DIGITAL SIGNAL PROCESSING

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Frequency domain sampling and Reconstruction of Discrete time signal:

DTS $\rightarrow$ equivalent Frq. domain $\rightarrow$ Fourier Transform
$x(n)$ $X(e^{jw})$

FT $\rightarrow$ sampled $\rightarrow$ a Frq. domain $\underline{X(K)}$ $\rightarrow$ DFT
$X(e^{jw})$

DFT $\rightarrow$ powerful tool

F.T. & DFT

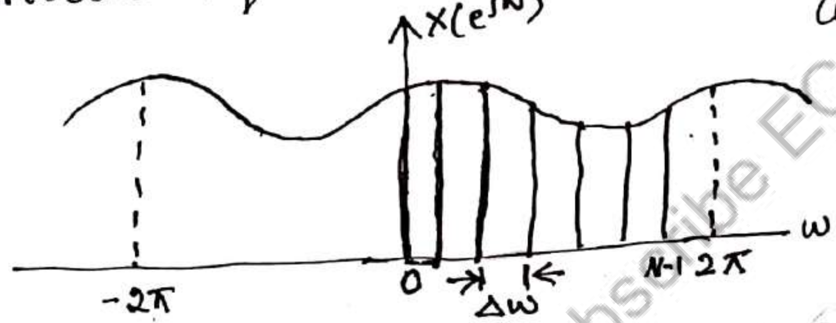
non-periodic discrete time signal $x(n) \rightarrow X(e^{jw})$

$$X(e^{jw}) = X(W) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-jwn} \longrightarrow ①$$

$\downarrow$

Periodic $\rightarrow$ fundamental frq range $0 < w < 2\pi$ rad

$\downarrow$ Continous fun



N equidistant samples $0 \leq w < 2\pi$

$$\Delta w = \frac{2\pi}{N}$$

in eqn $w = \frac{2\pi}{N}\cdot K$; $0 \leq K \leq N-1$

$K=0 \rightarrow 0$ $\quad K=1 \rightarrow \frac{2\pi}{N}$ $\quad K=2 \rightarrow \frac{4\pi}{N}$

$$X\left(\frac{2\pi}{N}\cdot K\right) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\frac{2\pi}{N}kn} \; ; \; 0 \leq K \leq N-1 \longrightarrow ②$$

$x(n) \rightarrow$ infinate no sumations.
$\therefore$ 'N' samples.

$$X\left(\frac{2\pi}{N}\cdot K\right) = \cdots + \sum_{n=-N}^{-1} x(n) e^{-j\frac{2\pi}{N}kn} + \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn} + \sum_{n=N}^{2N-1} x(n) e^{-j\frac{2\pi}{N}kn} + \cdots \longrightarrow ③$$

$$③ \Rightarrow X\left(\frac{2\pi}{N}\cdot K\right) = \sum_{l=-\infty}^{\infty} \sum_{n=lN}^{lN+N-1} x(n)\, e^{-j\frac{2\pi}{N}\cdot kn} \longrightarrow ④$$

$\boxed{n = n - lN} \Rightarrow lN = n - n$
$\boxed{lN = 0}$

$$X\left(\frac{2\pi}{N}\cdot K\right) = \sum_{l=-\infty}^{\infty} \sum_{n=0}^{N-1} x(n-lN)\, e^{-j\frac{2\pi}{N}\cdot kn}$$

$$X\left(\frac{2\pi}{N}\cdot K\right) = \sum_{n=0}^{N-1} \underbrace{\sum_{l=-\infty}^{\infty} x(n-lN)}_{\text{Periodic signal 'N'}\; x_p(n)} e^{-j\frac{2\pi}{N}\cdot kn}$$

Frequency domain Sampling and Reconstruction of Discrete time signal:

$$X\left(\frac{2\pi}{N}k\right) = \sum_{n=0}^{N-1} \underbrace{\sum_{l=-\infty}^{\infty} x(n-lN)}_{x_p(n)} e^{-j\frac{2\pi}{N}kn}$$

$$\boxed{X\left(\frac{2\pi}{N}k\right) = \sum_{n=0}^{N-1} x_p(n)\, e^{-j\frac{2\pi}{N}\cdot kn}} \rightarrow (5)$$

DFT eqn: Discrete Fourier Transform

$$\boxed{x_p(n) = \frac{1}{N}\sum_{k=0}^{N-1} X\left(\frac{2\pi}{N}\cdot k\right) e^{j\frac{2\pi}{N}kn}} \rightarrow (9)$$

$; 0 \le n \le N-1$

IDFT eqn:

Fourier Series

$$x_p(n) = \sum_{k=0}^{N-1} a_k\, e^{j\frac{2\pi}{N}kn} ; \quad 0 \le n \le N-1 \rightarrow (6)$$

$$a_k = \frac{1}{N}\sum_{n=0}^{N-1} x_p(n)\, e^{-j\frac{2\pi}{N}\cdot kn} ; \quad 0 \le k \le N-1 \rightarrow (7)$$

a_k → Fourier co-efficient.

Compare eqn (7) and eqn (5)

$$a_k = \frac{1}{N}\cdot X\left(\frac{2\pi}{N}\cdot k\right) ; \quad 0 \le k \le N-1 \rightarrow (8)$$

put eqn (8) in eqn (6)

$$x_p(n) = \sum_{k=0}^{N-1} \frac{1}{N}\cdot X\left(\frac{2\pi}{N}\cdot k\right) e^{j\frac{2\pi}{N}kn} ; \quad 0 \le n \le N-1$$

(9) ⇒ gives the reconstruction of Periodic Signal $x_p(n)$ from the samples of F.T. $X(e^{j\omega})$

DFT and IDFT equations:

DFT → Discrete Fourier Transform

IDFT → Inverse Discrete Fourier Transform.

DFT → $x(n)$; $0 \le n \le N-1$

$$\boxed{DFT\{x(n)\} = X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}\cdot Kn}}$$

$0 \le K \le N-1$

eqn ⑤ $X\left(\frac{2\pi}{N}K\right) = \sum_{n=0}^{N-1} x_p(n)\, e^{-j\frac{2\pi}{N}\cdot KN}$

$$\boxed{DFT\{x(n)\} = X(K) = \sum_{n=0}^{N-1} x(n)\, W_N^{Kn}}$$

$0 \le K \le N-1$

$W_N = e^{-j\frac{2\pi}{N}}$ → phase Factor (or) Twiddle Factor

$X(K)$ → Sequence in frq domain Lenth - 'N'

$x(n)$ → Discrete time signal

IDFT,

$$\boxed{IDFT\{X(K)\} = x(n) = \frac{1}{N}\sum_{K=0}^{N-1} X(K)\, e^{j\frac{2\pi}{N}Kn}}$$

$0 \le n \le N-1$

eqn ⑨ $x_p(n) = \frac{1}{N}\sum_{K=0}^{N-1} X\left(\frac{2\pi}{N}K\right) e^{j\frac{2\pi}{N}\cdot Kn}$.

$$\boxed{IDFT\{X(K)\} = x(n) = \frac{1}{N}\sum_{K=0}^{N-1} X(K)\, W_N^{-Kn}}$$

$0 \le n \le N-1$

Problems - DFT

Compute the N-Point DFT of the signal given by,

ⓐ $x(n) = \delta(n)$

$$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}Kn}$$

$$X(K) = \sum_{n=0}^{N-1} \delta(n)\, e^{-j\frac{2\pi}{N}Kn}$$

$$= \delta(0)\cdot \underbrace{e^{-j\frac{2\pi}{N}K\cdot 0}}_{1} \qquad \left\{\begin{array}{l}\delta(n) = 1;\ n=0\\ \quad\ \ = 0;\ n\neq 0\end{array}\right.$$

$$\boxed{X(K) = 1}$$

ⓑ $x(n) = \delta(n-n_0); \quad 0 < n_0 < N$

$$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}Kn}$$

$$= \sum_{n=0}^{N-1} \delta(n-n_0)\, e^{-j\frac{2\pi}{N}Kn}$$

$n - n_0 = 0$

$n = n_0$

$\delta(n-n_0) = 1;\ n = n_0$

$\quad = 0;\ n \neq n_0$

$$X(K) = 1\cdot e^{-j\frac{2\pi}{N}K\cdot n_0}$$

$$\boxed{X(K) = e^{-j\frac{2\pi}{N}Kn_0}}$$

ⓒ $x(n) = a^n; \quad 0 \leq n \leq N-1$

$$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}Kn}$$

$$= \sum_{n=0}^{N-1} a^n e^{-j\frac{2\pi}{N}Kn} = \sum_{n=0}^{N-1}\left[a\cdot e^{-j\frac{2\pi}{N}K}\right]^n$$

$$\boxed{\sum_{N_1}^{N_2} a^n = \frac{a^{N_1} - a^{N_2+1}}{1-a};\ a \neq 1}$$

$$X(K) = \frac{1 - \left[a\, e^{-j\frac{2\pi}{N}K}\right]^N}{1 - a\cdot e^{-j\frac{2\pi}{N}K}}$$

$a\cdot e^{-j2\pi K}$

$e^{-j2\pi K}$

$= \cos(2\pi K) - j\sin(2\pi K)$

$K = 0, 1, 2, \ldots$

$= 1 - 0 = 1$

$$\boxed{X(K) = \frac{1 - a^N}{1 - a\cdot e^{-j\frac{2\pi}{N}K}}}$$

Problems on DFT

Find the DFT of the sequence $x(n) = 1;\ 0 \le n \le 2$
$\qquad\qquad\qquad\qquad\qquad\qquad\quad 0;$ otherwise
for $N=4$. Sketch the magnitude and phase spectrum.

$e^{-j\theta} = \cos\theta - j\sin\theta$

$\cos\frac{2\pi k}{4} - j\sin\left(\frac{2\pi k}{4}\right)$

$-1.570 = -\frac{\pi}{2}$ comp. Radian

$1.570 = \frac{\pi}{2}$ $\quad (-j)$

$e^{-j\frac{2\pi}{4}k\cdot 2} = e^{-j\pi k} = e^{-j\pi}$

$\cos\pi - j\sin\pi$
(-1)

$N=4 \qquad x(n) = \{\overset{0}{1}\ \overset{1}{1}\ \overset{2}{1}\ \overset{3}{0}\}$

$$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}Kn}\ ;\ 0 \le K \le N-1$$

$$X(K) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{2\pi}{4}Kn}\ ;\ 0 \le K \le 3$$

$$X(K) = x(0) + x(1)\, e^{-j\frac{2\pi}{4}K} + x(2)\, e^{-j\frac{2\pi}{4}K\cdot 2} + x(3)\, e^{-j\frac{2\pi}{4}K\cdot 3}$$

$\underline{K=0}$
$X(0) = 1 + [1\times 1] + [1\times 1] + [0\times 1]$
$\boxed{X(0) = 3}$

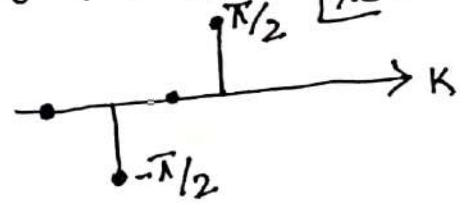
$\underline{K=1}$ $\quad X(1) = 1 + [1\times(-j)] + [1\times(-1)] + [0]$
$\boxed{X(1) = -j}$

$\underline{K=2}$
$X(2) = 1 + [1\times(-1)] + [1\times(1)] + 0$
$\boxed{X(2) = 1}$

$\underline{K=3}$
$X(3) = 1 + [1\times(j)] + [1\times(-1)] + 0$
$\boxed{X(3) = j}$

$\therefore\ X(K) = \{3, -j, 1, j\}$

Magnitude : $|X(K)| = \{3, 1, 1, 1\}$
Phase : $\angle X(K) = \{0, -\frac{\pi}{2}, 0, \frac{\pi}{2}\}$



Find IDFT of $X(K) = \{\underset{X(0)}{6}, \underset{X(1)}{-2+2j}, \underset{X(2)}{-2}, \underset{X(3)}{-2-2j}\}$

$e^{j\theta} = \cos\theta + j\sin\theta$

$e^{j\frac{2\pi}{4}} = \cos\left(\frac{2\pi}{4}\right) + j\sin\left(\frac{2\pi}{4}\right)$

comp quad $(+j)$

$$x(n) = \frac{1}{N}\sum_{K=0}^{N-1} X(K)\, e^{j\frac{2\pi}{N}Kn} \; ; \; 0 \le n \le N-1$$

N=4

$$x(n) = \frac{1}{4}\sum_{K=0}^{3} X(K)\, e^{j\frac{2\pi}{4}Kn} \; ; \; 0 \le n \le 3$$

$$x(n) = \frac{1}{4}\left[X(0) + X(1)\, e^{j\frac{2\pi}{4}n} + X(2)\, e^{j\frac{2\pi}{4}\cdot 2n} + X(3)\, e^{j\frac{2\pi}{4}\cdot 3n}\right]$$

n=0

$$x(0) = \frac{1}{4}\left[X(0) + X(1) + X(2) + X(3)\right]$$

$$x(0) = \frac{1}{4}\left[6 + [-2+2j] + [-2] + [-2-2j]\right]$$

$$\boxed{x(0) = 0}$$

n=1

$$x(1) = \frac{1}{4}\left[X(0) + X(1)\, e^{j\frac{2\pi}{4}} + X(2)\, e^{j\frac{2\pi}{4}\cdot 2} + X(3)\, e^{j\frac{2\pi}{4}\cdot 3}\right]$$

$$x(1) = \frac{1}{4}\left[6 + [-2+2j](j) + (-2)(-1) + (-2-2j)(-j)\right]$$

$$\boxed{x(1) = 1}$$

$$x(n) = \{0, 1, 2, 3\}$$

n=2

$$x(2) = \frac{1}{4}\left[X(0) + X(1)\, e^{j\frac{2\pi}{4}\cdot 2} + X(2)\, e^{j\frac{2\pi}{4}\cdot 4} + X(3)\, e^{j\frac{2\pi}{4}\cdot 6}\right]$$

$$x(2) = \frac{1}{4}\left[6 + [-2+2j](-1) + -2[1] + [-2-2j](-1)\right]$$

$$\boxed{x(2) = 2}$$

n=3

$$x(3) = \frac{1}{4}\left[X(0) + X(1)\, e^{j\frac{2\pi}{4}\cdot 3} + X(2)\, e^{j\frac{2\pi}{4}\cdot 6} + X(3)\, e^{j\frac{2\pi}{4}\cdot 9}\right]$$

$$x(3) = \frac{1}{4}\left[6 + [-2+2j](-j) + -2[-1] + [-2-2j](j)\right]$$

$$\boxed{x(3) = 3}$$

DFT as Linear Transformation [Matrix Method]

$$X(K) = \sum_{n=0}^{N-1} x(n)\, W_N^{Kn} \quad ; \quad 0 \le K \le N-1$$

$W_N \rightarrow$ Twiddle factor (or) phase factor.

$W_N = e^{-j\frac{2\pi}{N}}$; $W_N^K \rightarrow$ periodic 'N'

Put $n = 0, 1, 2 \ldots\ldots N-1$

$$X(K) = x(0)\cdot 1 + x(1)\, W_N^{K\cdot 1} + x(2)\, W_N^{K\cdot 2} + \ldots\ldots + x(N-1)\, W_N^{K\cdot(N-1)}$$

$K=0$

$$X(0) = x(0) + x(1)\cdot 1 + x(2)\cdot 1 + \ldots\ldots + x(N-1)\cdot 1$$

$K=1$

$$X(1) = x(0) + x(1)\, W_N^{1} + x(2)\, W_N^{2} + \ldots\ldots + x(N-1)\, W_N^{1\cdot(N-1)}$$

$\vdots \qquad \vdots \qquad \vdots$

$K=N-1$

$$X(N-1) = x(0) + x(1)\, W_N^{(N-1)\cdot 1} + x(2)\cdot W_N^{(N-1)2} + \ldots\ldots + x(N-1)\, W_N^{(N-1)(N-1)}$$

Matrix

$$\begin{bmatrix} X(0) \\ X(1) \\ \vdots \\ X(N-1) \end{bmatrix}_{N\times 1} = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & W_N & W_N^2 & \cdots & W_N^{(N-1)} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & W_N^{(N-1)} & W_N^{2(N-1)} & \cdots & W_N^{(N-1)(N-1)} \end{bmatrix}_{N\times N} \begin{bmatrix} x(0) \\ x(1) \\ \vdots \\ x(N-1) \end{bmatrix}_{N\times 1}$$

$$\boxed{X_N = W_N \cdot x_N} \rightarrow ① \text{ DFT}$$

Pre multiply by W_N^{-1}

$$W_N^{-1} X_N = \cancel{W_N^{-1}} \cdot \cancel{W_N}\, x_N$$

$$W_N^{-1} X_N = x_N \Rightarrow \boxed{x_N = W_N^{-1} X_N} \rightarrow ②$$

IDFT.

$$x(n) = \frac{1}{N} \sum_{n=0}^{N-1} X(K)\, W_N^{-Kn}$$

$$x(n) = \frac{1}{N} \sum_{n=0}^{N-1} X(K) \left(W_N^{Kn}\right)^{*} \qquad W_N^{*} \rightarrow \text{complex conjugate } W_N$$

$$\boxed{x_N = \frac{1}{N} X_N\, W_N^{*}} \rightarrow ③ \text{ IDFT}$$

compare ② & ③

$$W_N^{-1} = \frac{1}{N} W_N^{*}$$

Find 4-Point DFT of $x(n) = \{1, 0, 0, 1\}$ using Linear Transform | Matrix Method.

$$X_N = W_N x_N \quad \therefore N = 4$$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ 1 & W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ 2 & W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ 3 & W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{array} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix}$$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1+j \\ 0 \\ 1-j \end{bmatrix}$$

$$X(K) = \{2, \underline{1+j}, 0, 1-j\}$$

verify comp $(1 \angle 0 A) + (0 \angle -90 A) + (0 \angle -180 A) + (1 \angle -270 A)$

Method 1

$$W_N = e^{-j\frac{2\pi}{4}} \qquad W_N^0 = 1 \Rightarrow \underline{W_4^0 = 1}$$

$$W_4^1 = e^{-j\frac{2\pi}{4}\cdot 1} = \cos\left(\frac{2\pi}{4}\right) - j\sin\left(\frac{2\pi}{4}\right) = -j$$

$W_4^2 = -1$

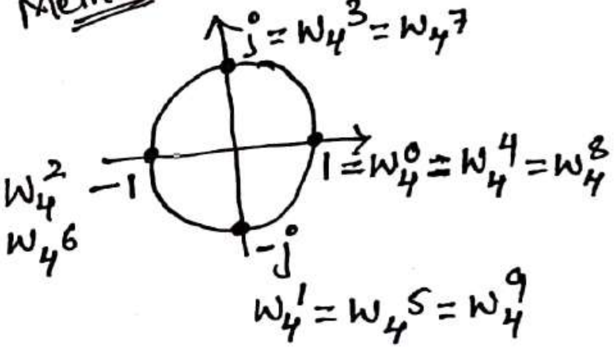
$W_4^3 = +j$

$W_4^4 = 1$

$W_4^6 = -1$

$W_4^9 = -j$

Method 2



$$\frac{360^\circ}{4} = 90^\circ$$

Find 6-Point DFT of $x(n) = \{1,1,0,0,0,2\}$ using Matrix method [linear Transformation]

$$W_N = e^{-j\frac{2\pi}{N}} \; ; \; W_N^0 = 1 \Rightarrow W_6^0 = 1$$

$$W_6^1 = e^{-j\frac{2\pi}{N}\cdot 1} = \cos\left(\frac{2\pi}{6}\right) - j\sin\left(\frac{2\pi}{6}\right) = 0.5 - 0.866j$$

$W_6^2 = -0.5 - 0.866j$

$W_6^3 = -1$

$W_6^4 = -0.5 + 0.866j$

$W_6^5 = 0.5 + 0.866j$

$W_6^6 = 1$

$W_6^8 = -0.5 - 0.866j$

$W_6^9 = -1$

$W_6^{10} = -0.5 + 0.866j$

$W_6^{12} = 1$

$W_6^{15} = -1$

$W_6^{16} = -0.5 + 0.866j$

$W_6^{20} = -0.5 - 0.866j$

$W_6^{25} = 0.5 - 0.866j$

$X_N = W_N x_N \quad \therefore N = 6$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \end{bmatrix} = \begin{array}{c c} & \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} W_6^0 & W_6^0 & W_6^0 & W_6^0 & W_6^0 & W_6^0 \\ W_6^0 & W_6^1 & W_6^2 & W_6^3 & W_6^4 & W_6^5 \\ W_6^0 & W_6^2 & W_6^4 & W_6^6 & W_6^8 & W_6^{10} \\ W_6^0 & W_6^3 & W_6^6 & W_6^9 & W_6^{12} & W_6^{15} \\ W_6^0 & W_6^4 & W_6^8 & W_6^{12} & W_6^{16} & W_6^{20} \\ W_6^0 & W_6^5 & W_6^{10} & W_6^{15} & W_6^{20} & W_6^{25} \end{bmatrix} \end{array} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \\ x(4) \\ x(5) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0.5-0.866j & -0.5-0.866j & -1 & -0.5+0.866j & 0.5+0.866j \\ 1 & -0.5-0.866j & -0.5+0.866j & 1 & -0.5-0.866j & -0.5+0.866j \\ 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -0.5+0.866j & -0.5-0.866j & 1 & -0.5+0.866j & -0.5-0.866j \\ 1 & 0.5+0.866j & -0.5+0.866j & -1 & -0.5-0.866j & 0.5-0.866j \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 2.5+0.866j \\ -0.5+0.866j \\ -2 \\ -0.5-0.866j \\ 2.5-j0.866j \end{bmatrix}$$

$$X(K) = \{4, 2.5+0.866j, -0.5+0.866j, -2, -0.5-0.866j, 2.5-0.866j\}$$

Find 8 point DFT of $x(n) = \{1, 1, 2, 2, 3, 3, 4, 4\}$ using Matrix method [Linear Transformation].

$$X_N = W_N x_N \quad ; N = 8.$$

$$
\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \\ X(6) \\ X(7) \end{bmatrix}
=
\begin{array}{c c}
 & \begin{array}{cccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\
\begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} &
\begin{bmatrix}
W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 \\
W_8^0 & W_8^1 & W_8^2 & W_8^3 & W_8^4 & W_8^5 & W_8^6 & W_8^7 \\
W_8^0 & W_8^2 & W_8^4 & W_8^6 & W_8^8 & W_8^{10} & W_8^{12} & W_8^{14} \\
W_8^0 & W_8^3 & W_8^6 & W_8^9 & W_8^{12} & W_8^{15} & W_8^{18} & W_8^{21} \\
W_8^0 & W_8^4 & W_8^8 & W_8^{12} & W_8^{16} & W_8^{20} & W_8^{24} & W_8^{28} \\
W_8^0 & W_8^5 & W_8^{10} & W_8^{15} & W_8^{20} & W_8^{25} & W_8^{30} & W_8^{35} \\
W_8^0 & W_8^6 & W_8^{12} & W_8^{18} & W_8^{24} & W_8^{30} & W_8^{36} & W_8^{42} \\
W_8^0 & W_8^7 & W_8^{14} & W_8^{21} & W_8^{28} & W_8^{35} & W_8^{42} & W_8^{49}
\end{bmatrix}
\end{array}
\begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \\ x(4) \\ x(5) \\ x(6) \\ x(7) \end{bmatrix}
$$

Method-1

$W_N = e^{-j\frac{2\pi}{N}}$

$W_N^0 = 1 \Rightarrow W_8^0 = 1$

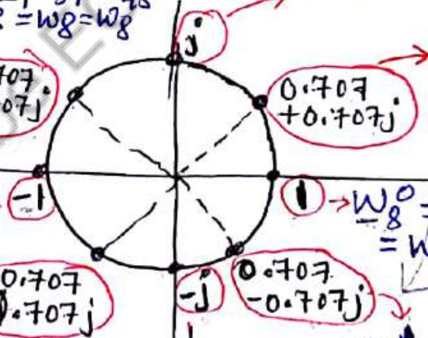
$W_8^1 = e^{-j\frac{2\pi}{8}\cdot 1}$
$= \cos\left(\frac{2\pi}{8}\right) - j\sin\left(\frac{2\pi}{8}\right)$
$= 0.707 - 0.707j$

$W_8^2 = e^{-j\frac{2\pi}{8}\cdot 2}$
$= \cos\left(\frac{4\pi}{8}\right) - j\sin\left(\frac{4\pi}{8}\right)$
$= -j$

Method 2

$W_8^5 = W_8^{13} = W_8^{21} = W_8^{29} = W_8^{37} = W_8^{45}$

$W_8^6 = W_8^{14} = W_8^{22} = W_8^{30} = W_8^{38} = W_8^{46}$



$W_8^7 = W_8^{15} = W_8^{23} = W_8^{31}$
$= W_8^{39} = W_8^{47}$

$W_8^4 = W_8^{12} = W_8^{20} = W_8^{28}$
$= W_8^{36} = W_8^{44}$

$W_8^0 = W_8^8 = W_8^{16} = W_8^{24}$
$= W_8^{32} = W_8^{40} = W_8^{48}$

$W_8^3 = W_8^{11} = W_8^{19}$
$= W_8^{27} = W_8^{35} = W_8^{43}$

$W_8^1 = W_8^9 = W_8^{17} = W_8^{25} = W_8^{33} = W_8^{41} = W_8^{49}$

$W_8^2 = W_8^{10} = W_8^{18}$
$= W_8^{26} = W_8^{33} = W_8^{41} = W_8^{49}$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \\ X(6) \\ X(7) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0.707-0.707j & -j & -0.707-0.707j & -1 & -0.707+0.707j & j & 0.707+0.707j \\ 1 & -j & -1 & j & 1 & -j & -1 & j \\ 1 & -0.707+0.707j & j & 0.707-0.707j & -1 & 0.707+0.707 & -j & -0.707-0.707j \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -0.707-0.707j & -j & 0.707+0.707j & -1 & 0.707-0.707j & j & -0.707+0.707j \\ 1 & j & -1 & -j & 1 & j & -1 & j \\ 1 & 0.707+0.707j & j & -0.707-0.707j & -1 & -0.707+0.707j & -j & 0.707-0.707j \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \\ 2 \\ 3 \\ 3 \\ 4 \\ 4 \end{bmatrix} = \begin{bmatrix} 20 \\ -2+4.82j \\ -2+2j \\ -2+0.828j \\ 0 \\ -2-0.828j \\ -2-2j \\ -2-4.82j \end{bmatrix}$$

$$X(K) = \{20, -2+4.82j, -2+2j, -2+0.828j, 0, -2-0.828j, -2-2j, -2-4.82j\}$$

Find DFT using Calculator: [Verification method]

$x(n) = \{1, 0, 0, 1\}$ 4-Point DFT

complex

$(1 \angle -0 A) + (0 \angle -90 A) + (0 \angle -180 A) + (1 \angle -270 A)$

$X(0) = 2$
$X(1) = 1 + j$
$X(2) = 0$
$X(3) = 1 - j$

$\frac{360}{4} = 90°$

$\frac{360}{6} = 60°$

$x(n) = \{1, 1, 0, 0, 0, 2\}$ 6-Point DFT

$(1 \angle -0 A) + (1 \angle -60 A) + (0 \angle -120 A) + (0 \angle -180 A)$
$+ (0 \angle -240 A) + (2 \angle -300 A)$

$X(K) = \{4, 2.5 + 0.866j, -0.5 + 0.866j,$
$-2, -0.5 - 0.866j, 2.5 - 0.866j\}$

$x(n) = \{1, 1, 2, 2, 3, 3, 4, 4\}$ 8-Point DFT

$(1 \angle -0 A) + (1 \angle -45 A) + (2 \angle -90 A) + (2 \angle -135 A) + (3 \angle -180 A) + (3 \angle -225 A) + (4 \angle -270 A)$
$+ (4 \angle -315 A)$

$A \to 0$ to 7

$X(K) = \{20, -2 + 4.82j, -2 + 2j, -2 + 0.828j, 0, -2 - 0.828j$
$-2 - 2j, -2 - 4.828j\}$

For $X(K) = \{2, 1+j, 0, 1-j\}$, find 4-point IDFT using Matrix method.

