 0.364\,x\,10^{-6}\,m^2/s$

$\mu_l = \rho\,\gamma$ ie $\mu_l = 974\,x\,0.364\,x\,10^{-6} = 3.545\,x\,10^{-4}$
$k = 0.6687\,W/mK$

Heat transfer coefficient for Horizontal cylinder of multiple tubes arranged in array from data hand book

$$h = 0.728\left[\frac{k^3 \rho^2 g\,h_{fg}}{\mu_l\,N\,D(T_V - T_s)}\right]^{0.25}$$

$where\ N$ = number of horizontal rows placed one above the other (irrespective of the number of tubes in horizontal row)
$Noumber\ of\ tubes\ 225 \in 15\,x\,15\,array$ ie 15 horizontal rows and in each row there are 15 tubes in each horizontal row. Therefore N=15
$T_V = 85^0\,C\,T_s$ = $75^0\,C$
$D = 1.27\,cm = 0.0127\,m$ Total
Number of Tubes =225

$$h = 0.728\left[\frac{0.6687^3\,x\,974^2\,x\,9.81\,x\,2293\,x\,10^3}{3.545\,x\,10^{-4}\,x\,15\,x\,0.0127(85-75)}\right]^{0.25}$$

N=15 (no of horizontal rows)
$h = ¿$ 7177.52W/m²K
Rate of heat transfer
$Q = hA\,(T_V - T_s)$
$A = \pi DLxno\ of\ tubes$
No of tubes =15x15=225; L =1m
$Q = 7177.52\,x\,(\pi x\,0.0127\,x\,1\,x\,225)\,x\,(85-75)$
$Q = ¿$ 644333.17Watts
15 horizontal rows

15

tubes in each Horizontal row

Mass of steam condensed

$$\acute{m}=\frac{Q}{h_{fg}}$$

$$\acute{m}=\frac{644333.17}{2293\,x\,10^{3}}$$ kg/s

$$\acute{m}=0.281\,kg/s$$