$X(0)\ X(1)\ X(2)\ X(3)$

$$x_N = \frac{1}{N} W_N^* X_N \quad \therefore N = 4$$

$4\times1 \quad 4\times4 \quad 4\times1$

$$W_N = e^{-j\frac{2\pi}{N}}$$

$$W_N^* = e^{j\frac{2\pi}{N}} ; \quad W_N^{*0} = 1 \Rightarrow W_4^{*0} = 1$$

$$W_4^{*1} = e^{j\frac{2\pi}{4}\cdot 1} = \cos\left(\frac{2\pi}{4}\right) + j\sin\left(\frac{2\pi}{4}\right) = j$$

comp rad

$$W_4^{*2} = e^{j\frac{2\pi}{4}\cdot 2} = \cos(\pi) + j\sin(\pi) = -1$$

$$W_4^{*3} = -j \quad W_4^{*4} = 1 \quad W_4^{*6} = -j$$

$$W_4^{*9} = j$$

$$\begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix} = \frac{1}{4} \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \quad 0 \qquad 1 \qquad 2 \qquad 3 \\ \begin{bmatrix} W_4^{*0} & W_4^{*0} & W_4^{*0} & W_4^{*0} \\ W_4^{*0} & W_4^{*1} & W_4^{*2} & W_4^{*3} \\ W_4^{*0} & W_4^{*2} & W_4^{*4} & W_4^{*6} \\ W_4^{*0} & W_4^{*3} & W_4^{*6} & W_4^{*9} \end{bmatrix} \end{array} \begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 2 \\ 1+j \\ 0 \\ 1-j \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 4 \\ 0 \\ 0 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\boxed{x(n) = \{1, 0, 0, 1\}}$$

Assignment:

① 6-Point IDFT:

$X(K) = \{4, 2.5+0.866j, -0.5+0.866j, -2, -0.5-0.866j, 2.5-0.866j\}$

$$\boxed{x(n) = \{1, 1, 0, 0, 0, 2\}}$$

② 8-Point IDFT:

$X(K) = \{20, -2+4.828j, -2+2j, -2+0.828j, 0, -2-0.828j, -2-2j, -2-4.828j\} \rightarrow$

$$\boxed{x(n) = \{1, 1, 2, 2, 3, 3, 4, 4\}}$$

Properties of DFT.

1. Linearity:

If $DFT\{x_1(n)\} = X_1(K)$

$DFT\{x_2(n)\} = X_2(K)$

then,

$$DFT\{a_1 x_1(n) + a_2 x_2(n)\} = a_1 X_1(K) + a_2 X_2(K)$$

Proof:

$$DFT\{x(n)\} = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn} \; ; \; 0 \leq K \leq N-1$$

$$DFT\{a_1 x_1(n) + a_2 x_2(n)\} = \sum_{n=0}^{N-1} \{a_1 x_1(n) + a_2 x_2(n)\} e^{-j\frac{2\pi}{N}kn}$$

$$= \sum_{n=0}^{N-1} a_1 x_1(n) e^{-j\frac{2\pi}{N}kn} + \sum_{n=0}^{N-1} a_2 x_2(n) e^{-j\frac{2\pi}{N}kn}.$$

$$= a_1 \sum_{n=0}^{N-1} x_1(n) e^{-j\frac{2\pi}{N}kn} + a_2 \sum_{n=0}^{N-1} x_2(n) e^{-j\frac{2\pi}{N}kn}.$$

$$= a_1 \cdot X_1(K) + a_2 X_2(K)$$

Properties of DFT.

2. Periodicity :

If $DFT\{x(n)\} = X(K)$

then

$x(n+N) = x(n)$; for all n

$X(K+N) = X(K)$; for all K

Proof

$$x(n) = \frac{1}{N}\sum_{K=0}^{N-1} X(K)\, e^{j\frac{2\pi}{N}Kn} ; \quad 0 \le n \le N-1$$

$$x(n+N) = \frac{1}{N}\sum_{K=0}^{N-1} X(K)\, e^{j\frac{2\pi}{N}K(n+N)}$$

$$= \frac{1}{N}\sum_{K=0}^{N-1} X(K)\, e^{j\frac{2\pi}{N}Kn} \cdot \cancel{e^{j\frac{2\pi}{N}KN}}^{\,1}$$

$$\left[e^{j2\pi K} = 1\right]$$

$$= \frac{1}{N}\sum_{K=0}^{N-1} X(K)\, e^{j\frac{2\pi}{N}Kn}.$$

$$x(n+N) = x(n)$$

$$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}Kn} ; \quad 0 \le K \le N-1$$

$$X(K+N) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}(K+N)n}$$

$$= \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}Kn} \cdot \cancel{e^{-j\frac{2\pi}{N}Nn}}^{\,1}$$

$$\left[e^{-j2\pi n} = 1\right]$$

$$= \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}Kn}.$$

$$X(K+N) = X(K)$$

Properties of DFT

3. Circular Time Shift:

If $DFT\{x(n)\} = X(K)$

then $DFT\{x((n-n_0))_N\} = X(K)\, e^{-j\frac{2\pi}{N}Kn_0}$

Proof: IDFT

$$x(n) = \frac{1}{N}\sum_{K=0}^{N-1} X(K)\, e^{j\frac{2\pi}{N}Kn} \quad ; \quad 0 \le n \le N-1$$

put $n = n - n_0$

$$x(n-n_0) = \frac{1}{N}\sum_{K=0}^{N-1} X(K)\, e^{j\frac{2\pi}{N}K(n-n_0)}$$

$$= \underbrace{\frac{1}{N}\sum_{K=0}^{N-1} X(K)\, e^{j\frac{2\pi}{N}Kn}}\cdot e^{-j\frac{2\pi}{N}Kn_0}$$

$$x(n-n_0) = x(n)\cdot e^{-j\frac{2\pi}{N}Kn_0}$$ Take DFT on Both sides.

$$DFT\{x(n-n_0)\} = \underbrace{DFT\{x(n)\}}_{X(K)}\cdot e^{-j\frac{2\pi}{N}Kn_0}$$

$$DFT\{x(n-n_0)\} = X(K)\cdot e^{-j\frac{2\pi}{N}\cdot Kn_0}$$

Properties of DFT:

4. Circular Frequency Shift:

If $DFT\{x(n)\} = X(K)$

then $DFT\{x(n)\, e^{j\frac{2\pi}{N}ln}\} = X((K-l))_N$

Proof:- DFT : $X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}Kn}$; $0 \le K \le N-1$

Put $K = K-l$

$$X((K-l))_N = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}(K-l)n}$$

$$= \underbrace{\sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}Kn}} \cdot e^{+j\frac{2\pi}{N}ln}.$$

$$= X(K) \cdot e^{j\frac{2\pi}{N}ln}$$

$$X((K-l))_N = DFT\{x(n) \cdot e^{j\frac{2\pi}{N}ln}\}$$

Properties of DFT:

5. Time reversal

If $DFT\{x(n)\} = X(K)$

then, $DFT\{x((-n))_N = x(N-n)\} = X((-K))_N = X(N-K)$

Proof:- $DFT\{x(N-n)\} = \sum_{n=0}^{N-1} x(N-n)\, e^{-j\frac{2\pi}{N}Kn}$

Put $m = N-n$ $\quad$ $m=N$

$\Rightarrow n = N-m$ $\quad$ $m = \cancel{N}-\cancel{N}+1 \Rightarrow m=1$

$$= \sum_{m=N}^{1} x(m)\cdot e^{-j\frac{2\pi}{N}K(N-m)} = \sum_{m=1}^{N} x(m)\cdot e^{j\frac{2\pi}{N}Km}\cdot e^{-j\frac{2\pi}{N}\cdot \cancel{KN}}$$

1

$$= \sum_{m=1}^{N} x(m)\cdot e^{j\frac{2\pi}{N}Km}\cdot e^{-j\frac{2\pi}{N}\cdot Nm} \overset{1}{=} \sum_{m=0}^{N-1} x(m)\cdot e^{-j\frac{2\pi}{N}(N-K)m}$$

$X(\underset{N-K}{K}) = X(N-K)$

$$DFT\{x(N-n)\} = X(N-K) = X((-K))_N$$

Concept of Time Reversal:

$x(n) \rightarrow$ N-point sequence

Time Reversal.

$$x((-n))_N = x(N-n)^*$$

Ex:- $x(n) = \{1, -1, 3, 5\}$ $n = 0$ to 3

$y(n) = x((-n))_4 = x(4-n)$

$n=0$ $y(0) = x(4) = x(4-4) = x(0) = 1$

$n=1$ $y(1) = x(3) = 5$

$n=2$ $y(2) = x(2) = 3$

$n=3$ $y(3) = x(1) = -1$

$$y(n) = \{1, 5, 3, -1\}$$

$x(n) = \{1, 2, 3, 4, 5, 6, 7, 8\}$ $n = 0$ to 7

$y(n) = x((-n))_8 = x(8-n)$

$n=0$ $y(0) = x(8) = x(8-8) = x(0) = 1$

$n=1$ $y(1) = x(7) = 8$

$n=2$ $y(2) = x(6) = 7$

$n=3$ $y(3) = x(5) = 6$

$n=4$ $y(4) = x(4) = 5$

$n=5$ $y(5) = x(3) = 4$

$n=6$ $y(6) = x(2) = 3$

$n=7$ $y(7) = x(1) = 2$

$y(n) = \{1, 8, 7, 6, 5, 4, 3, 2\}$.

Properties of DFT

6. Circular Convolution:

If $DFT\{x(n)\} = X(K)$

then $DFT\{y(n) = x_1(n) \circledN x_2(n)\} = Y(K) = X_1(K) \cdot X_2(K)$

$\circledast \rightarrow \circledN \rightarrow$ circular convolution.

Proof:- $Y(K) = X_1(K) \cdot X_2(K)$

$$y(n) = \frac{1}{N}\sum_{K=0}^{N-1} Y(K) \cdot e^{j\frac{2\pi}{N}Kn}$$

$$\therefore y(n) = \frac{1}{N}\sum_{K=0}^{N-1}\left[X_1(K)\cdot X_2(K)\right]\cdot e^{j\frac{2\pi}{N}Kn}$$

$$X_1(K) = \sum_{m=0}^{N-1} x_1(m)\, e^{-j\frac{2\pi}{N}\cdot Km}$$

$$X_2(K) = \sum_{l=0}^{N-1} x_2(l)\, e^{-j\frac{2\pi}{N}\cdot Kl}.$$

$$y(n) = \frac{1}{N}\sum_{K=0}^{N-1}\sum_{m=0}^{N-1} x_1(m)\, e^{-j\frac{2\pi}{N}Km} \cdot \sum_{l=0}^{N-1} x_2(l)\, e^{-j\frac{2\pi}{N}\cdot Kl} \cdot e^{j\frac{2\pi}{N}\cdot Kn}$$

$$\boxed{y(n) = x_1(n) \circledN x_2(n)}$$

Rearranging.

$$y(n) = \frac{1}{N}\sum_{m=0}^{N-1} x_1(m) \cdot \sum_{l=0}^{N-1} x_2(l)\left[\sum_{K=0}^{N-1} e^{j\frac{2\pi}{N}(n-m-l)K}\right] \rightarrow (1)$$

W.K.T.

$$\sum_{K=0}^{N-1}\alpha^K = \frac{1-\alpha^N}{1-\alpha} \quad ; \alpha \neq 1$$

$$= N \quad ; \alpha = 1$$

$$\alpha = e^{j\frac{2\pi}{N}(n-m-l)} \qquad n-m-n+m$$

$\alpha = 1 \quad ; l = n-m$

$\alpha \neq 1 \quad ; l \neq n-m$

$$= \frac{1 - e^{j\frac{2\pi}{N}(n-m-l)\cdot N}}{1 - e^{j\frac{2\pi}{N}(n-m-l)}} = 0 \quad ; l \neq n-m$$

$$= N \quad ; l = n-m$$

$$y(n) = \frac{1}{N}\sum_{m=0}^{N-1} x_1(m) \cdot \sum_{l=0}^{N-1} x_2(l) \cdot N \qquad (l = n-m)$$

$$y(n) = \frac{1}{\cancel{N}}\sum_{m=0}^{N-1} x_1(m)\cdot x_2(n-m)\cdot \cancel{N}$$

Properties of DFT:

7. Modulation (or) Multiplication.

If $DFT\{x_1(n)\} = X_1(K)$

& $DFT\{x_2(n)\} = X_2(K)$

then,

$$DFT\{x_1(n) \cdot x_2(n)\} = \frac{1}{N}\left[X_1(K) \, (N) \, X_2(K)\right]$$

Proof: $DFT\{x(n)\} = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N} \cdot Kn}$

$$DFT\{x_1(n) \cdot x_2(n)\} = \sum_{n=0}^{N-1} [x_1(n) \cdot x_2(n)] \cdot e^{-j\frac{2\pi}{N}Kn}$$

let $x_2(n) = \frac{1}{N}\sum_{l=0}^{N-1} X_2(l) e^{j\frac{2\pi}{N}ln}$

$$= \sum_{n=0}^{N-1} x_1(n) \cdot \frac{1}{N}\sum_{l=0}^{N-1} X_2(l) e^{j\frac{2\pi}{N}ln} \cdot e^{-j\frac{2\pi}{N} \cdot Kn}$$

Rearrange

$$= \frac{1}{N}\sum_{l=0}^{N-1} X_2(l) \cdot \underbrace{\sum_{n=0}^{N-1} x_1(n) \cdot e^{-j\frac{2\pi}{N}(K-l)n}}_{X_1(K-l)}$$

W.K.T.

$$X(K) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}Kn}.$$

$$X(K-l) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}(K-l)n}.$$

$$= \frac{1}{N}\sum_{l=0}^{N-1} X_2(l) X_1(K-l)$$

$$= \frac{1}{N}\sum_{l=0}^{N-1} X_1(K-l) X_2(l).$$

$$\boxed{DFT\{x_1(n) \cdot x_2(n)\} = \frac{1}{N}\left[X_1(K) \, (N) \, X_2(K)\right]}$$

Properties of DFT:

8. PARSEVAL'S THEOREM

If DFT $\{x_1(n)\} = X_1(K)$

& DFT $\{x_2(n)\} = X_2(K)$

then,
$$\sum_{n=0}^{N-1} x_1(n) \cdot x_2^*(n) = \frac{1}{N} \sum_{K=0}^{N-1} X_1(K) \cdot X_2^*(K)$$

Proof :-
$$x_2(n) = \frac{1}{N} \sum_{K=0}^{N-1} X_2(K) e^{j\frac{2\pi}{N}Kn}.$$

$$x_2^*(n) = \frac{1}{N} \sum_{K=0}^{N-1} X_2^*(K) \cdot e^{-j\frac{2\pi}{N}Kn}.$$

$$\sum_{n=0}^{N-1} x_1(n) \cdot x_2^*(n) = \sum_{n=0}^{N-1} x_1(n) \cdot \frac{1}{N} \sum_{K=0}^{N-1} X_2^*(K) e^{-j\frac{2\pi}{N}Kn}$$

Rearrange

$$= \frac{1}{N} \sum_{K=0}^{N-1} X_2^*(K) \cdot \underbrace{\sum_{n=0}^{N-1} x_1(n) \cdot e^{-j\frac{2\pi}{N}Kn}}_{X_1(K)}$$

$$= \frac{1}{N} \sum_{K=0}^{N-1} X_2^*(K) \cdot X_1(K)$$

$$\sum_{n=0}^{N-1} x_1(n) \cdot x_2^*(n) = \frac{1}{N} \sum_{K=0}^{N-1} X_1(K) \cdot X_2^*(K)$$

If $x_1(n) = x_2(n)$

$$\sum_{n=0}^{N-1} x_1(n) \cdot x_1^*(n) = \frac{1}{N} \sum_{K=0}^{N-1} X_1(K) \cdot X_1^*(K)$$

$$\underbrace{\sum_{n=0}^{N-1} |x_1(n)|^2}_{\text{Energy of } x(n)} = \frac{1}{N} \sum_{K=0}^{N-1} |X_1(K)|^2$$

Properties of DFT:

9. Circular Correlation Property

If $DFT\{x(n)\} = X(K)$

and $DFT\{y(n)\} = Y(K)$

then,

$$DFT\{r_{xy}(l)\} = R_{xy}(K) = X(K) \cdot Y^*(K)$$

Proof:- $$r_{xy}(l) = \underbrace{\sum_{n=0}^{N-1} x(n) \cdot y^*((n-l))_N}$$

$$r_{xy}(l) = x(l) \; N \; y^*((-l))_N$$

Take DFT on b.s.

$$DFT\{r_{xy}(l)\} = DFT\{x(l) \; N \; y^*((-l))_N\}$$

$$\boxed{R_{xy}(K) = X(K) \cdot Y^*(K)} \rightarrow \text{Cross Correlation.}$$

if $x(n) = y(n)$

$$R_{xx}(K) = X(K) \cdot X^*(K)$$

$$\boxed{R_{xx}(K) = |X(K)|^2} \rightarrow \text{auto correlation.}$$

We have,

$$DFT\{x^*((-n))_N\} = X^*(K)$$

$$DFT\{y^*((-l))_N\} = Y^*(K)$$

10. Symmetry Property:

$x(n) \longrightarrow X(K)$ complex value.

$$x(n) = x_R(n) + j\,x_I(n)$$

$$X(K) = X_R(K) + j\,X_I(K)$$

$$X(K) = \sum_{n=0}^{N-1} x(n)\,e^{-j\frac{2\pi}{N}kn} \quad ; \quad 0 \le K \le N-1$$

$$X_R(K) + j\,X_I(K) = \sum_{n=0}^{N-1} \left[x_R(n) + j\,x_I(n)\right] e^{-j\frac{2\pi}{N}kn}$$

$$= \sum_{n=0}^{N-1} \left[x_R(n) + j\,x_I(n)\right]\left[\cos\frac{2\pi}{N}kn - j\sin\frac{2\pi}{N}kn\right]$$

$$= \sum_{n=0}^{N-1} x_R(n)\cos\frac{2\pi}{N}kn - j\,x_R(n)\sin\frac{2\pi}{N}kn + j\,x_I(n)\cos\frac{2\pi}{N}kn + x_I(n)\sin\frac{2\pi}{N}kn.$$

$$X_R(K) + j\,X_I(K) = \sum_{n=0}^{N-1} \left\{\left[x_R(n)\cos\frac{2\pi}{N}kn + x_I(n)\sin\frac{2\pi}{N}kn\right] - j\left[x_R(n)\sin\frac{2\pi}{N}kn - x_I(n)\cos\frac{2\pi}{N}kn\right]\right\}$$

$$\boxed{X_R(K) = \sum_{n=0}^{N-1} \left[x_R(n)\cos\frac{2\pi}{N}kn + x_I(n)\sin\frac{2\pi}{N}kn\right]}$$

$$\boxed{X_I(K) = -\sum_{n=0}^{N-1} \left[x_R(n)\sin\frac{2\pi}{N}kn - x_I(n)\cos\frac{2\pi}{N}kn\right]}$$

$$x(n) = \frac{1}{N}\sum_{K=0}^{N-1} X(K)\,e^{j\frac{2\pi}{N}kn}$$

$$x_R(n) + j\,x_I(n) = \frac{1}{N}\sum_{K=0}^{N-1} \left[X_R(K) + j\,X_I(K)\right] e^{j\frac{2\pi}{N}kn}.$$

$$= \frac{1}{N}\sum_{K=0}^{N-1} \left[X_R(K)\cos\frac{2\pi}{N}kn + j\,X_R(K)\sin\frac{2\pi}{N}kn + j\,X_I(K)\cos\frac{2\pi}{N}kn - X_I(K)\sin\frac{2\pi}{N}kn\right]$$

$$= \frac{1}{N}\sum_{K=0}^{N-1} \left\{\left[X_R(K)\cos\frac{2\pi}{N}kn - X_I(K)\sin\frac{2\pi}{N}kn\right] + j\left[X_R(K)\sin\frac{2\pi}{N}kn + X_I(K)\cos\frac{2\pi}{N}kn\right]\right\}$$

$$\boxed{x_R(n) = \frac{1}{N}\sum_{K=0}^{N-1} \left[X_R(K)\cos\frac{2\pi}{N}kn - X_I(K)\sin\frac{2\pi}{N}kn\right]}$$

$$\boxed{x_I(n) = \frac{1}{N}\sum_{K=0}^{N-1} \left[X_R(K)\sin\frac{2\pi}{N}kn + X_I(K)\cos\frac{2\pi}{N}kn\right]}$$

$$X(K) = \sum_{n=0}^{N-1}\left[x_R(n)\cos\frac{2\pi}{N}Kn + x_I(n)\sin\frac{2\pi}{N}Kn\right] - j\sum_{n=0}^{N-1}\left[x_R(n)\sin\frac{2\pi}{N}Kn - x_I(n)\cos\frac{2\pi}{N}Kn\right]$$

(i) Real & Even Sequence:

$x(n)$ $\qquad x_I(n) = 0$ & $x(n) = x_R(n)$

$$\boxed{X(K) = \sum_{n=0}^{N-1} x(n)\cos\frac{2\pi}{N}Kn}$$

$$\boxed{\begin{aligned} X_R(K) &= \sum_{n=0}^{N-1} x(n)\sin\frac{2\pi}{N}Kn. \\ X_I(K) &= \sum_{n=0}^{N-1} x(n)\cos\frac{2\pi}{N}Kn. \end{aligned}}$$

(ii) Real & odd Sequence

$x(n)$ $\qquad x_I(n) = 0$ & $x(n) = x_R(n)$

$$\boxed{X(K) = -j\sum_{n=0}^{N-1} x(n)\sin\frac{2\pi}{N}Kn}$$

(iii) Purely Imaginary

$x(n)$ $\qquad x_R(n) = 0$ & $x(n) = j\,x_I(n)$

$$X_R(K) + jX_I(K) = \sum_{n=0}^{N-1} x_I(n)\sin\frac{2\pi}{N}Kn + j\sum_{n=0}^{N-1} x_I(n)\cos\frac{2\pi}{N}Kn.$$

Properties of DFT :

11. Duality :

If $x(n) \xleftrightarrow[DFT]{N} X(K)$

then, $X(n) \xleftrightarrow[DFT]{N} N\left[x((-K))_N\right] = Nx(N-K)$

Proof:-

$$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}Kn}$$

replace K by n & n by K

$$X(n) = \sum_{K=0}^{N-1} x(K)\, e^{-j\frac{2\pi}{N}\cdot nK}\cdot 1$$

$$= \sum_{K=0}^{N-1} x(K)\, e^{-j\frac{2\pi}{N}Kn} \cdot e^{j\frac{2\pi}{N}Nn}$$

$$X(n) = \sum_{K=0}^{N-1} x(K)\cdot e^{j\frac{2\pi}{N}(N-K)n} \longrightarrow (1)$$

W.K.T

$$x(n) = \frac{1}{N}\sum_{K=0}^{N-1} X(K)\, e^{j\frac{2\pi}{N}Kn} \longrightarrow (2)$$

Compare eqn (1) & (2)

$$X(n) = N\cdot x(N-K) = N\left[x((-K))_N\right]$$

12. Circular Symmetry.

$DFT\{x(n)\} = X(K)$ then $DFT\{x_p(n)\} = X(K)$

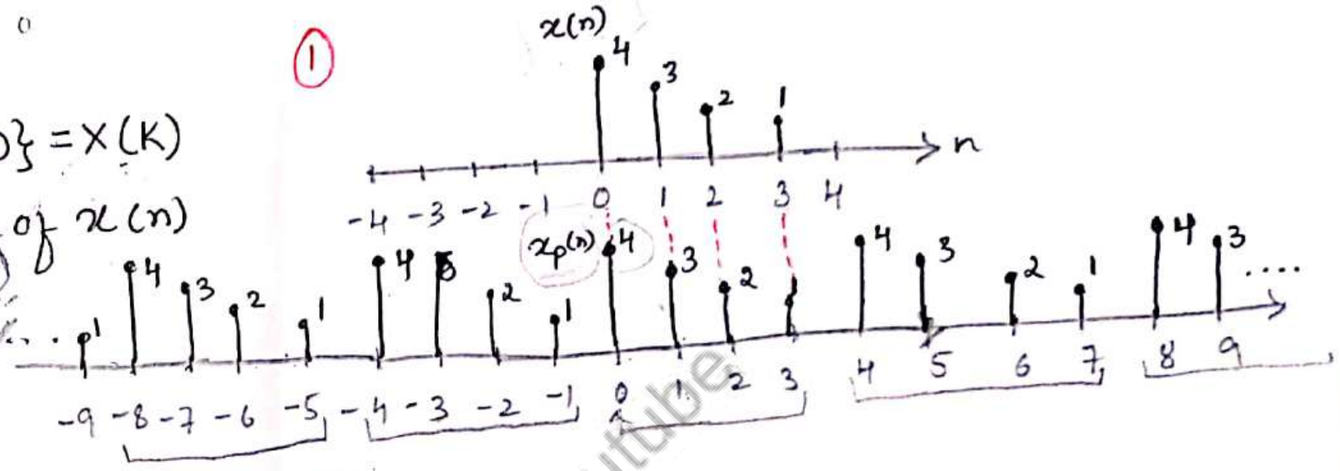
$x_p(n) \Rightarrow$ Periodic Representation of $x(n)$

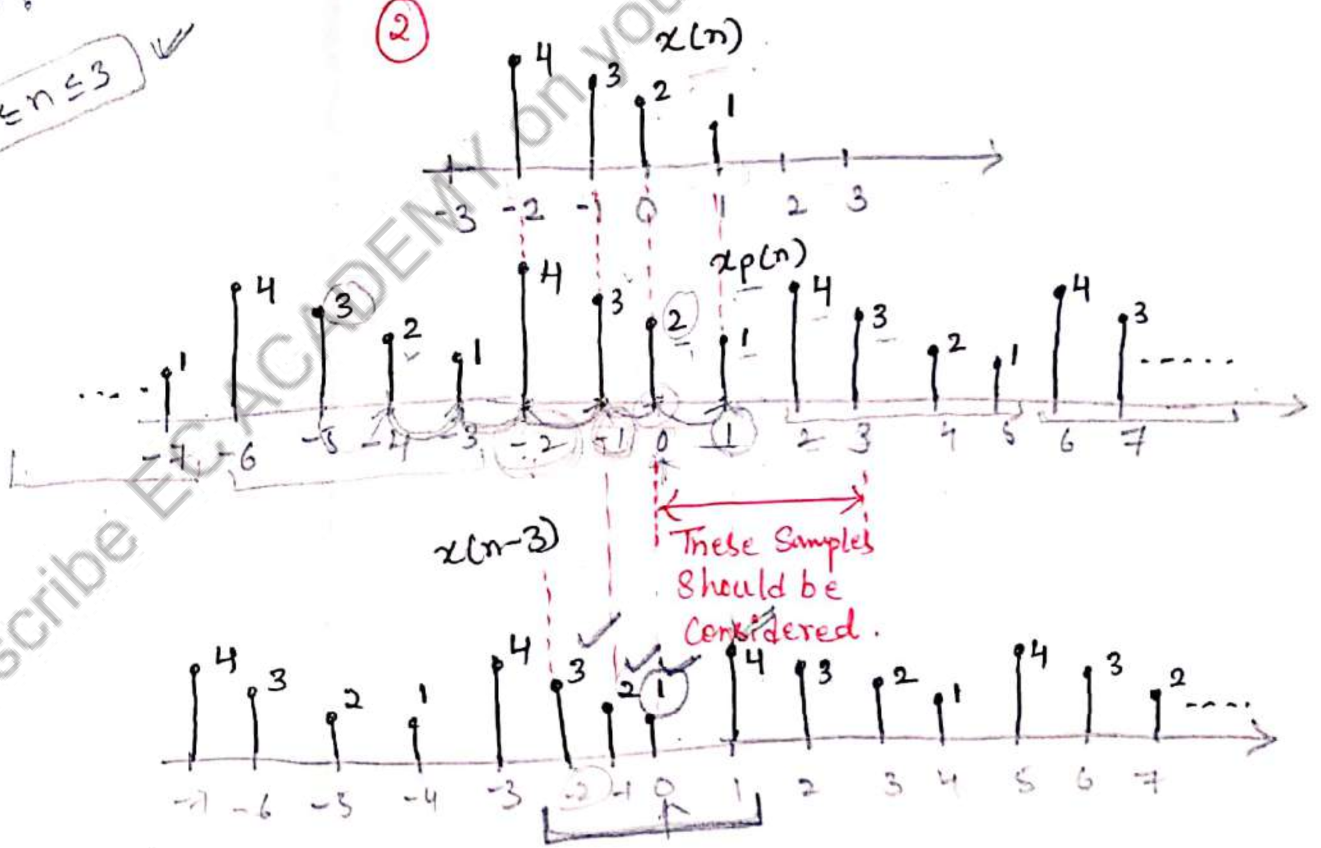
① $x(n) = \{4, 3, 2, 1\}$ $\quad 0 \le n \le N-1$

$x_p(n) = \{4, 3, 2, 1\}$ $\quad -2 \le n \le 1$

② $x(n) = \{4, 3, 2, 1\}$ DFT ?

$x_p(n) = \{2, 1, 4, 3\}$ $\quad 0 \le n \le 3$





Circular Time Shift

$x(n-3) = \{3, 2, 1, 4\}$

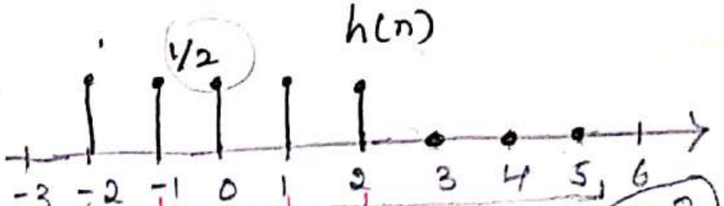
$x(n+2)$

Problem on Circular Symmetries

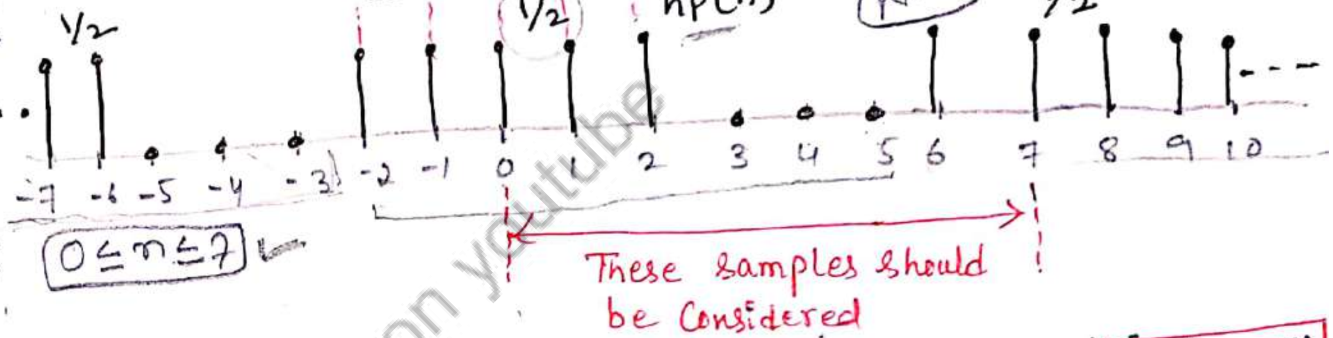
Find the DFT of $h(n)\begin{cases} 1/2 \,; & -2 \le n \le 2 \\ 0 \,; & \text{otherwise} \end{cases}$ $n = -2, -1, 0, 1, 2$

$\boxed{0 \le n \le N-1}$

Plot the magnitude & phase response for $N=8$



$h(n) = \{1/2, 1/2, \underset{\uparrow}{1/2}, 1/2, 1/2\}$ — DF?



$h(n) = h_p(n) = \{\underset{\uparrow}{1/2}, 1/2, 1/2, 0, 0, 0, 1/2, 1/2\}$ $\boxed{0 \le n \le 7}$

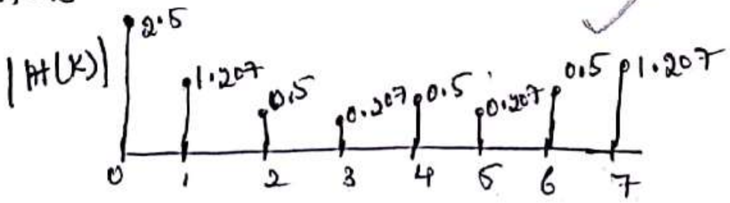
$$H(K) = \sum_{k=0}^{N-1} h(n)\, W_N^{kn} \Rightarrow \underset{8\times1}{H_N} = \underset{8\times8}{W_N}\, \underset{8\times1}{h_N} \quad \therefore N=8$$

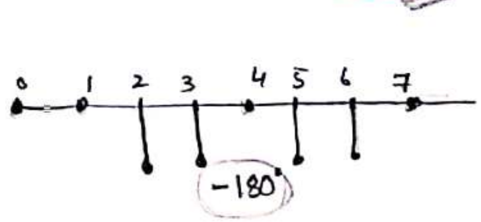
$$\begin{bmatrix} H(0) \\ H(1) \\ H(2) \\ H(3) \\ H(4) \\ H(5) \\ H(6) \\ H(7) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0.707-0.707j & -j & -0.707-0.707j & -1 & -0.707+0.707j & j & 0.707+0.707j \\ 1 & -j & -1 & j & 1 & -j & -1 & j \\ 1 & -0.707+0.707j & j & 0.707-0.707j & -1 & 0.707+0.707j & -j & -0.707-0.707j \\ 1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 \\ 1 & -0.707-0.707j & -j & 0.707+0.707j & -1 & 0.707-0.707j & j & -0.707+0.707j \\ 1 & j & -1 & -j & 1 & j & -1 & j \\ 1 & 0.707+0.707j & j & -0.707-0.707j & -1 & -0.707+0.707j & -j & 0.707-0.707j \end{bmatrix} \begin{bmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 0 \\ 0 \\ 0 \\ 1/2 \\ 1/2 \end{bmatrix} = \begin{bmatrix} 5/2 \\ 1.207 \\ -1/2 \\ -0.207 \\ 1/2 \\ -0.207 \\ -1/2 \\ 1.207 \end{bmatrix}$$

$\boxed{\text{only in compx}}$

Magnitude & phase

$5/2 \angle 0°$
$1.207 \angle 0°$
$1/2 \angle -180°$
$0.207 \angle -180°$
$1/2 \angle 0°$
$+0.207 \angle -180°$
$1/2 \angle 180°$
$1.207 \angle 0°$



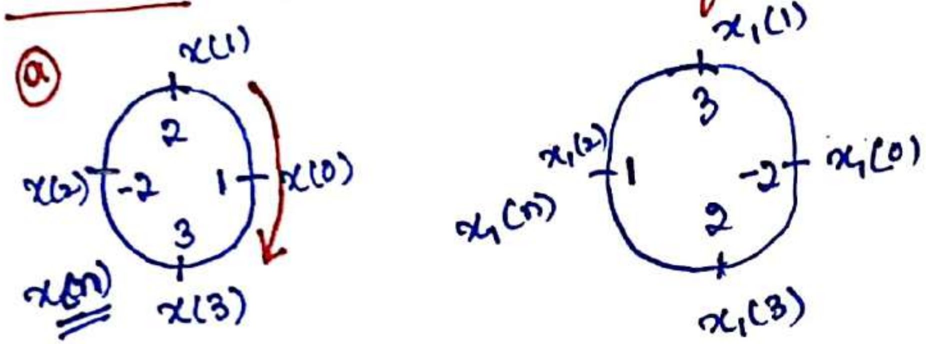


Let $x(n) = \{1, 2, -2, 3\}$. Find

$x_1(n) = x((n+2))_4$; $0 \le n \le 3$

1-Method :- Circular time shift.

ⓐ



$x_1(n) = \{-2, \underline{3}, 1, 2\}$ $\boxed{N=4}$

ⓑ $x_1(n) = x((n+2))_4 = x(4+n+2)$

$n=0$ $x_1(0) = x(6) = x(6-4) = x(2) = -2$

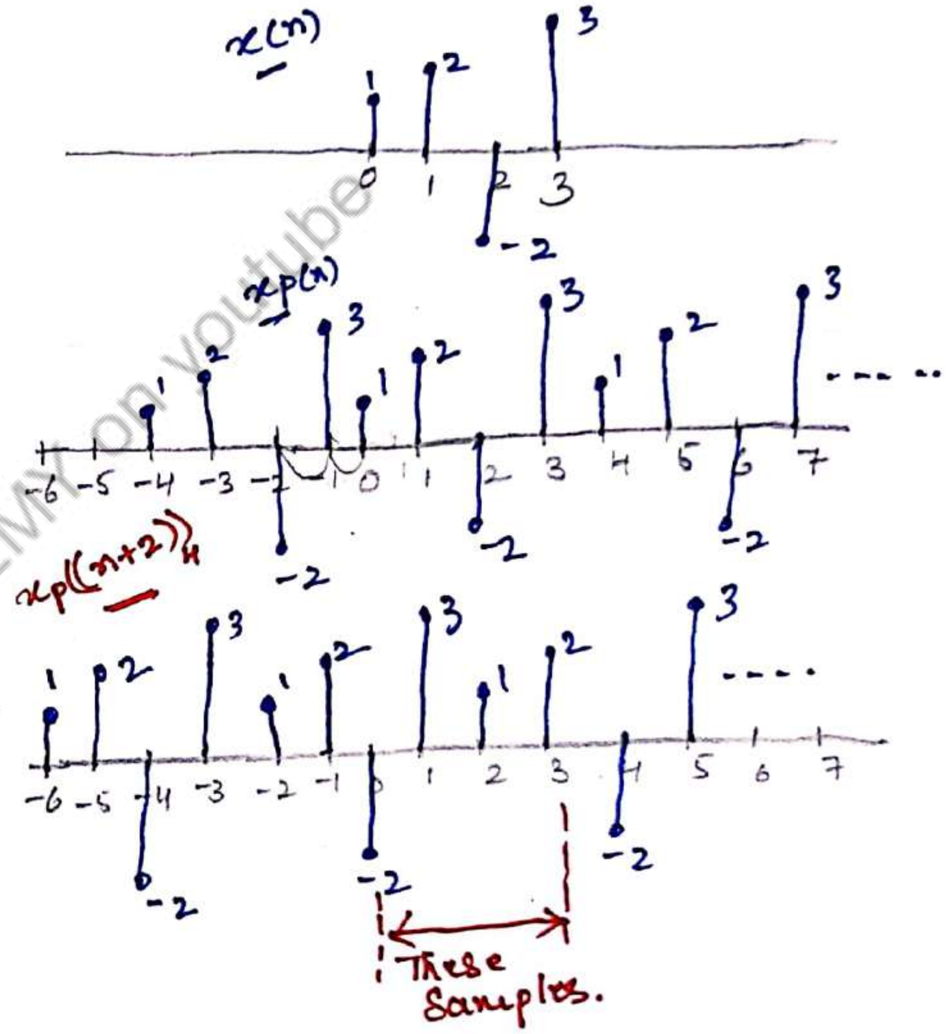
$n=1$ $x_1(1) = x(7) = x(7-4) = x(3) = 3$

$n=2$ $x_1(2) = x(8) = x(8-4) = x(4) = x(4-4)$
$= x(0) = 1$

$n=3$ $x_1(3) = x(9) = x(9-4) = x(5) = x(5-4)$
$= x(1) = 2$

$x(n) = \{-2, \underline{3}, 1, 2\}$.

2-Method :- Circular symmetry



$x_1(n) = \{-2, \underline{3}, 1, 2\}$

The First five points of 8-Point DFT $X(K)$ are
$\{0.25, 0.125-j0.3018, 0, 0.125-j0.518, 0\}$
Determine the remaining three point.
Estimate the value of $x(0)$.

$X(0) = 0.25$

$X(1) = 0.125 - j0.3018$

$X(2) = 0$

$X(3) = 0.125 - j0.518$ $\quad \boxed{N=8}$

$X(4) = 0$

$X(5) = ?$

$X(6) = ?$

$X(7) = ?$

Symmetry property

$$\boxed{X(N-K) = X^*(K)}$$

$K=3$

$X(8-K) = X^*(K)$

$X(8-3) = X^*(3)$

$X(5) = X^*(3)$

$$\boxed{X(5) = 0.125 + j0.518}$$

$K=2$

$X(8-2) = X^*(2)$

$X(6) = X^*(2) \Rightarrow \boxed{X(6) = 0}$

$K=1$

$X(8-1) = X^*(1)$

$X(7) = X^*(1)$

$$\boxed{X(7) = 0.125 + j0.3018}$$

IDFT,

$$x(n) = \frac{1}{N} \sum_{K=0}^{N-1} X(K) \underbrace{e^{j2\pi Kn/N}}_{1 \text{ for } n=0}$$

$$x(0) = \frac{1}{N} \sum_{K=0}^{N-1} X(K) \cdot 1$$

$$x(0) = \frac{1}{8} \sum_{K=0}^{7} X(K)$$

$$x(0) = \frac{1}{8}\left[0.25 + 0.125 - j0.3018 + 0 + 0.125 - j0.518 + 0 + 0.125 + j0.518 + 0 + 0.125 + j0.3018\right]$$

$$\boxed{x(0) = 0.09375}$$

Assignment:- $X(0) = 0.25$, $X(1) = 0.125 - j0.3018$, $X(6) = X(4) = 0$, $X(5) = 0.125 - j0.0518$. Determine the remaining samples.

Let $X(K)$ denotes N-point DFT of the N-point sequence $x(n)$

(i) Show that if $x(n)$ satisfies the relation $x(n) = -x(N-1-n)$ then $X(0)=0$

(ii) Show that with N even & if, $x(n) = x(N-1-n)$ then $X(\frac{N}{2}) = 0$

(i) $x(n) = -x(N-1-n) \rightarrow$ odd sequence

$$X(K) = -j\sum_{n=0}^{N-1} x(n)\sin\left[\frac{2\pi K n}{N}\right] \text{---- Symmetry Property.}$$

$$X(0) = -j\sum_{n=0}^{N-1} x(n)\sin\left[\frac{2\pi \times 0\, n}{N}\right]$$

$$\boxed{X(0) = 0}$$

(ii) $x(n) = x(N-1-n) \rightarrow$ even sequence

$$X(K) = \sum_{n=0}^{N-1} x(n)\cos\left[\frac{2\pi K n}{N}\right] \text{---- Symmetry Property.}$$

$$X\left(\frac{N}{2}\right) = \sum_{n=0}^{N-1} x(n)\cos\left[\frac{2\pi \frac{N}{2}\cdot n}{N}\right] = \sum_{n=0}^{N-1} x(n)\cos(\pi n)$$

$\underline{N=4}$

$$X\left(\frac{N}{2}\right) = \sum_{n=0}^{3} x(n)\cos(\pi n)$$

$$= \cancel{x(0)} - \cancel{x(1)} + \cancel{x(2)} - \cancel{x(3)}$$

$x(n) = x(N-1-n)$ $\quad \underline{N=4}$

$x(n) = x(3-n)$

$\underline{n=0}$ $\quad x(0) = x(3)$

$\underline{n=1}$ $\quad x(1) = x(2)$

$$\boxed{X\left(\frac{N}{2}\right) = 0}$$

Find the circular convolution of

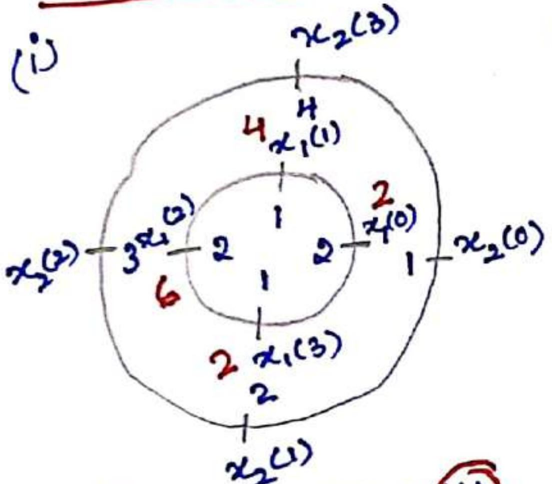
$x_1(n) = \{2, 1, 2, 1\}$ & $x_2(n) = \{1, 2, 3, 4\}$

Circular Convolution,

$$y(m) = \sum_{n=0}^{N-1} x_1(n) \cdot x_2((m-n))_N$$

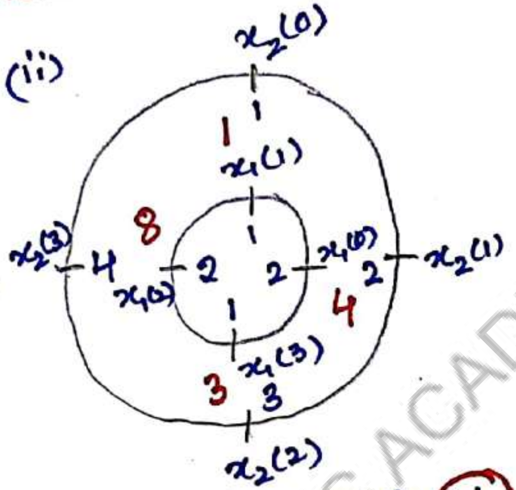
Method-1: [Stockham's method]

(i)



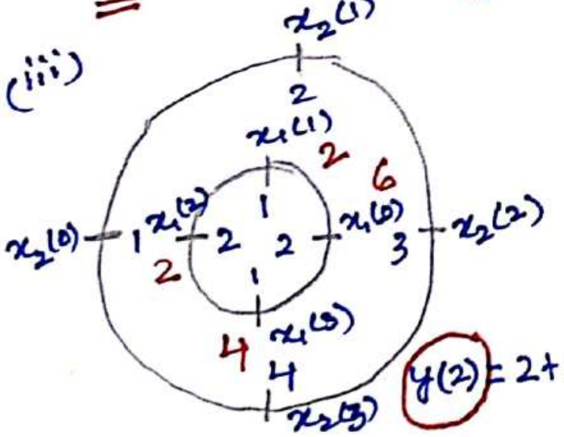
$y(0) = 2+4+6+2 = 14$

(ii)



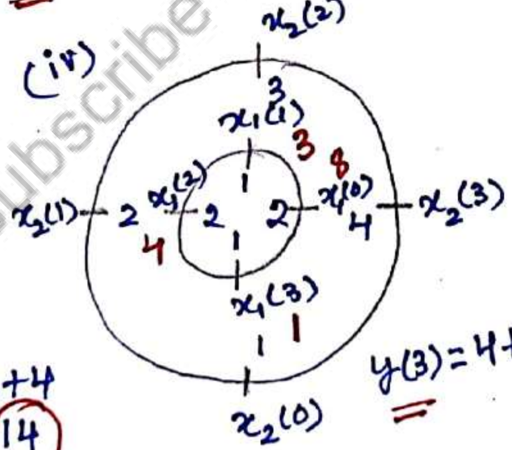
$y(1) = 1+4+3+8 = 16$

(iii)



$y(2) = 2+2+6+4 = 14$

(iv)



$y(3) = 4+3+8+1 = 16$

$y(n) = \{14, 16, 14, 16\}$

Method-2 [Matrix method]

$$\begin{bmatrix} 1 & 4 & 3 & 2 \\ 2 & 1 & 4 & 3 \\ 3 & 2 & 1 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 14 \\ 16 \\ 14 \\ 16 \end{bmatrix}$$

Method-3 [Tab method]

2 1 2 1	1 4 3 2	2+4+6+2 = 14
2 1 2 1	2 1 4 3	4+1+8+3 = 16
2 1 2 1	3 2 1 4	6+2+2+4 = 14
2 1 2 1	4 3 2 1	8+3+4+1 = 16

$y(n) = \{14, 16, 14, 16\}$

Find the linear convolution of
$x_1(n) = \{2, 1, 2, 1\}$ & $x_2(n) = \{1, 2, 3, 4\}$

$4+4-1 = 7$

$x_1(n) \Rightarrow$				2	1	2	1
$x_2(n) \Rightarrow$				1	2	3	4
				8	4	8	4
			6	3	6	3	×
		4	2	4	2	×	×
	2	1	2	1	×	×	×
	2	5	10	16	12	11	4

$y(n) = \{2, 5, 10, 16, 12, 11, 4\}$.

16	12	11	4
	2	5	10
16	14	16	14

$y(n) = \{14, 16, 14, 16\}$

Compute the Circular Convolution of $x_1(n) = \{2,1,2,1\}$ & $x_2(n) = \{1,2,3,4\}$ using DFT and IDFT.

$$y(n) = \underbrace{x_1(n) \, (N) \, x_2(n)}_{} \longleftrightarrow \underbrace{X_1(K) \cdot X_2(K)}_{IDFT}$$

$X_N = W_N \cdot x_N \qquad N=4$

$X_4 = W_4 \cdot x_4$

$$\begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix} = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix}$$

(i) DFT $x_1(n)$.

$$\begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 2 \\ 0 \end{bmatrix}$$

$X_1(K)$

(ii) DFT $x_2(n)$

$$\begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

$X_2(K)$

$$X_1(K) \cdot X_2(K) = \begin{bmatrix} 6 \times 10 \\ 0 \times -2+2j \\ 2 \times -2 \\ 0 \times -2-2j \end{bmatrix} = \begin{bmatrix} 60 \\ 0 \\ -4 \\ 0 \end{bmatrix}$$

(iii) IDFT of $X_1(K) \cdot X_2(K)$

$x_N = \frac{1}{N} [W_N^*] X_N \qquad N=4$

$$\begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 60 \\ 0 \\ -4 \\ 0 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 56 \\ 64 \\ 56 \\ 64 \end{bmatrix}$$

$$= \begin{bmatrix} 14 \\ 16 \\ 14 \\ 16 \end{bmatrix}$$

$\therefore y(n) = \{14, 16, 14, 16\}$

Linear Filtering using DFT & IDFT :

$$y(n) = \sum_{K=-\infty}^{\infty} h(K)\, x(n-K) \longrightarrow (1)$$

$$\overset{F.T.}{Y(\omega)} = F\left\{ \sum_{K=-\infty}^{\infty} h(K)\, x(n-K) \right\} \longrightarrow (2)$$

Convolution property of F.T.

$$F\{x_1(n) * x_2(n)\} = X_1(\omega) \cdot X_2(\omega)$$

$$(2) \Rightarrow Y(\omega) = H(\omega) \cdot X(\omega) \longrightarrow (3)$$

W.K.T. $Y(K) = Y(\omega)\big|_{\omega = \frac{2\pi K}{N}}$

$X(K) = X(\omega)\big|_{\omega = \frac{2\pi}{N}K}$ $\quad K = 0, 1, \dots N-1$

$H(K) = H(\omega)\big|_{\omega = \frac{2\pi K}{N}}$

$$(3) \Rightarrow Y(K) = X(K) \cdot H(K);\quad K = 0, 1, \dots N-1 \longrightarrow (4)$$

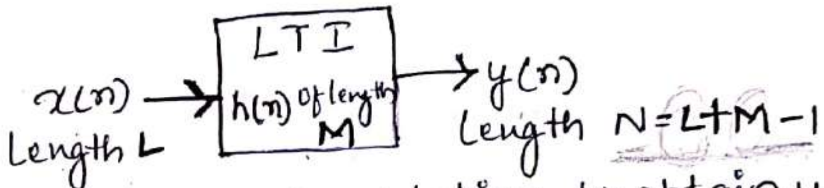
$$y(n) = IDFT\{Y(K)\} = IDFT\{X(K) \cdot H(K)\} \rightarrow (5)$$

Linear Convolution using Circular Convolution

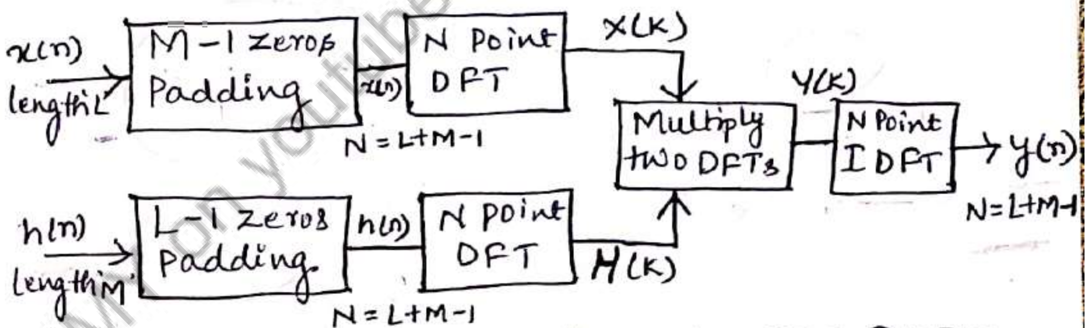
$$Y(K) = X(K) \cdot H(K)$$

From Circular Convolution: $x_1(n) \, (N) \, x_2(n) = X_1(K) \cdot X_2(K)$

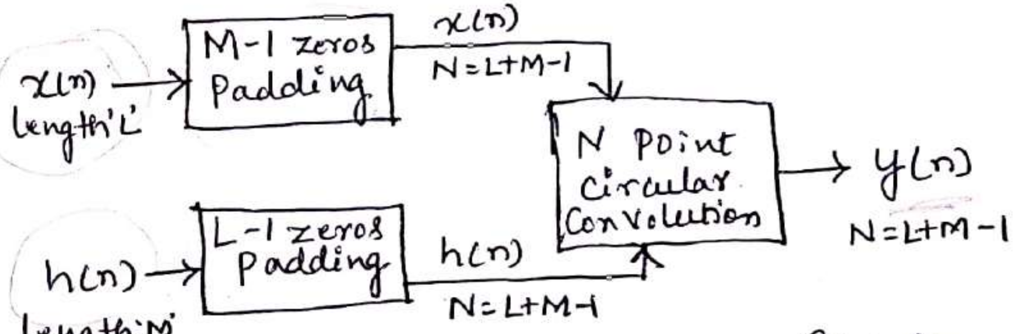
$$y(n) = x(n) \, (N) \, h(n) \qquad N = L+M-1.$$



(a) linear convolution to obtain y(n)



(b) y(n) obtained through DFT & IDFT



(c) Linear Convolution using Circular Convolution

Perform the following on the seq. $x(n) = \{1, 2, 3, 1\}$ and $h(n) = \{1, 1, 1\}$ (i) Linear Convolution. (ii) Circular Convolution (iii) Linear Convolution using circular convolution.

(i) Linear Convolution.

$L = 4 \quad M = 3 \qquad \therefore N = L + M - 1 = 4 + 3 - 1 = 6$

		1	2	3	1
			1	1	1
		1	2	3	1
	1	2	3	1	x
1	2	3	1	x	x
1	3	6	6	4	1

$y(n) = \{1, 3, 6, 6, 4, 1\}$

(ii) Linear Convolution using Circular Conv.

$h(n) = \{1, 1, 1, 0\}$

$$\begin{bmatrix} 1 & 1 & 3 & 2 \\ 2 & 1 & 1 & 3 \\ 3 & 2 & 1 & 1 \\ 1 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \\ 6 \\ 6 \end{bmatrix}$$

$y(n) = \{5, 4, 6, 6\}$

$6 - 4 = 2$

$y(n) = \{1, 3, 6, 6, 4, 1\}$

$y(n) = \{5, 4, 6, 6\}$

(iii) Linear conv. using circular conv.

$N = L + M - 1$, $M-1$ zeros to $x(n)$

$L-1$ zeros to $h(n)$

$M - 1 \Rightarrow 3 - 1 = 2$

$L - 1 \Rightarrow 4 - 1 = 3$

$x(n) = \{1, 2, 3, 1, 0, 0\}$

$h(n) = \{1, 1, 1, 0, 0, 0\}$.

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 3 & 2 \\ 2 & 1 & 0 & 0 & 1 & 3 \\ 3 & 2 & 1 & 0 & 0 & 1 \\ 1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 1 & 3 & 2 & 1 & 0 \\ 0 & 0 & 1 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 6 \\ 6 \\ 4 \\ 1 \end{bmatrix}$$

$y(n) = \{1, 3, 6, 6, 4, 1\}$

$x(n) = 1\delta(n) + 3\delta(n-1) + 2\delta(n-2) - \delta(n-3) + \delta(n-4)$

$h(n) = 2\delta(n) - \delta(n-1)$

$x(n) = \{1, 3, 2, -1, 1\}$

$h(n) = \{2, -1\}$

DFT for linear filtering of Long Duration Sequences.

① Overlap Save method:

→ h(n) of Length 'M' & x(n) → Segmented into blocks of 'L'

Step 1: Select Value of $N = 2^M$

Step 2: The length of h(n) is made 'N' by padding L-1 zeros. $[N = M+L-1]$

$$h(n) = \{h(0), h(1) \ldots h(M-1), \underbrace{0, 0 \ldots 0}\}$$

Step 3: The sequence x(n) is divided into sub sequences of leng 'N' as:

$$x_1(n) = \{\underbrace{0, 0, \ldots 0}_{M-1 \text{ zeros}}, x(0), x(1), \ldots x(L-1)\}$$

$$x_2(n) = \{\underbrace{x(L-M+1), \ldots x(L-1)}_{M-1 \text{ samples of } x_1(n)}, x(L), x(L+1) \ldots \ldots x(2L-1)\}$$

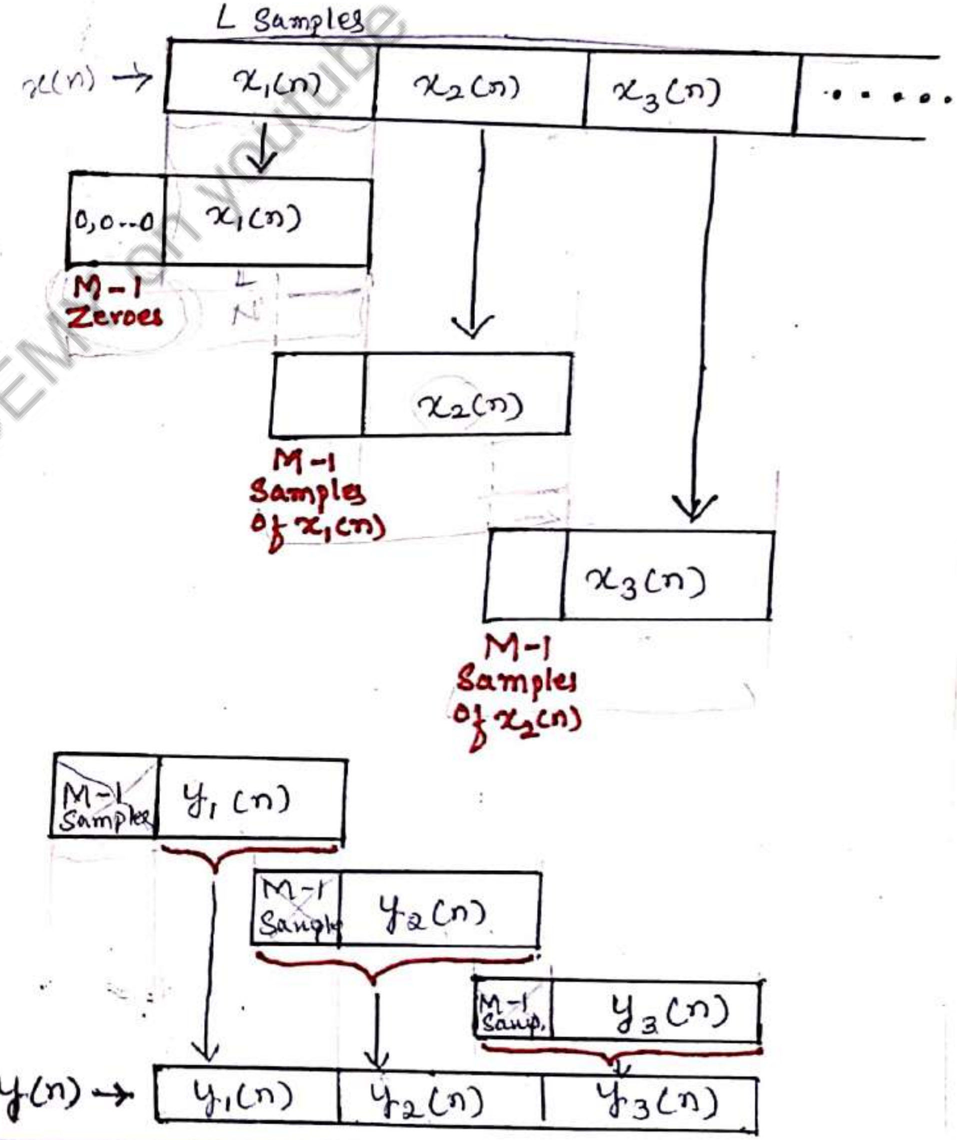
$$x_3(n) = \{\underbrace{x(2L-M+1), \ldots x(2L-1)}_{M-1 \text{ samples of } x_2(n)}, x(2L, x(2L+1) \ldots \ldots x(3L-1)\}$$

Step 4: Calculate $Y_1(K) = X_1(K) H(K)$

$\boxed{y_1(n) = x_1(n) ⓝ h(n)}$ $y_1(n) = \text{IDFT}\{Y_1(K)\}$

Step 5: Repeat step 4 to obtain $y_2(n)$, $y_3(n) \ldots$

Step 6: First M-1 Samples of $y_1(n), y_2(n)$, $y_3(n)$ are discarded and remaing samples are fitted one after the other to get the final sequance.



Scanned with CamScanner

Find $y(n)$ for $h(n) = \{1, 1, 1\}$ and $x(n) = \{3, -1, 0, 1, 3, 2, 0, 1, 2, 1\}$ using Overlap-save method.

$h(n) = \{1, 1, 1\} \quad \therefore M = 3$

$3 + 10 - 1 \Rightarrow 12$

Step 1: $N = 2^M = 2^3 \Rightarrow \boxed{N = 8}$

$N = M + L - 1 \Rightarrow L = N - M + 1 = 8 - 3 + 1 \Rightarrow \boxed{L = 6}$

$6 - 1 = 5$

Step 2:

$h(n) = \{1, 1, 1, \underbrace{0, 0, 0, 0, 0}_{L-1 \text{ zeros}}\}$

$3 - 1 = 2$

Step 3:

$x_1(n)\ \{\underbrace{0, 0}_{M-1 \text{ zeros}}, 3, -1, 0, 1, 3, 2\}$

$x_2(n)\ \{\underbrace{3, 2}_{M-1\ x_1(n)}, 0, 1, 2, 1, 0, 0\}$

Step 4: $y_1(n) = x_1(n) \, (N) \, h(n)$

$$\begin{bmatrix} 0 & 2 & 3 & 1 & 0 & -1 & 3 & 0 \\ 0 & 0 & 2 & 3 & 1 & 0 & -1 & 3 \\ 3 & 0 & 0 & 2 & 3 & 1 & 0 & -1 \\ -1 & 3 & 0 & 0 & 2 & 3 & 1 & 0 \\ 0 & -1 & 3 & 0 & 0 & 2 & 3 & 1 \\ 1 & 0 & -1 & 3 & 0 & 0 & 2 & 3 \\ 3 & 1 & 0 & -1 & 3 & 0 & 0 & 2 \\ 2 & 3 & 1 & 0 & -1 & 3 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ 3 \\ 2 \\ 2 \\ 0 \\ 4 \\ 6 \end{bmatrix}$$

$y_1(n) = \{\cancel{5}, \cancel{2}, 3, 2, 2, 0, 4, 6\}.$

Step 5: $y_2(n) = x_2(n) \, (N) \, h(n)$

$$\begin{bmatrix} 3 & 0 & 0 & 1 & 2 & 1 & 0 & 2 \\ 2 & 3 & 0 & 0 & 1 & 2 & 1 & 0 \\ 0 & 2 & 3 & 0 & 0 & 1 & 2 & 1 \\ 1 & 0 & 2 & 3 & 0 & 0 & 1 & 2 \\ 2 & 1 & 0 & 2 & 3 & 0 & 0 & 1 \\ 1 & 2 & 1 & 0 & 2 & 3 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 & 2 & 3 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 5 \\ 3 \\ 3 \\ 4 \\ 3 \\ 1 \end{bmatrix}$$

$y_2(n) = \{\cancel{3}, \cancel{5}, 5, 3, 3, 4, 3, 1\}$

Step 6:

$y(n) = \{3, 2, 2, 0, 4, 6, 5, 3, 3, 4, 3, 1\}$

DFT for Linear Filtering of Long duration Sequence

2. Overlap Add method

$h(n) \rightarrow$ length 'M' & $x(n) \rightarrow$ segmented into blocks.

Step 1: Select $N = 2^M$ $\quad h(n) = \{h(0), h(1) \ldots, \underbrace{0, 0 \ldots 0}_{L-1}\}$

Step 2: Length of h(n) is made 'N' by padding L-1 zeros $[N = M+L-1]$

Step 3: The seq. x(n) is divided into Sub seq. of length 'N'

$x_1(n) = \{x(0), x(1) \ldots x(L-1), \underbrace{0, 0 \ldots 0}_{M-1}\}$

$x_2(n) = \{x(L), x(L+1), \ldots x(2L-1), 0, 0 \ldots 0\}$

$x_3(n) = \{x(2L), x(2L+1), \ldots x(3L-1), 0, 0 \ldots 0\}$

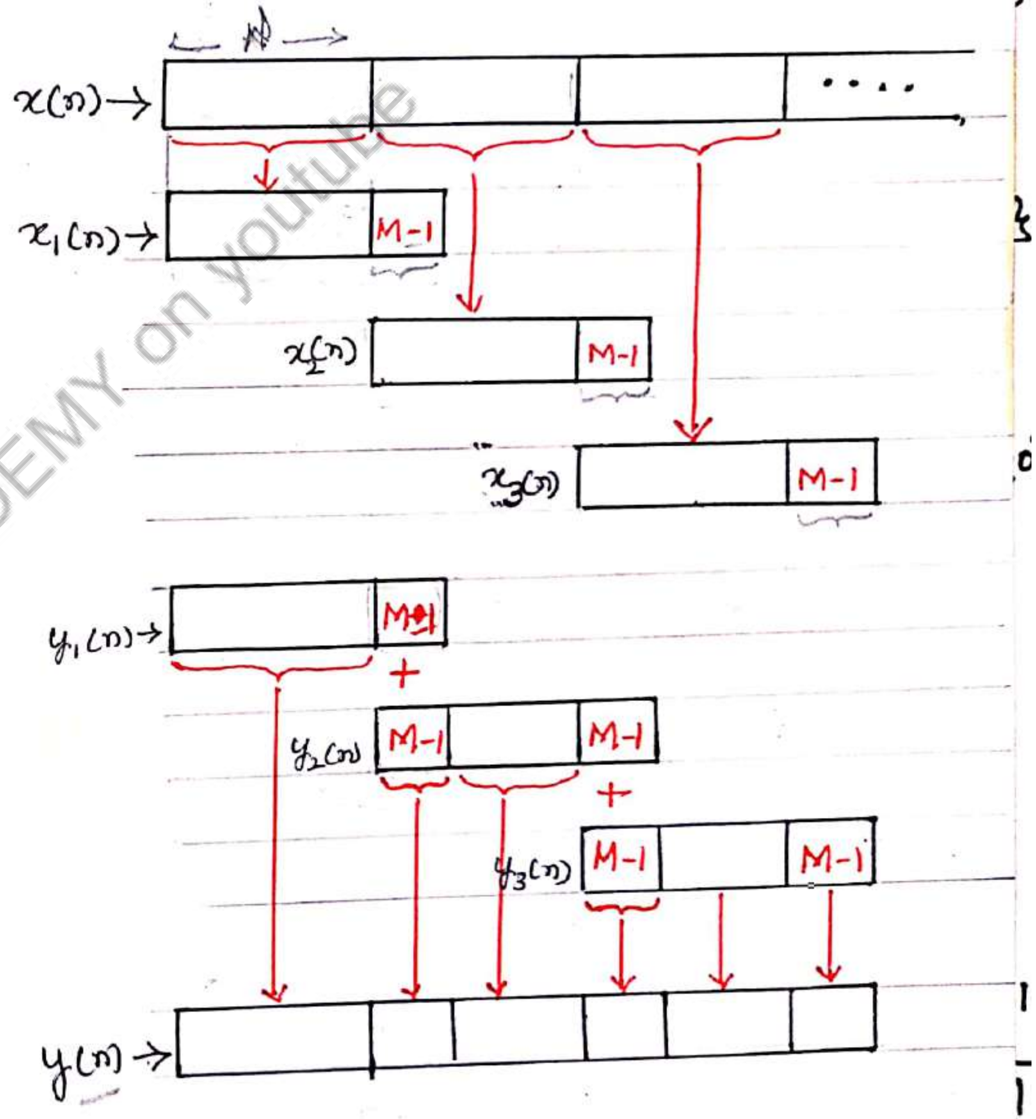
Step 4: Calculate $Y_1(K) = X_1(K) \cdot H(K)$

$y_1(n) = IDFT\{Y_1(K)\}$

(or) $y_1(n) = x_1(n) \, (N) \, h(n)$

Step 5: Repeat Step 4 to obtain $y_2(n), y_3(n) \ldots$

Step 6: Add all M-1 samples of each o/p sequence to first M-1 samples of succeeding o/p seq. Such seqs. are fitted one after another to get final seq.



For $h(n) = \{3,2,1,1\}$ & $x(n) = \{1,2,3,3,2,1,-1,-2,-3,5,6,-1,2,0,2,1\}$

Find the o/p using overlap Add method assume block length as 7.

$h(n) = \{3,2,1,1\}$ $\therefore M=4$

step1:- $N=7$ $\quad$ $\cancel{N=2^M}$

$N = M+L-1 \Rightarrow 7 = 4+L-1 \Rightarrow L=4$

$4-1 \Rightarrow 3$

step2:- $h(n) = \{3,2,1,1,\underbrace{0,0,0}_{L-1}\}$ $\quad$ $4-1 \Rightarrow 3$

step3: $x_1(n) = \{1,2,3,3,\underbrace{0,0,0}_{M-1}\}$

$x_2(n) = \{2,1,-1,-2,0,0,0\}$

$x_3(n) = \{-3,5,6,-1,0,0,0\}$

$x_4(n) = \{2,0,2,1,0,0,0\}$

Step4: $y_1(n) = x_1(n) \, (N) \, h(n)$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 3 & 3 & 2 \\ 2 & 1 & 0 & 0 & 0 & 3 & 3 \\ 3 & 2 & 1 & 0 & 0 & 0 & 3 \\ 3 & 3 & 2 & 1 & 0 & 0 & 0 \\ 0 & 3 & 3 & 2 & 1 & 0 & 0 \\ 0 & 0 & 3 & 3 & 2 & 1 & 0 \\ 0 & 0 & 0 & 3 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \underbrace{\begin{bmatrix} 3 \\ 8 \\ 14 \\ 18 \\ 11 \\ 6 \\ 3 \end{bmatrix}}_{y_1(n)}$$

Step5 $y_2(n) = x_2(n) \, (N) \, h(n)$

$= \{2,1,-1,-2,0,0,0\} \, (N) \, \{3,2,1,1,0,0,0\}$

$y_2(n) = \{6,7,1,-5,-4,-3,-2\}$

$y_3(n) = x_3(n) \, (N) \, h(n)$

$= \{-3,5,6,-1,0,0,0\} \, (N) \, \{3,2,1,1,0,0,0\}$

$y_3(n) = \{-9,9,25,11,9,5,-1\}$.

$y_4(n) = x_4(n) \, (N) \, h(n)$

$= \{2,0,2,1,0,0,0\} \, (N) \, \{3,2,1,1,0,0,0\}$

$y_4(n) = \{6,4,8,9,4,3,1\}$

Step6 $\quad M=4 \quad M-1 \Rightarrow 3$

$y_1(n) =$	3	8	14	18	11	6	3												
$y_2(n) =$					6	7	1	-5	-4	-3	-2								
$y_3(n) =$									-9	9	25	11	9	5	-1				
$y_4(n) =$													6	4	8	9	4	3	1
$y(n) =$	3	8	14	18	17	13	4	-5	-13	6	23	11	15	9	7	9	4	3	1

Radix-2 DIT-FFT Algorithm:

DIT → Decimation in Time
FFT → Fast Fourier Transform.

$x(n) \rightarrow$ lenth N

$x(n) = \{x(0), x(1), x(2), x(3) \ldots x(N-2), x(N-1)\}$

even indexed seq: $\{x(0), x(2), x(4) \ldots x(N-2)\}$

odd indexed seq: $\{x(1), x(3), x(5) \ldots x(N-1)\}$

W.K.T. N-Point DFT

$$X(K) = \sum_{n=0}^{N-1} x(n) W_N^{Kn} \; ; \; 0 \le K \le N-1 \longrightarrow (1)$$

eqn (1) → Decimation → even, odd seq.

$2r = N-2 \Rightarrow r = \frac{N}{2}-1$

$$X(K) = \sum_{\substack{n=0 \\ n \rightarrow \text{even}}}^{N-2} x(n) W_N^{Kn} + \sum_{\substack{n=1 \\ n \rightarrow \text{odd}}}^{N-1} x(n) W_N^{Kn} \longrightarrow (2)$$

$2r=0$, $r=0$ $\quad$ $2r+1=1$, $2r=0$, $r=0$ $\quad$ $2r+1 = N-1$, $2r = N-2$, $r = \frac{N}{2}-1$

Put $n = 2r$ in 1^{st} term, $n = 2r+1$ in 2^{nd} term.

$$X(K) = \sum_{r=0}^{N/2-1} \underbrace{x(2r)}_{g(r)} W_N^{2Kr} + \sum_{r=0}^{N/2-1} \underbrace{x(2r+1)}_{h(r)} W_N^{K(2r+1)} \longrightarrow (3)$$

$$X(K) = \sum_{r=0}^{N/2-1} g(r) W_N^{2Kr} + \sum_{r=0}^{N/2-1} h(r) \boxed{W_N^{K(2r+1)}}$$

$W_N^{2Kr} \cdot W_N^K$

$$\because W_N = e^{\frac{-j2\pi}{N}} \Rightarrow W_N^2 = e^{\frac{-j2\pi \cdot 2}{N}} = e^{\frac{-j2\pi}{N/2}} = W_{N/2}$$

Rearrange,

$$X(K) = \underbrace{\sum_{r=0}^{N/2-1} g(r) W_{N/2}^{Kr}}_{N/2 \text{ DFT eqn seq}} + W_N^K \underbrace{\sum_{r=0}^{N/2-1} h(r) W_{N/2}^{Kr}}_{N/2 \text{ DFT odd.}} \rightarrow (4)$$

$$X(K) = G(K) + W_N^K H(K) \; ; \; 0 \le K \le \frac{N}{2}-1 \rightarrow (5)$$

$G(K)$ & $H(K)$ → periodic with period $\frac{N}{2}$

$$X(K) = G\left(K-\frac{N}{2}\right) + W_N^K H\left(K-\frac{N}{2}\right) \; ; \; \frac{N}{2} \le K \le N-1 \rightarrow (6)$$

$G(K-4)$ $\quad$ $H(K-4)$

Ex:- $N=8$ $\quad \therefore K \rightarrow 0$ to 7

$K = 0$ to $3 \rightarrow (5)$ $\quad K = 4$ to 7 (6)

(5) ⇒

$K=0$; $X(0) = G(0) + W_8^0 H(0)$

$K=1$; $X(1) = G(1) + W_8^1 H(1)$

$K=2$; $X(2) = G(2) + W_8^2 H(2)$

$K=3$; $X(3) = G(3) + W_8^3 H(3)$

(6) ⇒

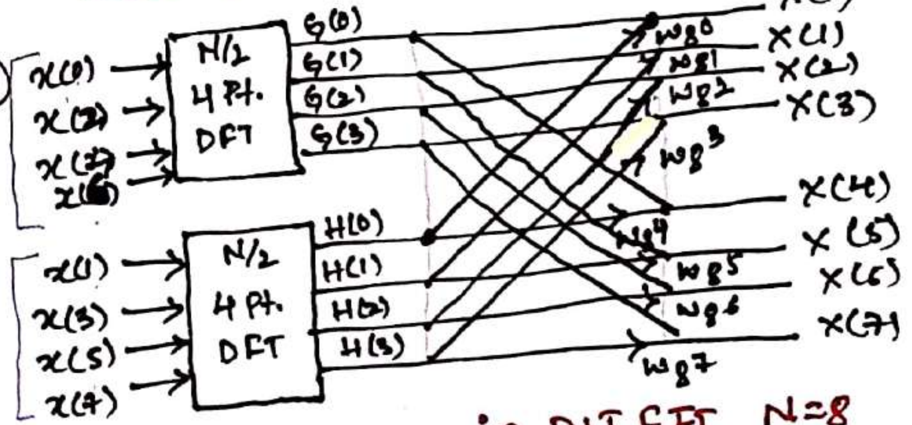
$K=4$; $X(4) = G(0) + W_8^4 H(0)$

$K=5$; $X(5) = G(1) + W_8^5 H(1)$

$K=6$; $X(6) = G(2) + W_8^6 H(2)$

$K=7$; $X(7) = G(3) + W_8^7 H(3)$

Signal flow graph



First stage in DIT FFT $N=8$

$G(K)$ & $H(K) \rightarrow \frac{N}{2}$ point

combination of two $\frac{N}{4}$ points.

$$G(K) = \sum_{r=0}^{N/2-1} g(r) W_{N/2}^{Kr} \rightarrow (7)$$

$$G(K) = \sum_{r=0}^{N/2-2} g(r) W_{N/2}^{Kr} + \sum_{r=1}^{N/2-1} g(r) W_{N/2}^{Kr}$$

$2l = \frac{N}{2}-2 \Rightarrow l = \frac{N}{4}-1$; $2l+1 = \frac{N}{2}-1$, $2l = \frac{N}{2}-2$, $l = \frac{N}{4}-1$; $2l = 0 \Rightarrow l = 0$; $2l+1 = 1 \Rightarrow l = 0$

$$g(r) = \{ g(0), g(1), g(2) \dots g(\tfrac{N}{2}-2), g(\tfrac{N}{2}-1) \}$$

Put $r = 2l$ in 1st term, $r = 2l+1$ in 2nd (7)

$$G(K) = \sum_{l=0}^{N/4-1} \underbrace{g(l)}_{a(l)} W_{N/2}^{2Kl} + \sum_{l=0}^{N/4-1} \underbrace{g(2l+1)}_{b(l)} \boxed{W_{N/2}^{K(2l+1)}}$$

$W_{N/2}^{2Kl} \cdot W_{N/2}^{K}$

$$G(K) = \underbrace{\sum_{l=0}^{N/4-1} a(l) W_{N/4}^{Kl}}_{A(K)} + W_{N/2}^{K} \underbrace{\sum_{l=0}^{N/4-1} b(l) W_{N/4}^{Kl}}_{B(K)}$$

$W_{N/2}^{2} = W_{N/4}$

$$G(K) = \underbrace{A(K)}_{N/4} + W_{N/2}^{K} \underbrace{B(K)}_{N/4}; \quad 0 \le K \le \frac{N}{4}-1 \rightarrow (8)$$

$$H(K) = C(K) + W_{N/2}^{K} D(K); \quad 0 \le K \le \frac{N}{4}-1 \rightarrow (9)$$

lly

$A(K), B(K), C(K)$ & $D(K) \rightarrow$ periodic $\frac{N}{4}$.

$x(0)$ $x(2)$ $x(4)$ $x(6)$

$x(1)$ $x(3)$ $x(5)$ $x(7)$

(8) $\Rightarrow$ $G(K) = A(K-\frac{N}{4}) + W_{N/2}^{K} B(K-\frac{N}{4})$; $\frac{N}{4} \le K \le \frac{N}{2}-1 \rightarrow (10)$

$A(K-2)$ $\quad B(K-2)$

(9) $\Rightarrow$ $H(K) = C(K-\frac{N}{4}) + W_{N/2}^{K} D(K-\frac{N}{4})$; $\frac{N}{4} \le K \le \frac{N}{2}-1 \rightarrow (11)$

$C(K-2)$ $\quad D(K-2)$

$K \Rightarrow 0$ & $1 \rightarrow$ eqn (8) & (9)

2 & $3 \rightarrow$ eqn (10) & (11)

eqn (8) $\Rightarrow$

$K=0;\ G(0) = A(0) + W_4^0 B(0)$

$K=1;\ G(1) = A(1) + W_4^1 B(1)$

eqn (10)

$K=2;\ G(2) = A(0) + W_4^2 B(0)$

$K=3;\ G(3) = A(1) + W_4^3 B(1)$

eqn (9)

$K=0;\ H(0) = C(0) + W_4^0 D(0)$

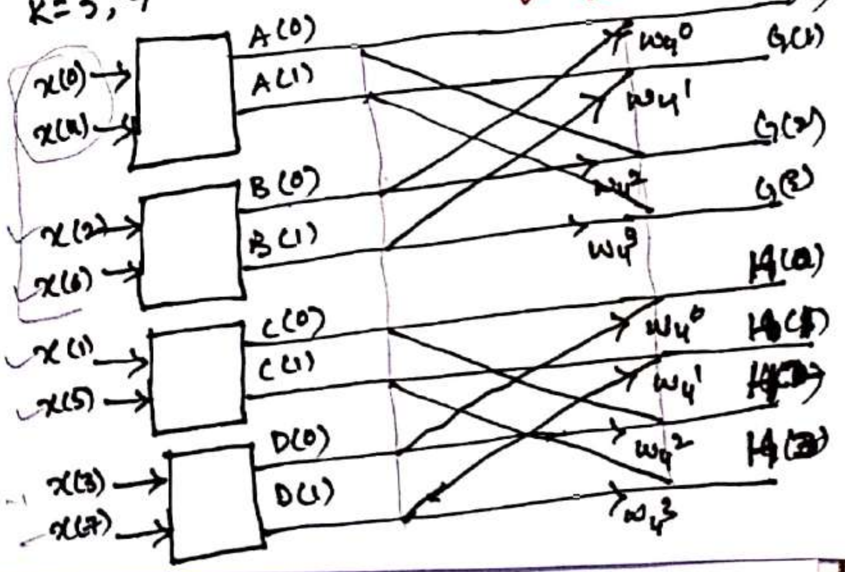
$K=1;\ H(1) = C(1) + W_4^1 D(1)$

eqn (11)

$K=2;\ H(2) = C(0) + W_4^2 D(0)$

$K=3;\ H(3) = C(1) + W_4^3 D(1)$

2nd stage



Radix-2 DIT-FFT Algorithm:

③

Each $\frac{N}{4}$ DFT as two $\frac{N}{8}$ point DFTs.

the 2-point DFT of $x(0)$ & $x(4)$

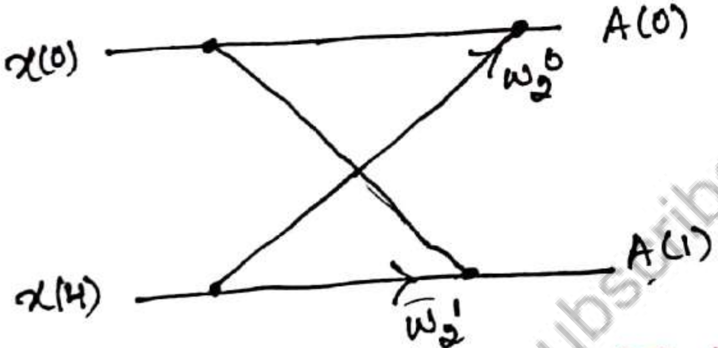
$$A(K) = \sum_{n=0}^{N/4-1} x(n) W_{N/4}^{Kn} \; ; \; 0 \le K \le \frac{N}{4} - 1$$

$\boxed{N=8}$

$$A(K) = \sum_{n=0}^{1} x(n) W_2^{Kn} \; ; \; 0 \le K \le 1$$

For ; $K=0 \Rightarrow A(0) = x(0) + W_2^0 x(4)$

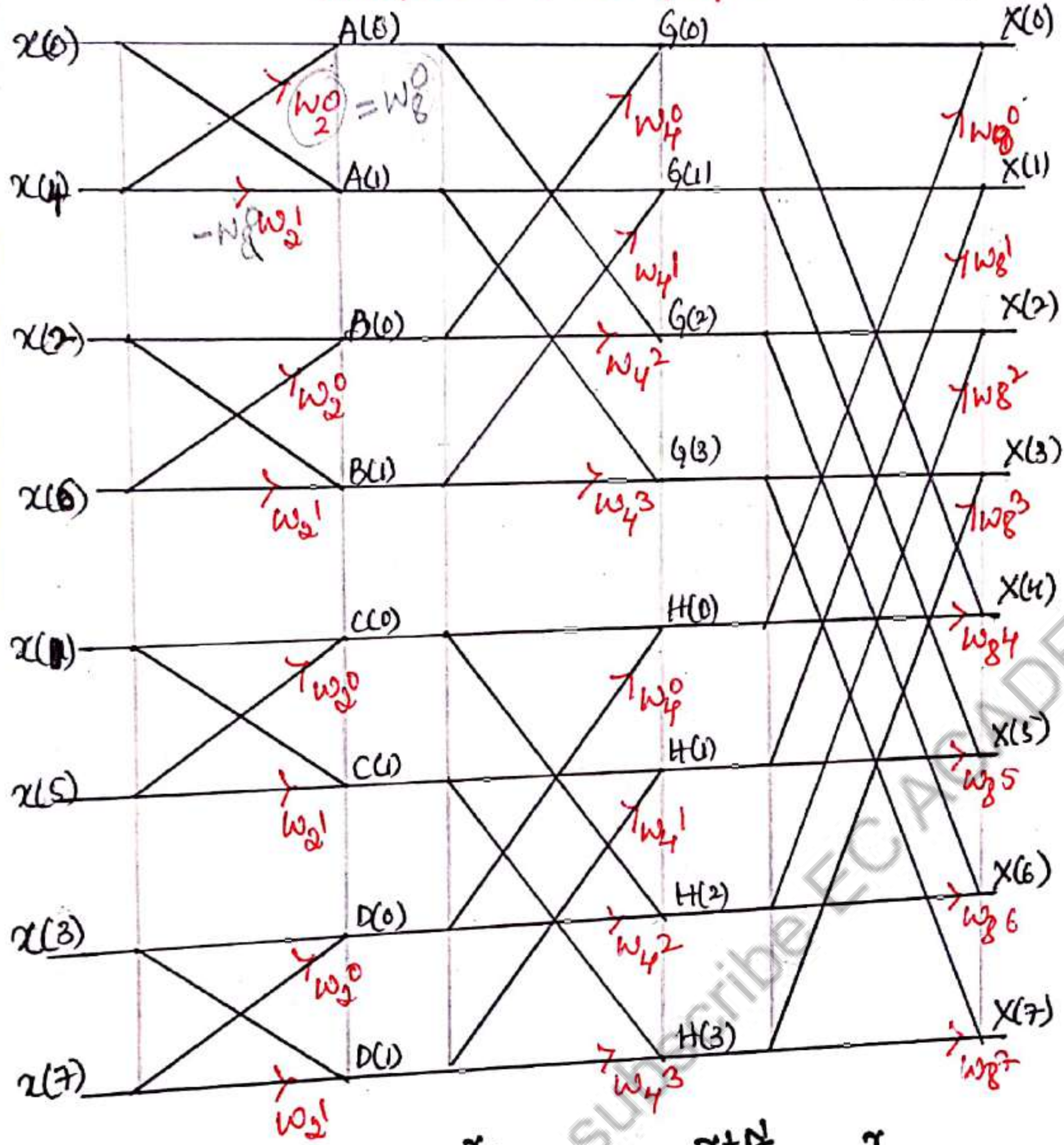
For ; $K=1 \Rightarrow A(1) = x(0) + W_2^1 x(4)$



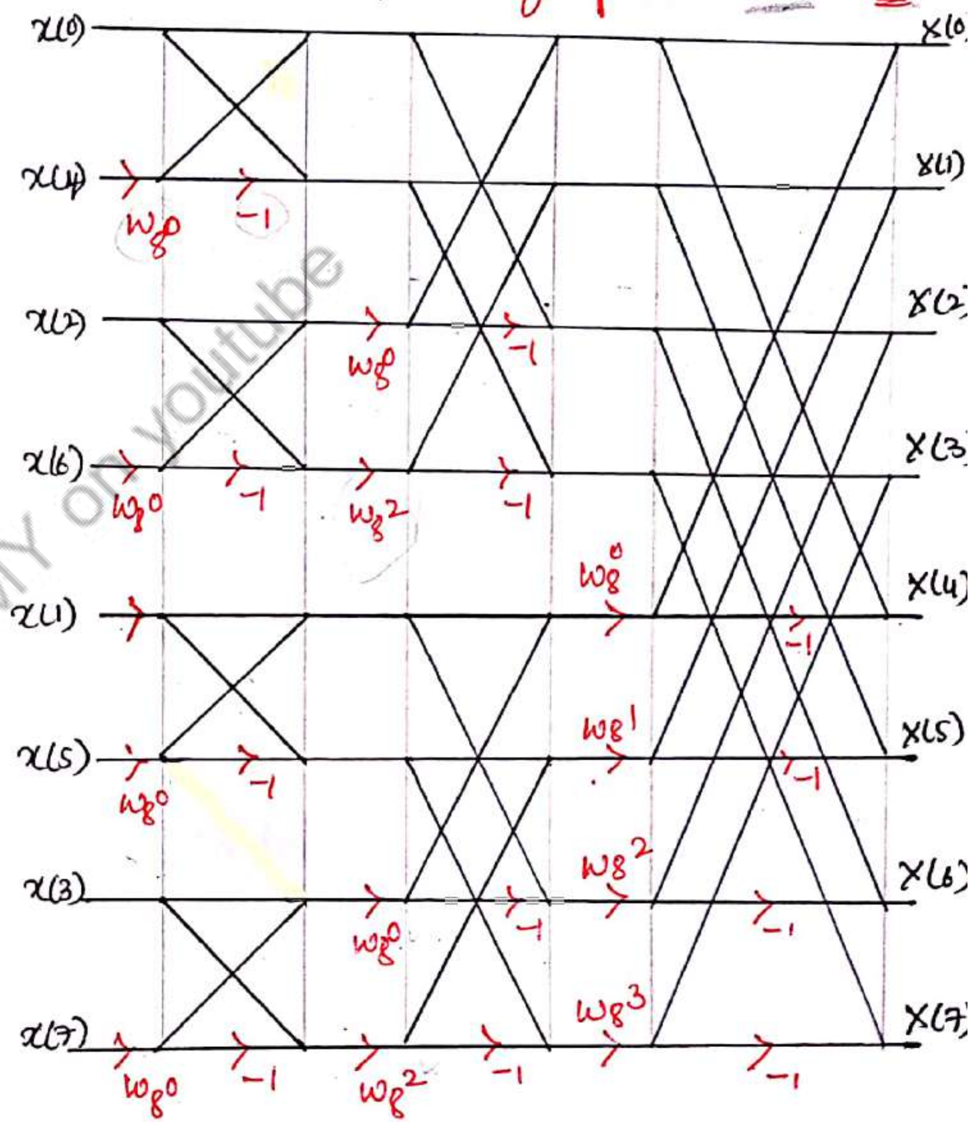
3rd stage in DIT FFT $N=8$

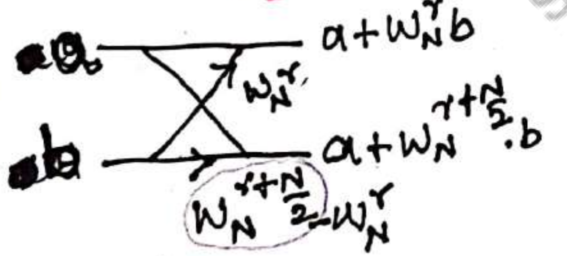
8
X(0)
X(1)
X(2)
X(3)
X(4)
X(5)
X(6)
X(7)

Complete Flow graph DIT-FFT ; N=8



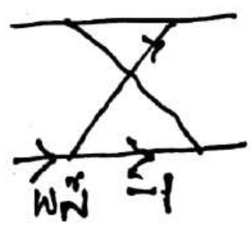
Reduced Flow graph: DIT-FFT N=8





$$W_N^{r+\frac{N}{2}} = -W_N^r$$

→ Symmetry



Computational Efficiency of fft over dft:

Direct Computation of DFT

no of complex addion $= N(N-1)$

no of complex multiplication $= N^2$

Radix-2 FFT

no of complex addion $= N \log_2 N$

no of complex multipliation $= \frac{N}{2} \log_2 N$

$$\%\ \text{Saving in Add.} = 100 - \frac{\text{no of addition FFT}}{\text{no of adition in DFT}} \times 100$$

$$\%\ \text{Saving in mul.} = 100 - \frac{\text{no of mul. FFT}}{\text{no of mul. DFT}} \times 100$$

Ex:- $N = 1024$

Direct Computation of DFT

no of complex addions $= 1024(1024-1) = 1047552$

no of complex mul. $= (1024)^2 = 1048576$

Radix-2 FFT.

no complex add $= N \log_2 N = 1024 \log_2 1024$

$\log_2 x = \frac{\ln x}{\ln 2}$

$$= 1024 \frac{\ln 1024}{\ln 2} = 10240$$

$$\text{no complex mul} = \frac{1024}{2} \log_2 1024$$

$$= 512 \frac{\ln 1024}{\ln 2} = 5120$$

$$\%\ \text{Saving in Add} = 100 - \left[\frac{10240}{1047552}\right] \times 100$$

$$= 99\%$$

$$\%\ \text{Savin in mul} = 100 - \left[\frac{5120}{1048576}\right] \times 100$$

$$= 99.5\%$$

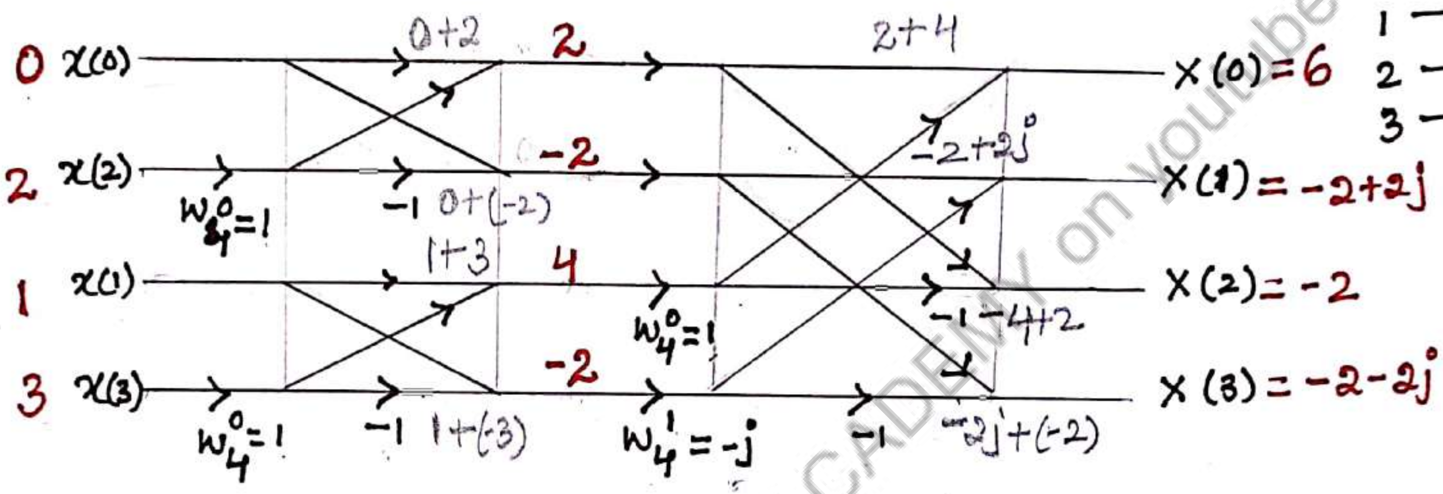
Given $x(n) = \{0, 1, 2, 3\}$, find $X(K)$ using DIT-FFT Algorithm.

$\therefore N = 4$ $\quad W_4^0 = 1 \quad W_4^1 = -j$

bit reversal

$4 = 2^{2}$

	BR
$0 \to 00$	$00 \to 0$
$1 \to 01$	$10 \to 2$
$2 \to 10$	$01 \to 1$
$3 \to 11$	$11 \to 3$

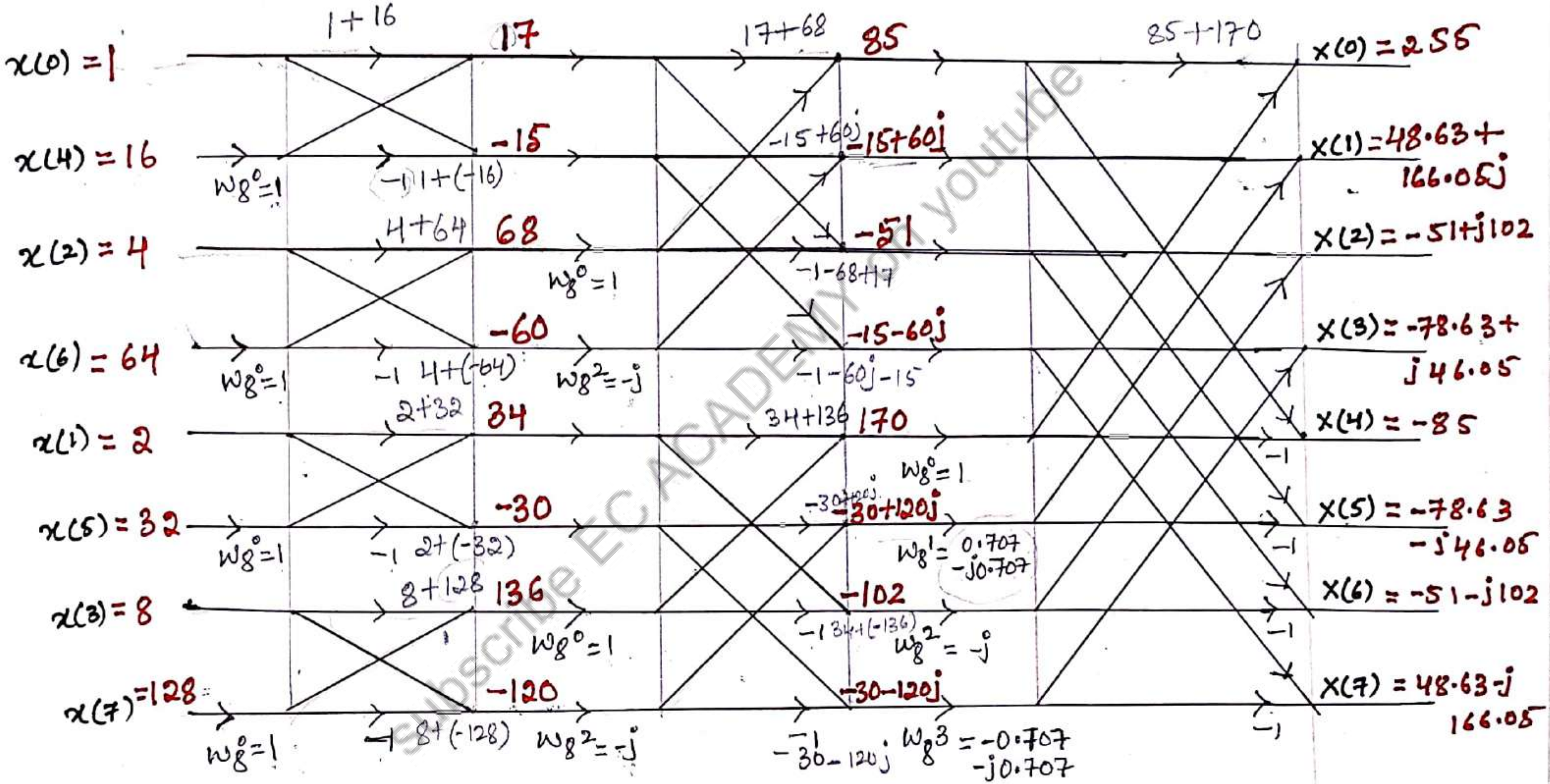


Flow-graph for DIT-FFT : N = 4

$\therefore X(K) = \{6, -2+2j, -2, -2-2j\}$

Given $x(n) = \{1, 2, 4, 8, 16, 32, 64, 128\}$
Find $X(K)$ using DIT-FFT.

$\therefore N = 8$



$\therefore X(K) = \{256, 48.63+j166.05, -51+j102, -78.63+j46.05, -85, -78.63-j46.05, -51-j102, 48.63-j166.05\}$

Radix-2 DIF FFT Algorithm:

DIF → Decimation in Frequency.

$$X(K) = \sum_{n=0}^{N-1} x(n)\, W_N^{Kn}; \quad 0 \le K \le N-1 \longrightarrow (1)$$

$$(1) \Rightarrow X(K) = \sum_{n=0}^{N/2-1} x(n)\, W_N^{Kn} + \sum_{n=N/2}^{N-1} x(n)\, W_N^{Kn}. \qquad n = n+\tfrac{N}{2}$$

$$= \sum_{n=0}^{N/2-1} x(n)\, W_N^{Kn} + \sum_{n=0}^{N/2-1} x\left(n+\tfrac{N}{2}\right) W_N^{K\left(n+\frac{N}{2}\right)}$$

$$= \sum_{n=0}^{N/2-1} x(n)\, W_N^{Kn} + W_N^{K\frac{N}{2}} \sum_{n=0}^{N/2-1} x\left(n+\tfrac{N}{2}\right) W_N^{Kn} \qquad \because W_N^{K\frac{N}{2}} = (-1)^K$$

$$= \sum_{n=0}^{N/2-1} x(n)\, W_N^{Kn} + (-1)^K \sum_{n=0}^{N/2-1} x\left(n+\tfrac{N}{2}\right) W_N^{Kn}$$

$$\therefore X(K) = \sum_{n=0}^{N/2-1} \left[x(n) + (-1)^K x\left(n+\tfrac{N}{2}\right)\right] W_N^{Kn} \longrightarrow (2)$$

Decompose $X(K)$ as even & odd index seq.
$K=2r$ $\quad K=(2r+1)$

$$X(2r) = \sum_{n=0}^{N/2-1} \left[x(n) + (-1)^{2r} x\left(n+\tfrac{N}{2}\right)\right] W_N^{2rn}$$

$$X(2r) = \sum_{n=0}^{N/2-1} \Big[\underbrace{x(n) + x\left(n+\tfrac{N}{2}\right)}_{g(n)}\Big] W_{N/2}^{rn}$$

$$X(2r) = \sum_{n=0}^{N/2-1} g(n)\, W_{N/2}^{rn}; \quad 0 \le r \le \tfrac{N}{2}-1 \longrightarrow (3)$$

$$X(2r+1) = \sum_{n=0}^{N/2-1} \left[x(n) + (-1)^{2r+1} x\left[n+\tfrac{N}{2}\right]\right] W_N^{(2r+1)n}$$

$$= \sum_{n=0}^{N/2-1} \Big[\underbrace{x(n) - x\left(n+\tfrac{N}{2}\right)}_{h(n)}\Big] W_N^{n} \cdot W_{N/2}^{rn} \longrightarrow (4)$$

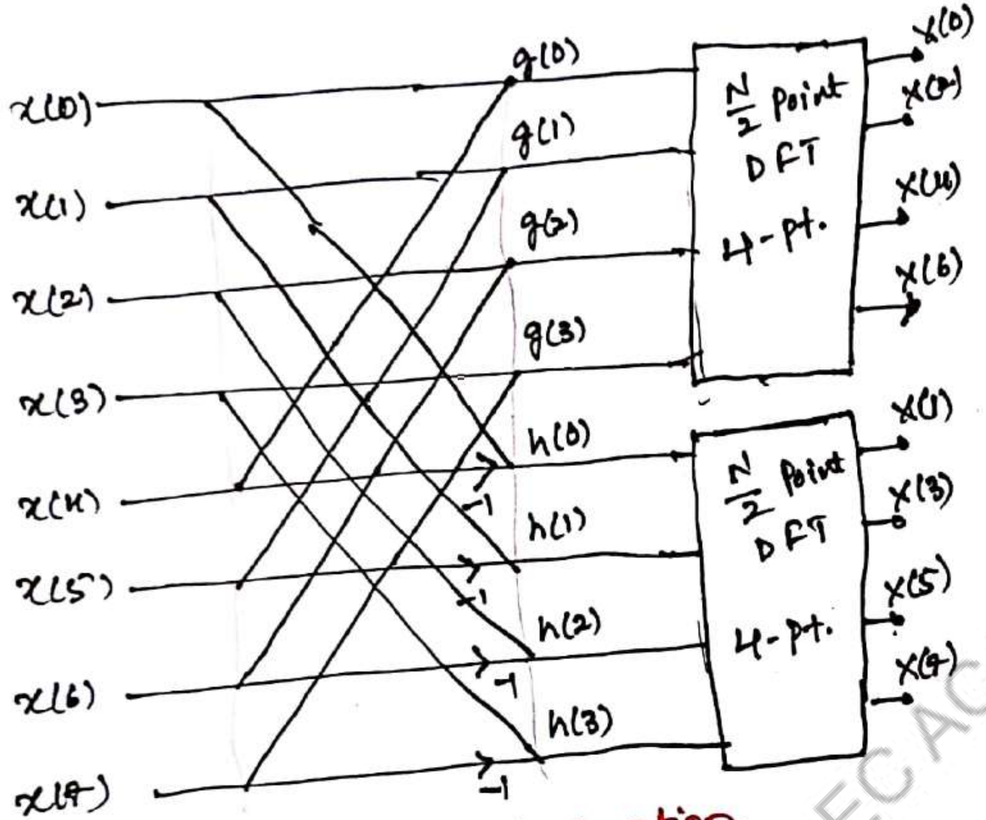
$$\therefore X(2r+1) = \sum_{n=0}^{N/2-1} h(n)\, W_N^{n} \cdot W_{N/2}^{rn}$$

$$\Rightarrow g(n) = x(n) + \underbrace{x\left(n+\tfrac{N}{2}\right)}_{(n+4)} \qquad N=8$$

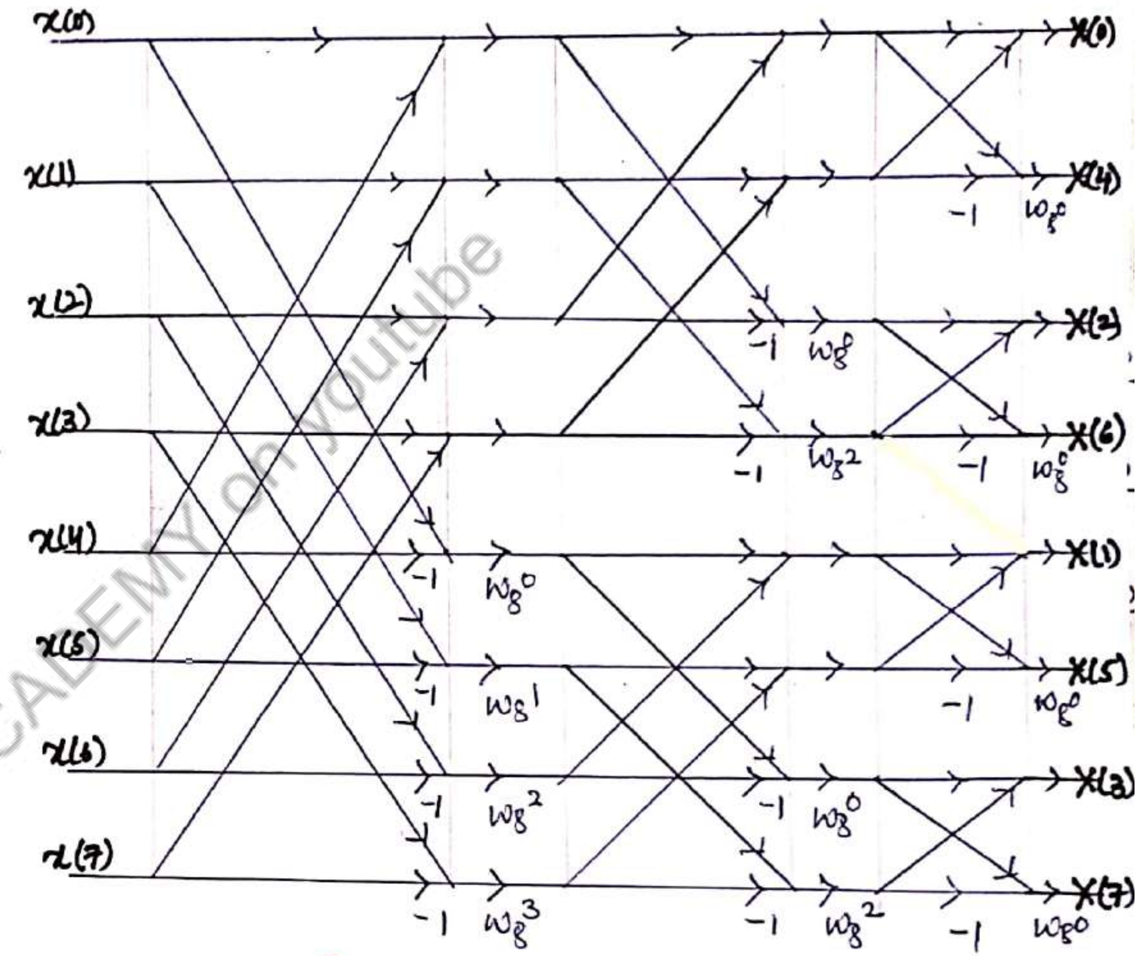
$$h(n) = x(n) - \underbrace{x\left(n+\tfrac{N}{2}\right)}_{(n+4)}$$

Put $n \rightarrow 0$ to 3

$n=0$: $g(0) = x(0) + x(4)$	$h(0) = x(0) - x(4)$
$n=1$: $g(1) = x(1) + x(5)$	$h(1) = x(1) - x(5)$
$n=2$: $g(2) = x(2) + x(6)$	$h(2) = x(2) - x(6)$
$n=3$: $g(3) = x(3) + x(7)$	$h(3) = x(3) - x(7)$

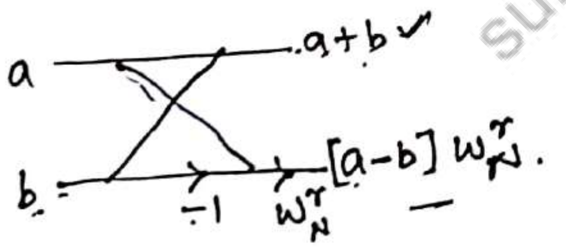


1st stage of decimation.



Reduced flow graph for N=8 DIF-FFT algorithm

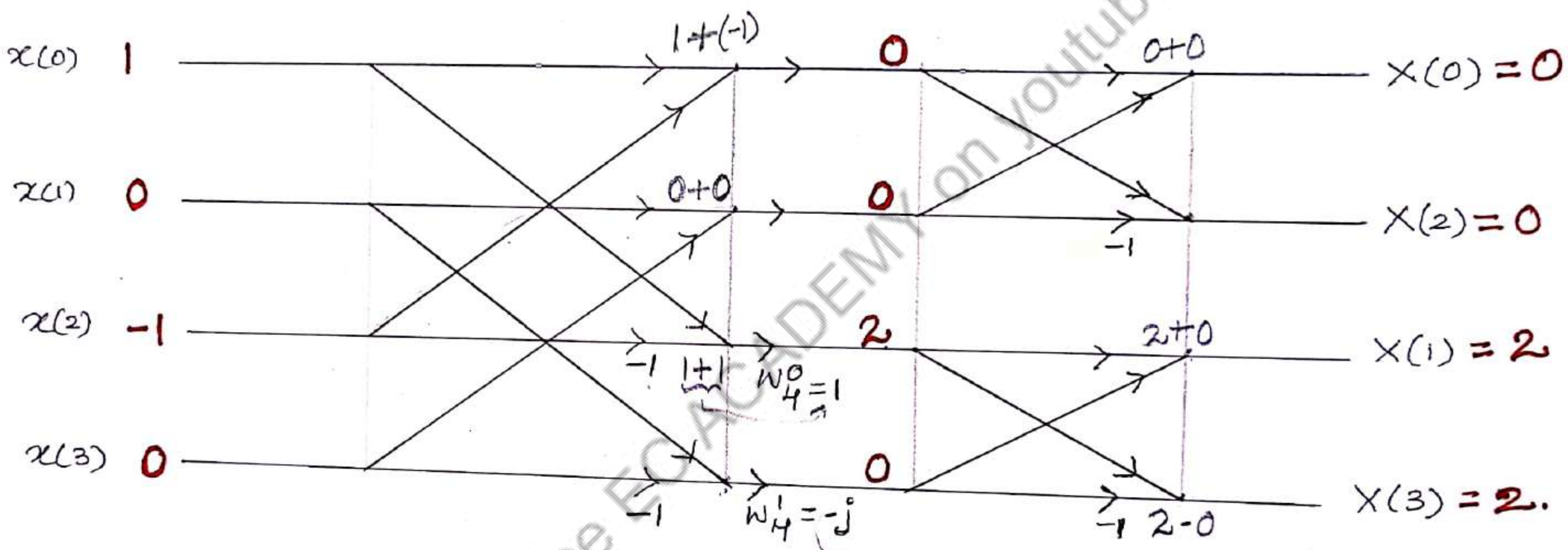
$\frac{N}{2} \rightarrow \frac{N}{4}$ point, 2-point DFT $\rightarrow$ N-point



Compute the DFT of $x(n) = \cos\frac{n\pi}{2}$ where $N=4$ using DIF-FFT.

$$x(n) = \cos\frac{n\pi}{2}$$

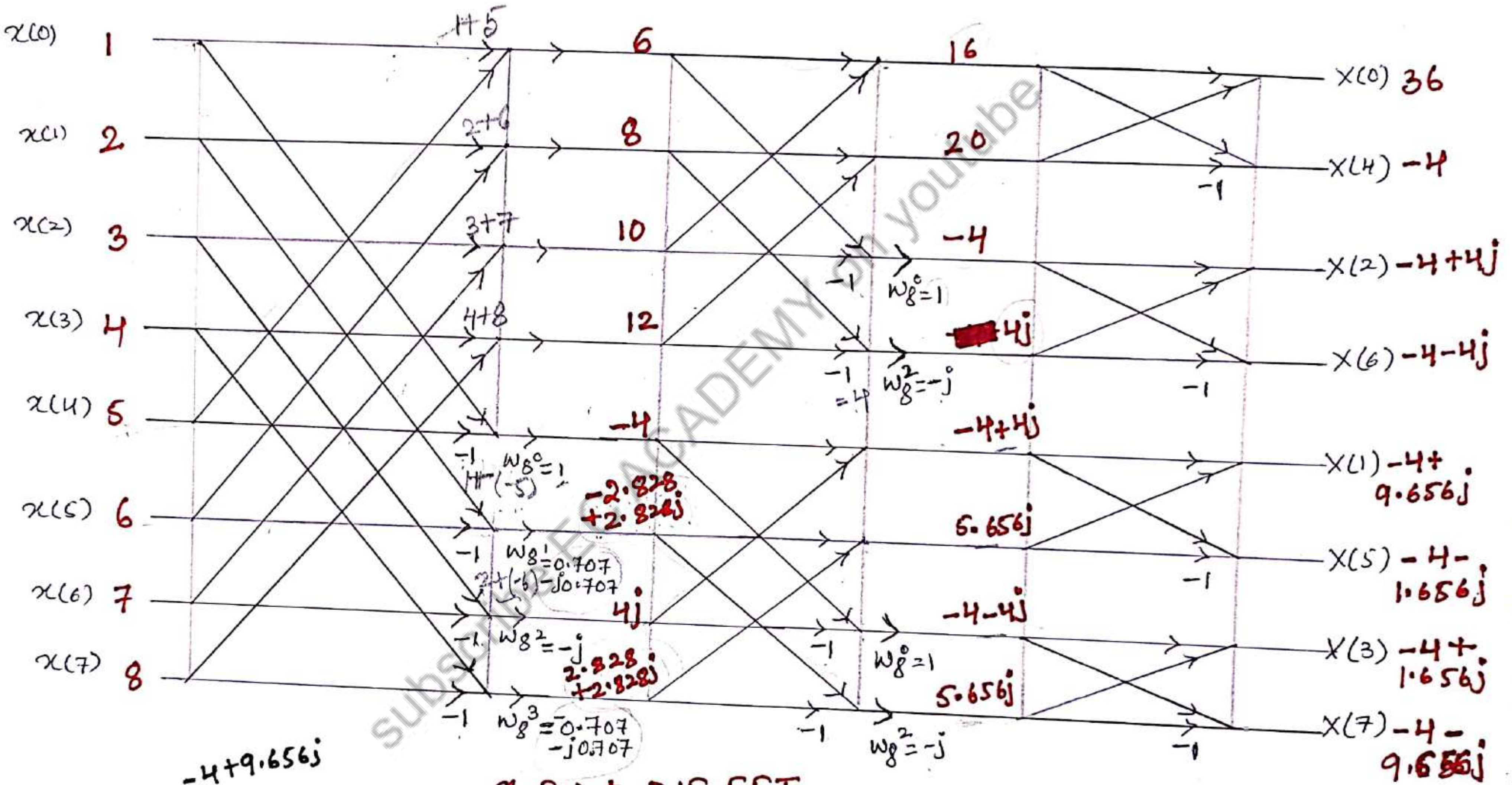
$n = 0$ to 3 ; $x(n) = \{1, 0, -1, 0\}$



Flow graph for 4-Point DIF FFT

$\therefore X(K) = \{0, 2, 0, 2\}$.

Given $x(n) = n+1$ for $0 \le n \le 7$ find $X(K)$ using DIF-FFT Algorithm. $x(n) = \{1, 2, 3, 4, 5, 6, 7, 8\}$



8-Point DIF FFT.

$-4+9.656j$

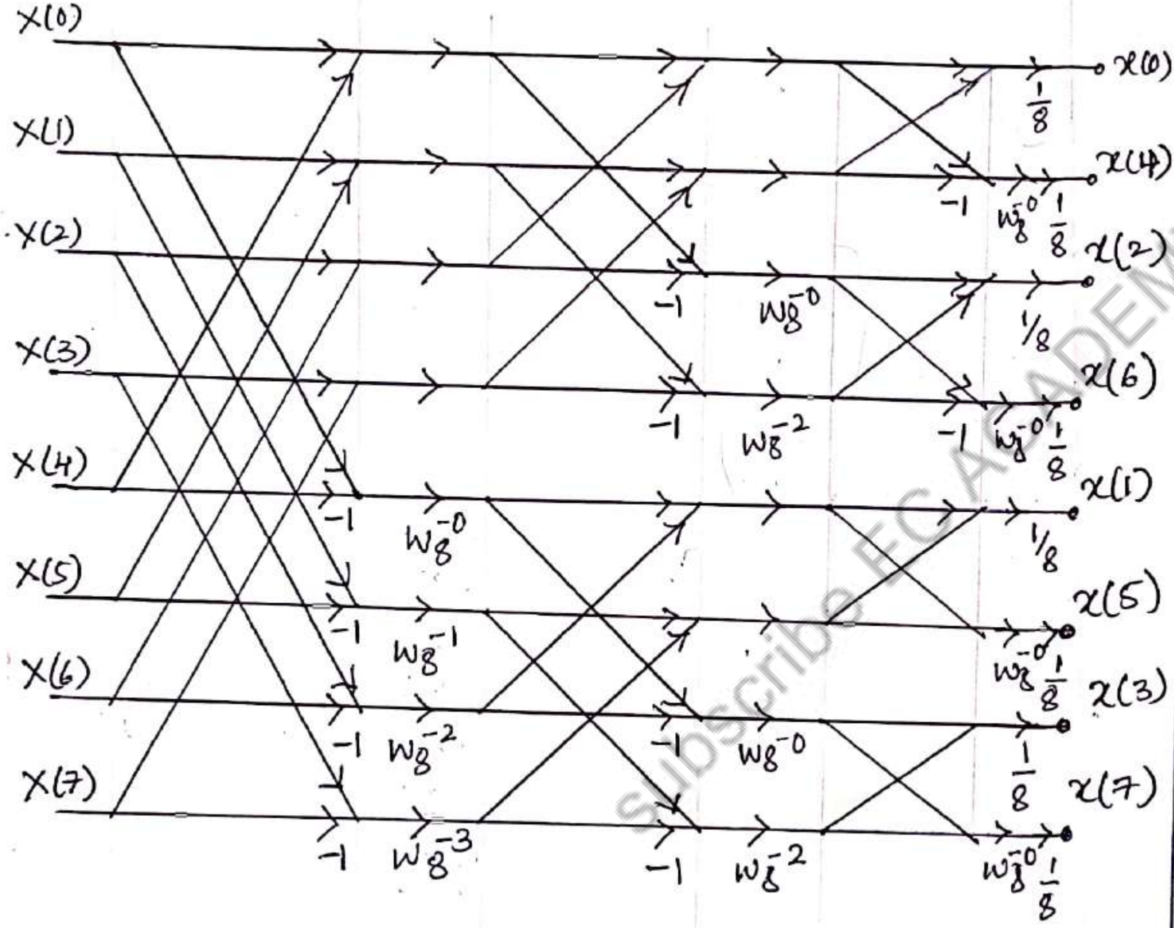
$\therefore X(K) = \{36, -4+4j, -4+1.656j, -4, -4-1.656j, -4-4j, -4-9.656j\}$

Scanned with CamScanner

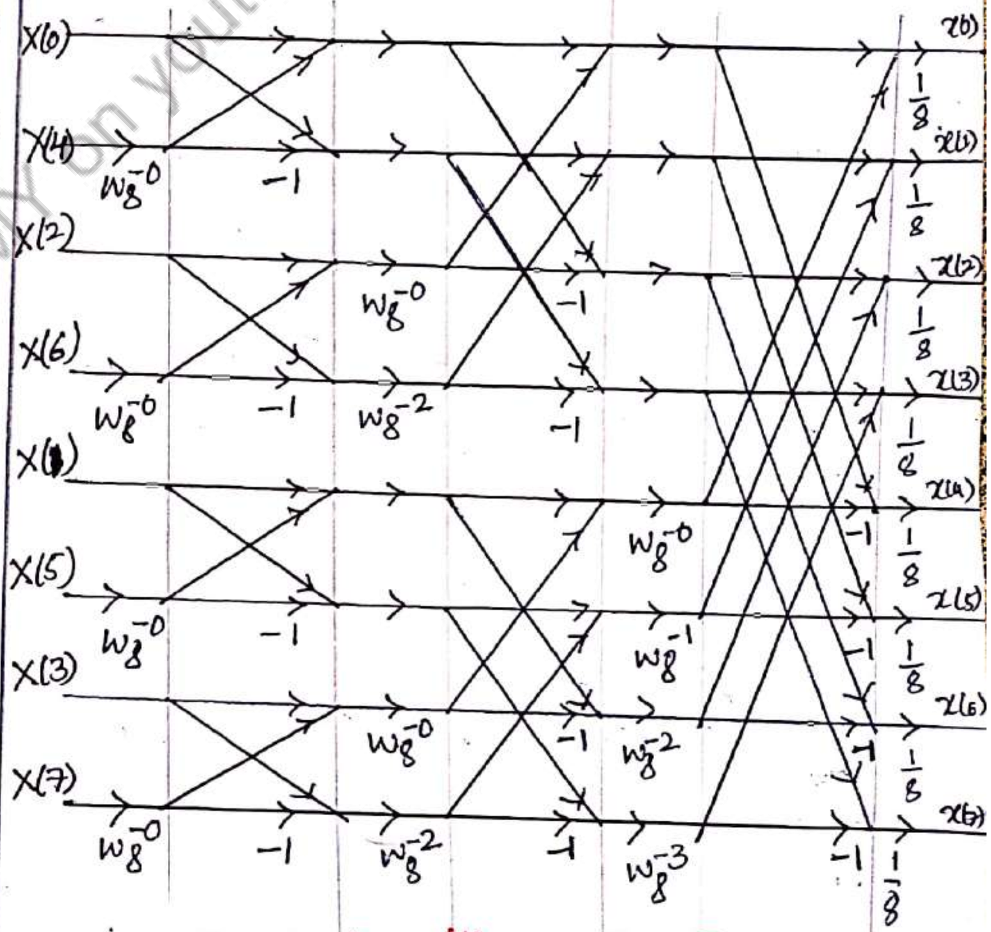
IDFT using FFT

IDFT ; $x(n) = \frac{1}{N} \sum_{K=0}^{N-1} X(K)\, W_N^{-Kn}$; $0 \le n \le N-1$

DFT ; $X(K) = \sum_{n=0}^{N-1} x(n)\, W_N^{Kn}$; $0 \le K \le N-1$

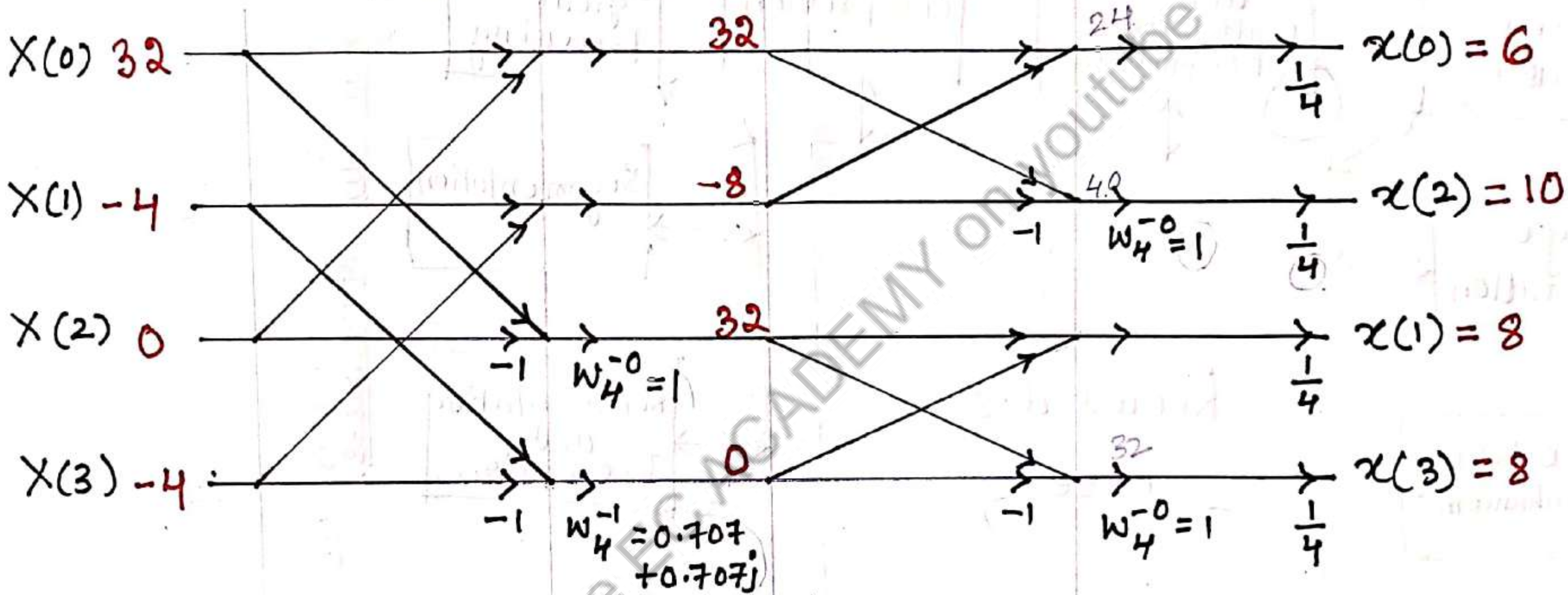


DIT-FFT algorithm for IDFT ; N=8



DIF-FFT algorithm for IDFT : N=8

For $X(K) = \{32, -4, 0, -4\}$ compute IDFT using DIT-FFT.

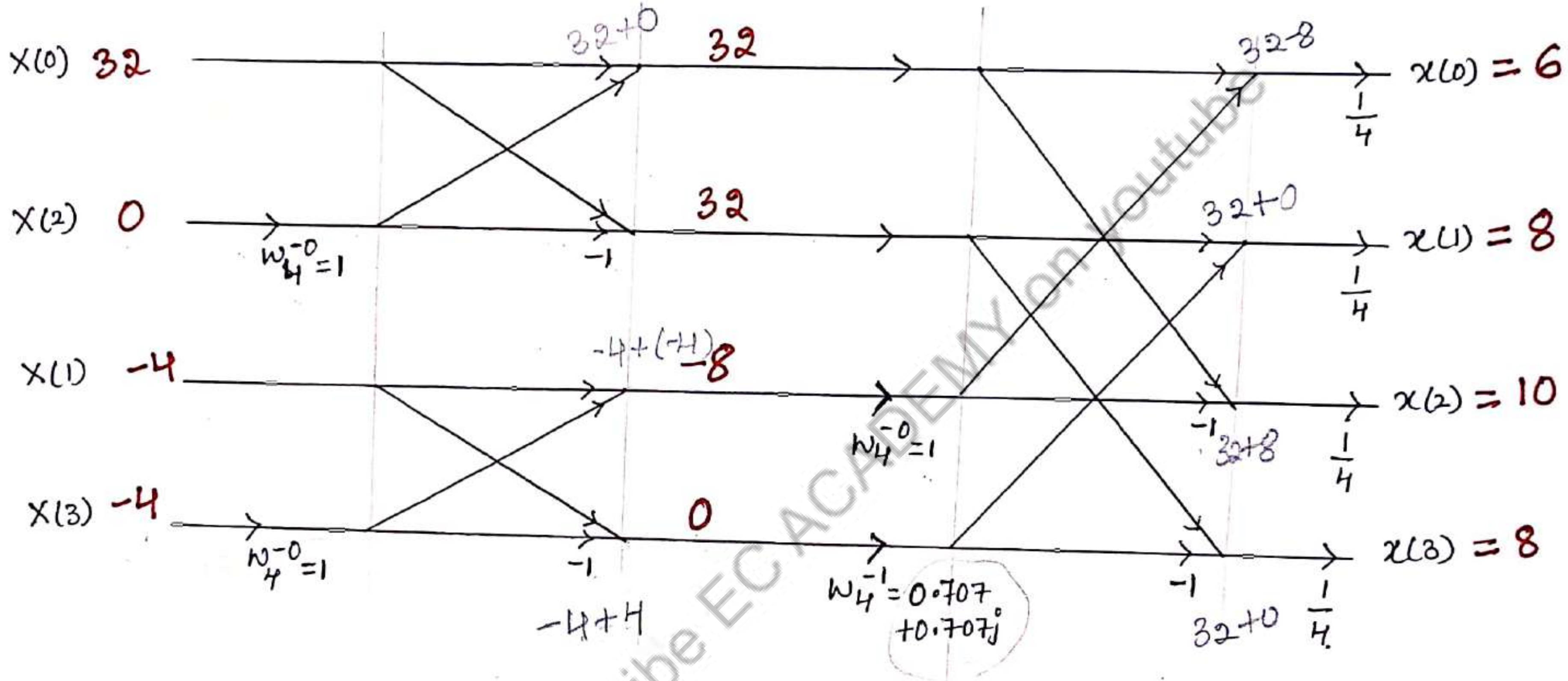


Flow graph for IDFT using DIT FFT

N=4

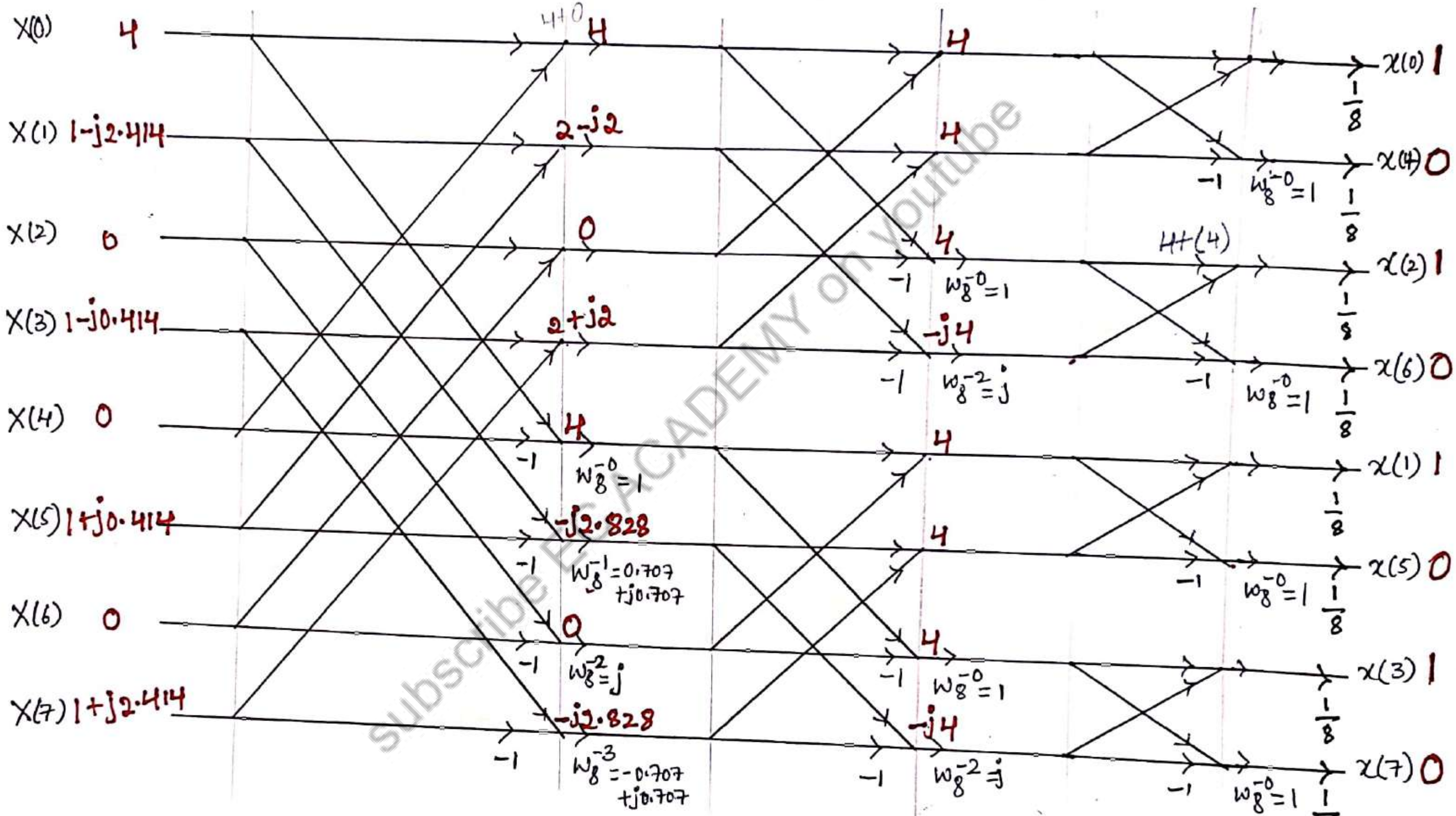
$\therefore x(n) = \{6, 8, 10, 8\}$

Compute $x(n)$ IDFT for $X(K) = \{32, -4, 0, -4\}$ using DIF-FFT



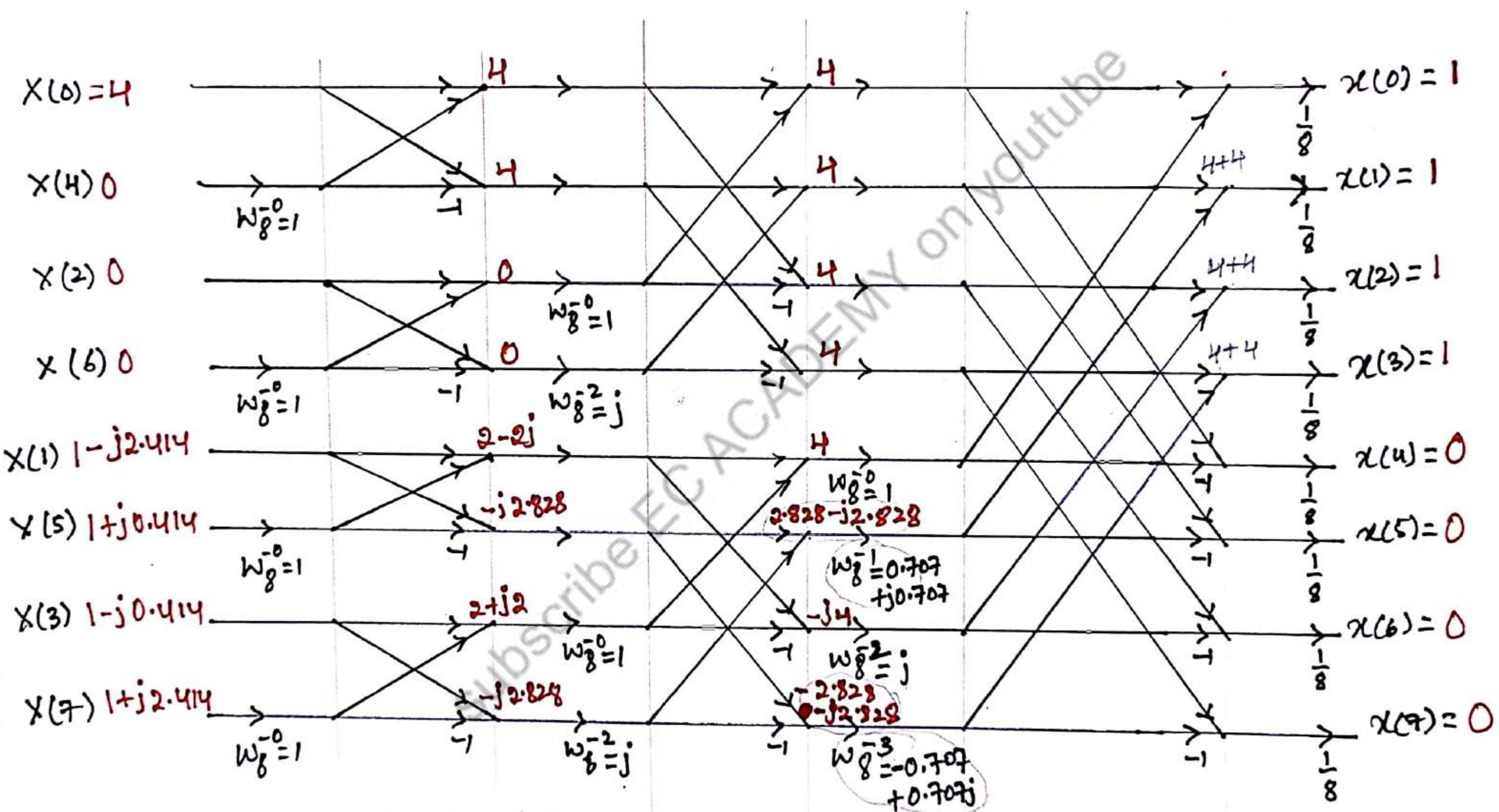
$\therefore x(n) = \{6, 8, 10, 8\}$

Find the seq. $x(n)$ Corresponding to 8-point DFT using DIT-FFT for $X(K) = \{4,\ 1-j2.414,\ 0,\ 1-j0.414,\ 0,\ 1+j0.414,\ 0,\ 1+j2.414\}$



$\therefore x(n) = \{1, 1, 1, 1, 0, 0, 0, 0\}$

Find the seq. x(n) Corresponding to 8-point IDFT using DIF-FFT for $X(K) = \{4, 1-j2.414, 0, 1-j0.414, 0, 1+j0.414, 0, 1+j2.414\}$



$\therefore x(n) = \{1, 1, 1, 1, 0, 0, 0, 0\}$

Introduction – Design of FIR Filter

→ Filter → Impulse response is finite.

→ output → depends only on present and (or) past values.

→ Applications → where linear phase is important.

Ex:- Data transmission, Speech processing, Correlation processing, Interpolation.

Characteristics:

(i) Impulse response → Finite length.

(ii) non-recursive FIR filter → Stable.

(iii) phase distortion of Frq response can be eliminated by FIR filter.

(iv) Implement a recursive FIR Filters

(v) Effect of Start-up transient have Small duration.

(vi) Quantization noise can be made negligible

Advantages:

→ Stable

→ Can be realized in both recursive & non-recursive.

→ exact linear phase.

→ Flexible.

→ low sensitive to quantization noise.

→ Efficiently realized in H/w.

Disadvantages:

→ complex

→ requires more filter co-efficients to be stored.

→ long duration impulse response require large amount of processing.

→ narrow transition band FIR filter requires more arithmetic operations & H/w components
↳ Costly.

Frequency Response of Linear Phase FIR Filter

$$h(n) \xleftrightarrow{DTFT} H(\omega)$$

Frq response of FIR filter.

$$H(\omega) = \underbrace{H_r(\omega)}_{\text{real part of } H(\omega)} e^{j\theta(\omega)}$$

(i) Symmetric Impulse response with M is even:

$$H_r(\omega) = \sum_{n=0}^{\frac{M}{2}-1} 2\,h(n) \cos\left[\omega\left(n - \frac{M-1}{2}\right)\right]$$

$$\theta(\omega) = \begin{cases} -\omega\left(\frac{M-1}{2}\right) & ; \; H_r(\omega) > 0 \\ -\omega\left(\frac{M-1}{2}\right) + \pi & ; \; H_r(\omega) < 0 \end{cases}$$

(ii) Symmetric Impulse response with M is odd:

$$H_r(\omega) = h\left(\frac{M-1}{2}\right) + \sum_{n=0}^{\frac{M-3}{2}} 2h(n) \cos\left(\omega\left[n - \frac{M-1}{2}\right]\right)$$

$$\theta(\omega) = \begin{cases} -\omega\left(\frac{M-1}{2}\right) & ; \; H_r(\omega) > 0 \\ -\omega\left(\frac{M-1}{2}\right) + \pi & ; \; H_r(\omega) < 0 \end{cases}$$

(iii) Anti Symmetric Impulse response with M is even:

$$H_r(\omega) = 2 \sum_{n=0}^{\frac{M}{2}-1} h(n) \sin\left(\omega\left[\frac{M-1}{2} - n\right]\right)$$

$$\theta(\omega) = \begin{cases} \frac{\pi}{2} - \omega\left(\frac{M-1}{2}\right) & ; \; H_r(\omega) > 0 \\ \frac{3\pi}{2} - \omega\left(\frac{M-1}{2}\right) & ; \; H_r(\omega) < 0 \end{cases}$$

(iv) Anti Symmetric Impulse response with M is odd:

$$H_r(\omega) = 2 \sum_{n=0}^{M-3/2} h(n) \sin\left(\omega\left[\frac{M-1}{2} - n\right]\right)$$

$$\theta(\omega) = \begin{cases} \frac{\pi}{2} - \omega\left(\frac{M-1}{2}\right) & ; \; H_r(\omega) > 0 \\ 3\frac{\pi}{2} - \omega\left(\frac{M-1}{2}\right) & ; \; H_r(\omega) < 0 \end{cases}$$

Impulse Response of Linear phase FIR Filter:

Symmetric:

$$h(n) = h(\underset{\substack{\downarrow \\ \text{order}}}{M}-1-n) \; ; \; 0 \le n \le M-1$$

Let $M = 8 \rightarrow$ even

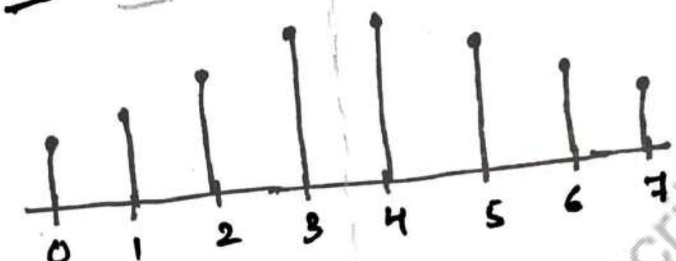
$$\therefore h(n) = h(8-1-n) \; ; \; 0 \le n \le 7$$

$n=0$ $h(0) = h(8-1-0) = h(7)$

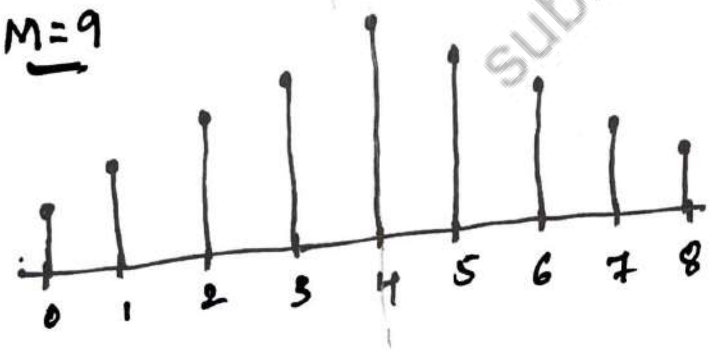
$n=1$ $h(1) = h(8-1-1) = h(6)$

$n=2$ $h(2) = h(8-1-2) = h(5)$

$n=3$ $h(3) = h(8-1-3) = h(4)$



$M=9$



Anti Symmetric:

$$h(n) = -h(M-1-n) \; ; \; 0 \le n \le M-1$$

Let $M = 8$

$$h(n) = -h(8-1-n) \; ; \; 0 \le n \le 7$$

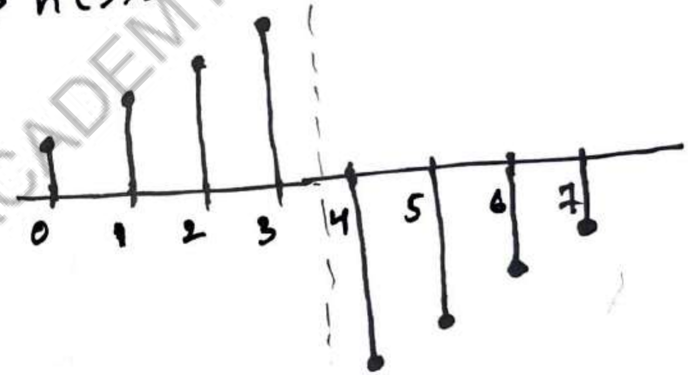
$$\therefore h(n) = -h(8-1-n)$$

$n=0$ $h(0) = -h(8-1-0) = -h(7)$

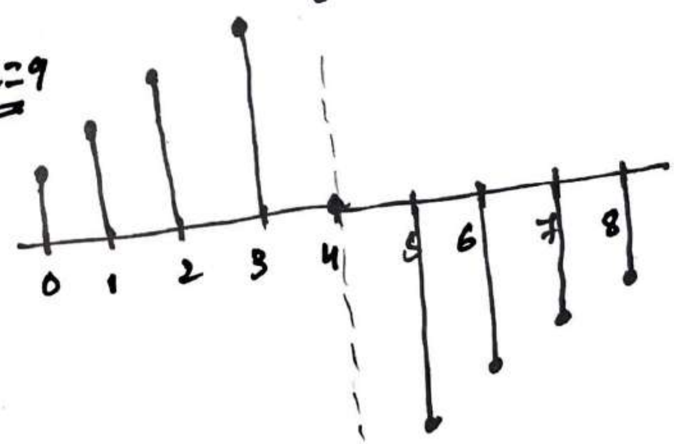
$n=1$ $h(1) = -h(8-1-1) = -h(6)$

$n=2$ $h(2) = -h(8-1-2) = -h(5)$

$n=3$ $h(3) = -h(8-1-3) = -h(4)$

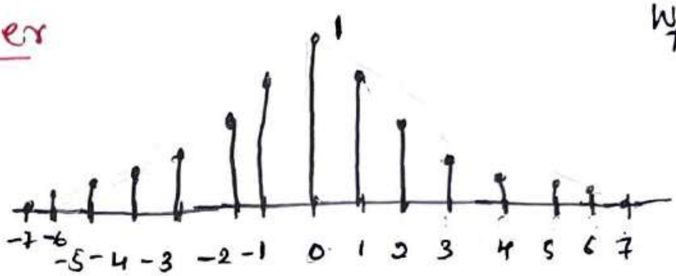


$M=9$

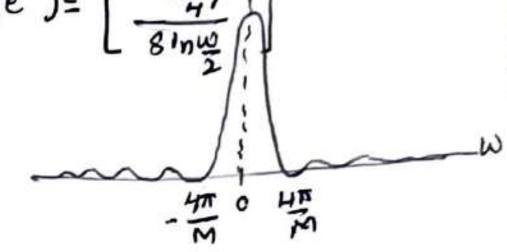


Design of Linear phase FIR filter using window method

(i) Different types of windows



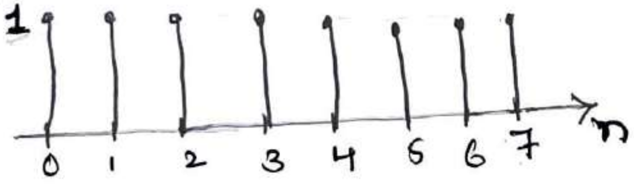
$$W_T(e^{j\omega}) = \left[\frac{\sin\left(\frac{M-1}{4}\right)\omega}{\sin\frac{\omega}{2}}\right]^2$$



(a) Rectangular window

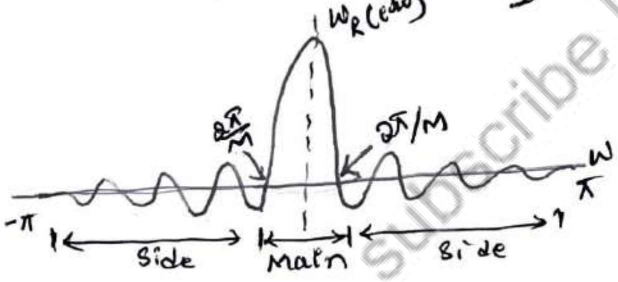
$$W_R(n) = 1 \; ; \; 0 \le n \le M-1$$
$$= 0 \; ; \; \text{otherwise.}$$

$M=8$ $W_R(n)=1 \; ; \; 0 \le n \le 7$



Spectrum: $W_R(e^{j\omega}) = \dfrac{\sin\frac{\omega M}{2}}{\sin\frac{\omega}{2}}$

$W_R(\omega)$

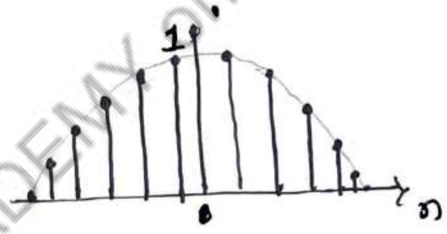


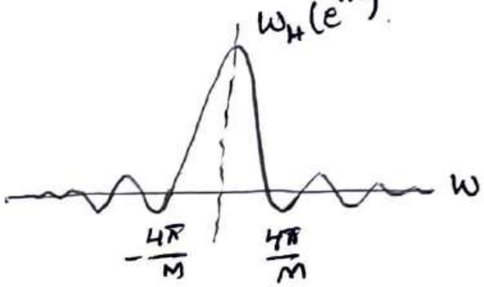
(b) Bartlett window [Triangular]

$$W_T(n) = 1 - \frac{2\left|n - \left(\frac{M-1}{2}\right)\right|}{M-1} \; ; \; 0 \le n \le M-1$$
$$= 0 \; ; \; \text{otherwise}$$

(c) Hanning window

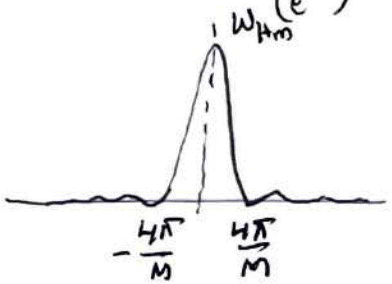
$$W_H(n) = 0.5 - 0.5\cos\frac{2\pi n}{M-1} \; ; \; 0 \le n \le M-1$$
$$= 0 \; ; \; \text{otherwise}$$





(d) Hamming window

$$W_{Hm}(n) = 0.54 - 0.46\cos\left(\frac{2\pi n}{M-1}\right) \; ; \; 0 \le n \le M-1$$
$$= 0 \; ; \; \text{otherwise.}$$



Procedure to design Linear Phase FIR Filters using windows.

$H_d(\omega) \rightarrow$ desired freq response

$H_d(\omega) \rightarrow$ F.T. of $h_d(n)$

$$\therefore H_d(\omega) = \sum_{n=0}^{\infty} h_d(n)\, e^{-j\omega n} \rightarrow (1)$$

$h_d(n) \rightarrow$ Inverse F.T. of $H_d(\omega)$

$$\therefore \underbrace{h_d(n)}_{\text{infinite duration}} = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(\omega)\, e^{j\omega n}\, d\omega \rightarrow (2)$$

$\underbrace{h_d(n)}_{\text{lenth } M} \rightarrow$ finate $\rightarrow$ multiply $h_d(n)$ by window sequance. 'M'

Example: Rectangular window.

$$W_R(n) = \begin{cases} 1 \,; & n = 0,1,2,\ldots M-1 \\ 0 \,; & \text{otherwise.} \end{cases} \rightarrow (3)$$

$h_d(n) \rightarrow$ Sample response $\rightarrow$ Infinate

$$\therefore h(n) = h_d(n)\, W_R(n) \rightarrow (4)$$

$h_d(n) \rightarrow$ desired Sample response. $\rightarrow$ Finate.

$$\therefore h(n) = \underbrace{\begin{cases} h_d(n) \,; & n = 0,1,2\ldots M-1 \\ 0 \,; & \text{otherwise} \end{cases}}_{\text{Windowing}} \rightarrow (5)$$

(4) $\Rightarrow h(n) = h_d(n) \cdot W_R(n)$

generally.

$h(n) = h_d(n) \cdot \underbrace{W(n)}_{\text{generalized window.}} \rightarrow$ unit sample response FIR.

Freq response,

$$H(\omega) = F.T.\{h_d(n) \cdot W(n)\}$$

$$H(\omega) = H_d(\omega) * W(\omega) \rightarrow (6)$$

Design the Symmetric FIR lowpass Filter whose

$$H_d(w) = \begin{cases} e^{-jw\tau} & ; \ |w| \le w_c \\ 0 & ; \ \text{otherwise} \end{cases}$$ with $M=7$ & $w_c = 1$ rad/sam.

use Rectangular window.

(i) obtain $h_d(n)$:

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(w)\, e^{jwn}\, dw \rightarrow ①$$

$$H_d(w) = \begin{cases} e^{-jw\tau} & ; \ -1 \le w \le 1 \\ 0 & ; \ \text{Otherwise.} \end{cases} \rightarrow ②$$

$$\therefore h_d(n) = \frac{1}{2\pi}\int_{-1}^{1} e^{-jw\tau}\cdot e^{jwn}\, dw = \frac{1}{2\pi}\int_{-1}^{1} e^{jw(n-\tau)}\, dw$$

$$= \frac{1}{2\pi}\left[\frac{e^{jw(n-\tau)}}{j(n-\tau)}\right]_{-1}^{1} = \frac{1}{2\pi}\left[\frac{e^{j(n-\tau)} - e^{-j(n-\tau)}}{j(n-\tau)}\right]$$

$$= \frac{1}{\pi(n-\tau)}\left[\frac{e^{j(n-\tau)} - e^{-j(n-\tau)}}{2j}\right] \qquad \because \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$h_d(n) = \frac{\sin(n-\tau)}{\pi(n-\tau)} \qquad n \ne \tau \rightarrow ③$$

if $n = \tau$

$$h_d(n) = \frac{1}{2\pi}\int_{-1}^{1} 1\cdot dw = \frac{1}{2\pi}[2] = \frac{1}{\pi} \rightarrow ④$$

$$h_d(n) = \begin{cases} \dfrac{\sin(n-\tau)}{\pi(n-\tau)} & ; \ n \ne \tau \\ \dfrac{1}{\pi} & ; \ n = \tau \end{cases} \rightarrow ⑤$$

determine the Value of τ

$$h(n) = h(M-1-n)$$

$$\because h(n) = h_d(n)\cdot w(n)$$

$$h_d(n)\, w(n) = h_d(M-1-n)\, w(n)$$

$$h_d(n) = h_d(M-1-n)$$

$$\frac{\sin(n-\tau)}{\pi(n-\tau)} = \frac{\sin(M-1-n-\tau)}{\pi(M-1-n-\tau)}$$

Determine the Filter coefficients $h_d(n)$ for desired frq. response of a low pass Filter given by,

$$H_d(e^{j\omega}) = \begin{cases} e^{-j2\omega} ; & -\frac{\pi}{4} \le \omega \le \frac{\pi}{4} \\ 0 ; & \frac{\pi}{4} \le |\omega| \le \pi \end{cases}$$

If we define new filter co-efficients by $h_d(n) \cdot w(n)$ where $w(n) = \begin{cases} 1 ; & 0 \le n \le 4 \\ 0 ; & \text{elsewhere} \end{cases}$

Determine $h(n)$ and also the frq. response $H(e^{j\omega})$ and compare with $H_d(e^{j\omega})$. Determine $H(e^{j\omega})$ using the Hamming window.

(i) obtain $h_d(n)$:

$$h_d(n) = \frac{1}{2\pi} \int_{-2\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} d\omega = \frac{1}{2\pi} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} e^{-j2\omega} \cdot e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi} \int_{-\pi/4}^{\pi/4} e^{j\omega(n-2)} d\omega = \frac{1}{2\pi} \left[\frac{e^{j\omega(n-2)}}{j(n-2)} \right]_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$$

$$= \frac{1}{2\pi} \left[\frac{e^{j\frac{\pi}{4}(n-2)} - e^{-j\frac{\pi}{4}(n-2)}}{j(n-2)} \right] = \frac{1}{\pi(n-2)} \left[\frac{e^{j\frac{\pi}{4}(n-2)} - e^{-j\frac{\pi}{4}(n-2)}}{2j} \right]$$

$$h_d(n) = \frac{\sin\frac{\pi}{4}(n-2)}{\pi(n-2)} ; \quad n \ne 2$$

if $n = 2$

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi/4}^{\pi/4} 1\, d\omega = \frac{1}{2\pi}\left[\frac{2\pi}{4}\right] = \frac{1}{4}$$

$$\therefore h_d(n) = \begin{cases} \dfrac{\sin\frac{\pi}{4}(n-2)}{\pi(n-2)} ; & n \ne 2 \\ \dfrac{1}{4} ; & n = 2 \end{cases}$$

(ii) obtain $h(n)$:

$$h(n) = h_d(n) \cdot w(n) ; \quad 0 \le n \le 4$$

$n=0$; $h(0) = h_d(0) = 0.159091$

$n=1$; $h(1) = h_d(1) = 0.224989$

$n=2$; $h(2) = h_d(2) = \frac{1}{4}$

$n=3$; $h(3) = h_d(3) = 0.224989$

$n=4$; $h(4) = h_d(4) = 0.159091$

(iii) Obtain $H(e^{jw})$: 59 (2)

$\because M=5$; $0 \le n \le 4$
$\hookrightarrow$ odd.

$$\therefore H(w) = e^{-jw\left(\frac{M-1}{2}\right)}\left\{h\left(\frac{M-1}{2}\right)+2\sum_{n=0}^{\frac{M-3}{2}} h(n)\cos w\left(n-\frac{M-1}{2}\right)\right\}$$

$$H(w) = e^{-j2w}\left[h(2)+2\sum_{n=0}^{1} h(n)\cos w(n-2)\right]$$

$$= e^{-j2w}\left[h(2)+2h(0)\cos w(-2)+2h(1)\cos w(-1)\right]$$

$$= e^{-j2w}\left[0.25+2\times 0.159091\cos 2w+2\times 0.224989\cos w\right]$$

$$= e^{-j2w}\left[0.25+0.318\cos 2w+0.45\cos w\right]$$

$$\boxed{|H(w)| = 0.25+0.318\cos 2w+0.45\cos w}$$

$$\angle H(w) = \begin{cases} -2w\ ; & |H(w)|>0 \\ -2w+\pi\ ; & |H(w)|<0 \end{cases}$$

$H(e^{jw})$ with $H_d(e^{jw}) \rightarrow$ different & will not same.

(iv) obtain $H(e^{j\omega})$ using Hamming window. $\underline{M=5}$ 9 ③

Hamming window $w(n) = 0.54 - 0.46\cos\left(\frac{2\pi n}{M-1}\right]$; $0 \le n \le M-1$

$$w(n) = 0.54 - 0.46\cos\left(\frac{\pi n}{2}\right) ; \quad 0 \le n \le 4$$

$h(n) = h_d(n) \cdot w(n)$

$h_d(0) = 0.15901$	$n=0$; $w(0) = 0.08$	$h(0) = 0.01273$
$h_d(1) = 0.224984$	$n=1$; $w(1) = 0.54$	$h(1) = 0.12149$
$h_d(2) = 0.25$	$n=2$; $w(2) = 1$	$h(2) = 0.25$
$h_d(3) = 0.224984$	$n=3$; $w(3) = 0.54$	$h(3) = 0.12149$
$h_d(4) = 0.159091$	$n=4$; $w(4) = 0.08$	$h(4) = 0.01273$

$\underline{M=5}$ → odd

$$H(\omega) = e^{-j2\omega}\left[h(2) + 2\sum_{n=0}^{1} h(n)\cos\omega(n-2)\right].$$

$$H(e^{j\omega}) = H(\omega) = e^{-j2\omega}\left[0.25 + 2\times 0.01273\cos 2\omega + 2\times 0.12149\cos\omega\right]$$

$$H(e^{j\omega}) = e^{-j2\omega}\left[0.25 + 0.02546\cos 2\omega + 0.243\cos\omega\right]$$

Design a FIR linear phase filter using Kaiser window to meet the following specifications

$$0.99 \le |H(e^{j\omega})| \le 1.01 \; ; \; 0 \le |\omega| \le 0.19\pi$$

$$|H(e^{j\omega})| \le 0.01 \; ; \; 0.21\pi \le |\omega| \le \pi$$

$$1-0.01 \le |H(e^{j\omega})| \le 1+0.01 \; ; \; 0 \le |\omega| \le \underbrace{0.19\pi}_{\omega_p}$$

$$|H(e^{j\omega})| \le 0.01 \; ; \; \underbrace{0.21\pi}_{\omega_s} \le |\omega| \le \pi$$

$\delta_1 = 0.01$ & $\delta_2 = 0.01$ $\quad \omega_p = 0.19\pi$ $\quad \omega_s = 0.21\pi$

ω_p ↓ Passband edge freq

ω_s ↓ Stopband edge freq

$$\therefore \Delta\omega = \omega_s - \omega_p = 0.21\pi - 0.19\pi = 0.02\pi$$

$$\delta = \text{min of } \delta_1 \& \delta_2 \Rightarrow \delta = 0.01$$

Attenuation $A = -20\log_{10}\delta = -20\log_{10} 0.01 = 40$

(i) Cut off freq ω_c:

$$\omega_c = \frac{\omega_p + \omega_s}{2} = \frac{0.19\pi + 0.21\pi}{2}$$

$$\omega_c = 0.2\pi$$

(ii) To obtain β & M:

$$\beta = \begin{cases} 0.1102(A-8.7) \; ; & A > 50 \\ 0.5842(A-21)^{0.4} + 0.07886(A-21) \; ; & 21 \le A \le 50 \\ 0 \; ; & A < 21 \end{cases}$$

$$\therefore \beta = 0.5842(40-21)^{0.4} + 0.07886(40-21) = 3.395$$

$$M = \frac{A-8}{2.285\Delta\omega} = \frac{40-8}{2.285(0.02\pi)} = 223$$

(iii) eqn for Kaiser window:

$$\alpha = \frac{M}{2} = \frac{223}{2} = 111.5$$

$$w_K(n) = \begin{cases} \dfrac{I_0\left\{\beta\left[1-\left[\dfrac{n-\alpha}{\alpha}\right]^2\right]^{1/2}\right\}}{I_0(\beta)} & ;\ 0 \le n \le M \\ 0 & ;\ \text{otherwise} \end{cases}$$

$$w(n) = \begin{cases} \dfrac{I_0\left\{3.395\left[1-\left[\dfrac{n-111.5}{111.5}\right]^2\right]^{1/2}\right\}}{I_0(3.395)} & ;\ 0 \le n \le 223 \\ 0 & ;\ \text{otherwise} \end{cases}$$

(iv) Obtain $h_d(n)$:

Ideal freq response

$$H_d(\omega) = \begin{cases} e^{-j\omega\left(\frac{M-1}{2}\right)} & ;\ -\omega_c \le \omega \le \omega_c \\ 0 & ;\ \text{otherwise} \end{cases}$$

I.F.T.

$$h_d(n) = \begin{cases} \dfrac{\sin\left[\omega_c\left[n-\frac{M-1}{2}\right]\right]}{\pi\left(n-\frac{M-1}{2}\right)} & ;\ n \ne \frac{M-1}{2} \\ \dfrac{\omega_c}{\pi} & ;\ n = \frac{M-1}{2} \end{cases}$$

$M = 223 \quad \therefore \underbrace{\frac{M}{2} = 111.5}$ →

$h_d(n)$

$$\therefore h_d(n) = \frac{\sin\left[0.2\pi\left[n-\frac{223}{2}\right]\right]}{\pi\left(n-\frac{223}{2}\right)}$$

$$h_d(n) = \frac{\sin\left[0.2\pi(n-111.5)\right]}{\pi(n-111.5)} \quad ;\ 0 \le n \le 223$$

(v) Obtain $h(n)$:

$$h(n) = h_d(n) \cdot w(n)$$

$$\therefore h(n) = \begin{cases} \dfrac{\sin(0.2\pi(n-111.5))}{\pi(n-111.5)} \cdot \dfrac{I_0\left\{3.395\left[1-\left[\dfrac{n-111.5}{111.5}\right]^2\right]^{1/2}\right\}}{I_0(3.395)} & ;\ 0 \le n \le 223 \\ 0 & ;\ \text{otherwise} \end{cases}$$

Design the Symmetric FIR lowpass filter whose

$$H_d(\omega) = \begin{cases} e^{-j\omega\alpha} & ; |\omega| \le \omega_c \\ 0 & ; \text{Otherwise} \end{cases}$$ with $M=7$ & $\omega_c = 1$ rad/sam.

Use Hanning window.

$n=0$; $h_d(0) = 0.01497$
$n=1$; $h_d(1) = 0.14472$
$n=2$; $h_d(2) = 0.26785$
$n=3$; $h_d(3) = \frac{1}{\pi}$
$n=4$; $h_d(4) = 0.26785$
$n=5$; $h_d(5) = 0.14472$
$n=6$; $h_d(6) = 0.1497$

Hanning window $W(n) = 0.5\left[1 - \cos\left(\frac{2\pi n}{M-1}\right)\right]$

$\because M=7$ $\quad W(n) = 0.5\left[1 - \cos\left(\frac{2\pi n}{6}\right)\right]$

$\Rightarrow W(n) = 0.5\left[1 - \cos\left(\frac{\pi n}{3}\right)\right]$ $\quad n = 0$ to 6 $\quad M=7$

Rad $\quad 0.5 \times (1 - \cos(\pi A \div 3)$

$n=0$; $w(0) = 0$
$n=1$; $w(1) = 0.25$
$n=2$; $w(2) = 0.75$
$n=3$; $w(3) = 1$
$n=4$; $w(4) = 0.75$
$n=5$; $w(5) = 0.25$
$n=6$; $w(6) = 0$

$h(n) = h_d(n) \cdot w(n)$

$\therefore \rightarrow h(0) = 0$
$\rightarrow h(1) = 0.03618$
$\rightarrow h(2) = 0.20089$
$h(3) = \frac{1}{\pi}$
$\rightarrow h(4) = 0.20089$
$\rightarrow h(5) = 0.03618$
$\rightarrow h(6) = 0$

$$\frac{-\sin(n-\tau)}{-\pi(n-\tau)} = \frac{\sin(M-1-n-\tau)}{\pi(M-1-n-\tau)} \quad \because -\sin\theta = \sin(-\theta)$$

$$\frac{\sin[-(n-\tau)]}{\pi(-(n-\tau))} = \frac{\sin(M-1-n-\tau)}{\pi(M-1-n-\tau)}$$

$$-(n-\tau) = M-1-n-\tau$$

$$-\cancel{n}+\tau = M-1-\cancel{n}-\tau$$

$$2\tau = M-1$$

$$\boxed{\tau = \frac{M-1}{2}}$$

(5) $\Rightarrow$

$$h_d(n) = \begin{cases} \dfrac{\sin\left(n-\frac{M-1}{2}\right)}{\pi\left(n-\frac{M-1}{2}\right)} & ; n \neq \frac{M-1}{2} \\ \dfrac{1}{\pi} & ; n = \frac{M-1}{3} \end{cases}$$

$\because M = 7$

$$h_d(n) = \begin{cases} \dfrac{\sin(n-3)}{\pi(n-3)} & ; n \neq 3 \\ \dfrac{1}{\pi} & ; n = 3 \end{cases}$$

Put $n = 0$ to 6

$n=0$; $h_d(0) = 0.01497$

$n=1$; $h_d(1) = 0.14472$

$n=2$; $h_d(2) = 0.26785$

$n=3$; $h_d(3) = \frac{1}{\pi}$

$n=4$; $h_d(4) = 0.26785$

$n=5$; $h_d(5) = 0.14472$

$n=6$; $h_d(6) = 0.01497$

(ii) $\because$ $h(n) = h_d(n) \cdot w(n)$ $\quad w_n(n) = \begin{cases} 1 & ; 0 \le n \le 6 \\ 0 & ; \text{other} \end{cases}$

$$h(n) = \begin{cases} h_d(n) & ; 0 \le n \le 6 \\ 0 & ; \text{otherwise} \end{cases}$$

Co-efficients of FIR Filter

$\rightarrow h(0) = 0.01497$

$\rightarrow h(1) = 0.14472$

$\rightarrow h(2) = 0.26785$

$h(3) = \frac{1}{\pi}$

$\rightarrow h(4) = 0.26785$

$\rightarrow h(5) = 0.14472$

$\rightarrow h(6) = 0.01497$

Symmetric

$h(n) = h(6-n)$

Design of Linear Phase FIR filters using Frequency Sampling:

Desired frq response $\rightarrow H_d(\omega)$

This frq response is sampled at 'M' points $\rightarrow \boxed{\omega = \frac{2\pi}{M} k}$

$k = 0, 1, 2 \ldots M-1$

Discrete fourier tansform

$$H(k) = H_d(\omega) \; ; \; k = 0, 1, 2 \ldots M-1$$

$$H(k) = H_d\left[\frac{2\pi}{M} k\right] ; \; k = 0, 1, 2 \ldots M-1$$

$H(k) \rightarrow$ M-point DFT.

Take IDFT of $H(k)$ to get $h(n)$

$h(n) \rightarrow$ unit sample response of FIR filter.

If $M \Rightarrow$ odd:

$$h(n) = \frac{1}{M}\left[H(0) + 2\sum_{k=1}^{\frac{M-1}{2}} Re\left\{H(k)\, e^{j\frac{2\pi}{N}kn}\right\}\right]$$

If $M \Rightarrow$ even:

$$h(n) = \frac{1}{M}\left[H(0) + 2\sum_{k=1}^{\frac{M}{2}-1} Re\left\{H(k)\, e^{j\frac{2\pi}{N}kn}\right\}\right]$$

Window

$$H_d(\omega) \xrightarrow{IDFT} \underbrace{h_d(n)}_{\text{infinite}} \xrightarrow{\text{finite}} \overbrace{h_d(n) \cdot w(n)}^{h(n)}$$

Digital Filter Structure:

I. Direct Form Structure:

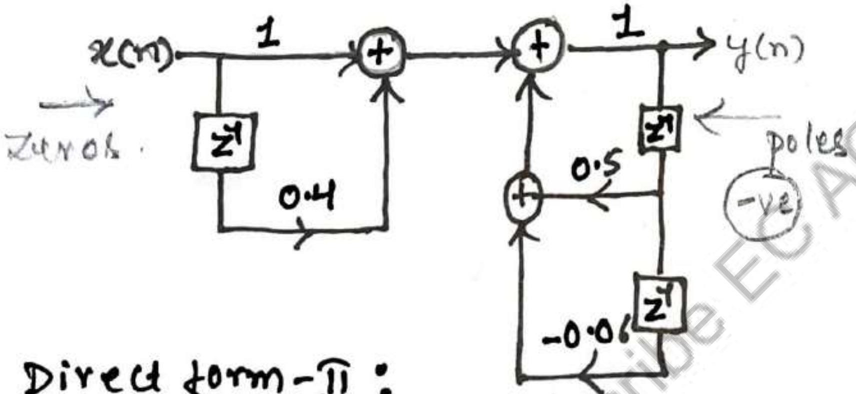
ⓐ Direct form - I

ⓑ Direct form - II

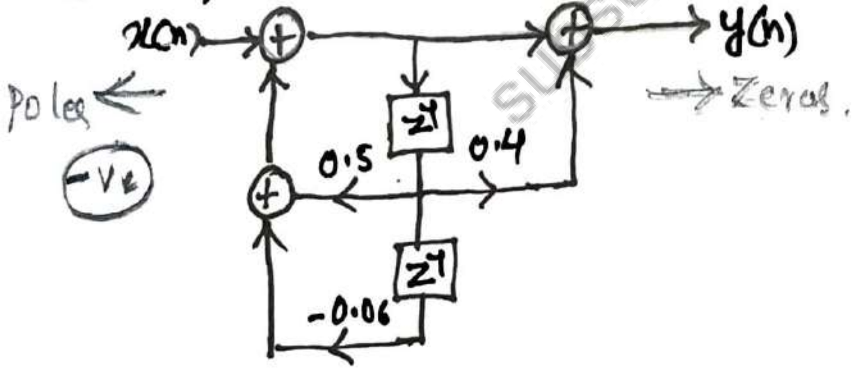
① $H(z) = \dfrac{1+0.4z^{-1}}{1-0.5z^{-1}+0.06z^{-2}}$ Zeros poles

z^{-n} ↳ delay.

Direct form - I:



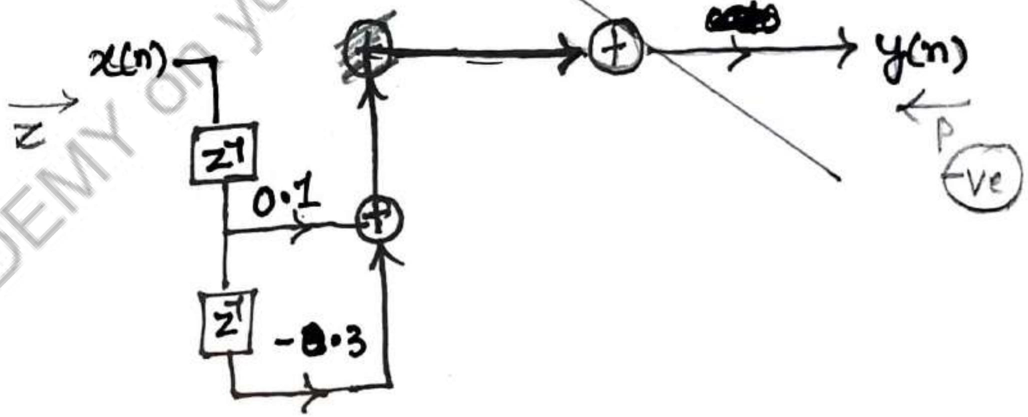
Direct form - II:



② $H(z) = \dfrac{z^{-1}-3z^{-2}}{(10-z^{-1})(1+0.5z^{-1}+0.5z^{-2})}$

$H(z) = \dfrac{z^{-1}-3z^{-2}}{10+4z^{-1}+4.5z^{-2}-0.5z^{-3}}$

Direct form I:

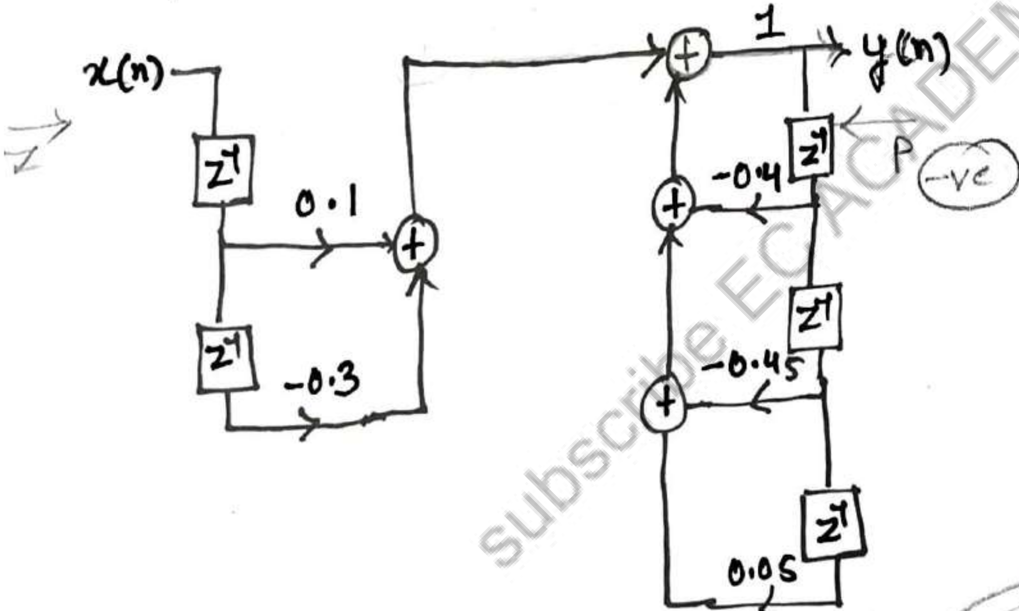


(2) $$H(z) = \frac{z^{-1} - 3z^{-2}}{(10 - z^{-1})(1 + 0.5z^{-1} + 0.5z^{-2})}$$

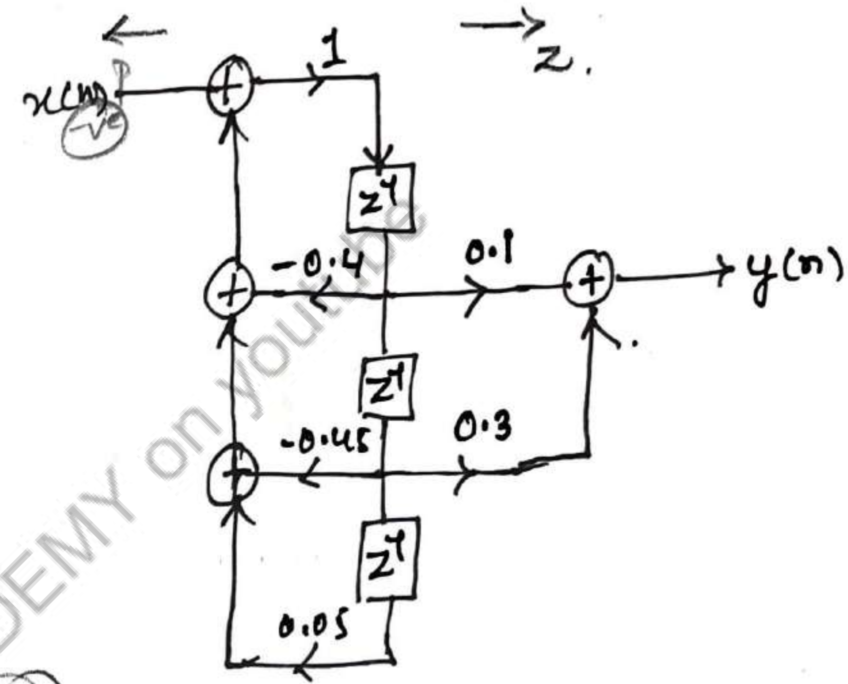
$$H(z) = \frac{z^{-1} - 3z^{-2}}{10 + 4z^{-1} + 4.5z^{-2} - 0.5z^{-3}}$$

$$H(z) = \frac{0.1z^{-1} + 0.3z^{-2}}{1 + 0.4z^{-1} + 0.45z^{-2} - 0.05z^{-3}}$$

Direct form I :



Direct form II :



(3) $y(n) - \frac{1}{4}y(n-1) + \frac{1}{8}y(n-2) = x(n) + \frac{1}{2}x(n-2)$

Take Z-transform

$$Y(z) - \frac{1}{4}z^{-1}Y(z) + \frac{1}{8}z^{-2}Y(z) = X(z) + \frac{1}{2}z^{-2}X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + 1/2\, z^{-2}}{1 - \frac{1}{4}z^{-1} + \frac{1}{8}z^{-2}}$$

Determine the impulse response $h(n)$ of a filter having desired frq response

$$H_d(e^{j\omega}) = \begin{cases} e^{-j(N-1)\omega/2} & ; 0 \le |\omega| \le \frac{\pi}{2} \\ 0 & ; \frac{\pi}{2} \le |\omega| \le \pi \end{cases}$$

$M = N = 7$, use frq sampling approach.

(i) Desired frq response:

$$N=7 \Rightarrow H_d(e^{j\omega}) = \begin{cases} e^{-j3\omega} & ; 0 \le |\omega| \le \frac{\pi}{2} \\ 0 & ; \frac{\pi}{2} \le |\omega| \le \pi \end{cases}$$

(ii) Sample $H_d(e^{j\omega})$:

Put $\omega = \frac{2\pi k}{N}$; $k = 0, 1, 2 \ldots N-1$

For $N=7 \Rightarrow \omega = \frac{2\pi k}{7}$; $k = 0, 1, 2 \ldots 6$

$$e^{-j3\cdot\frac{2\pi k}{7}} = e^{-j\frac{6\pi k}{7}}$$

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\frac{6\pi k}{7}} & ; 0 \le \frac{2\pi k}{7} \le \frac{\pi}{2} \\ 0 & ; \frac{\pi}{2} \le \frac{2\pi k}{7} \le \pi \end{cases}$$

$\frac{2\pi k}{7} = \frac{\pi}{2} \Rightarrow \boxed{k = \frac{7}{4}}$ $\quad \frac{2\pi k}{7} = \frac{\pi}{2} \Rightarrow \boxed{k = \frac{7}{4}}$

$\frac{2\pi k}{7} = \pi \Rightarrow \boxed{k = \frac{7}{2}}$

$$H_d(e^{j\omega}) = H_d(k) = \begin{cases} e^{-j\frac{6\pi k}{7}} & ; 0 \le k \le \frac{7}{4} \\ 0 & ; \frac{7}{4} \le k \le \frac{7}{2} \end{cases}$$

(iii) To obtain $h(n)$:

$M = N = 7 \rightarrow$ odd

$$h(n) = \frac{1}{M}\left[H(0) + 2\sum_{k=1}^{\frac{M-1}{2}} \text{Re}\left\{H(k)e^{j\frac{2\pi}{N}kn}\right\}\right]$$

$\frac{M-1}{2} \Rightarrow \frac{7-1}{2} \Rightarrow 3$; $H(0) = 1$

$$h(n) = \frac{1}{7}\left[1 + 2\cdot\sum_{k=1}^{3} \text{Re}\left\{e^{-j\frac{6\pi k}{7}} \cdot e^{j2\pi kn/7}\right\}\right]$$

$$h(n) = \frac{1}{7}\left[1 + 2\sum_{k=1}^{3} \text{Re}\left\{e^{-j2\pi k(3-n)/7}\right\}\right]$$

$e^{-j\theta} = \underbrace{\cos\theta}_{Re} - j\underbrace{\sin\theta}_{im} \Rightarrow \text{Re}\left[e^{-j\theta}\right] = \cos\theta$

$$h(n) = \frac{1}{7}\left[1 + 2\sum_{k=1}^{3} \cos\left[\frac{2\pi k(3-n)}{7}\right]\right]$$

; $n = 0, 1, 2 \ldots 6$

Determine $h(n)$ of a filter having

$$H_d(e^{j\omega}) = \begin{cases} e^{-j(N-1)\omega/2} & ; 0 \le |\omega| \le \frac{\pi}{2} \\ 0 & ; \frac{\pi}{2} \le |\omega| \le \pi \end{cases}$$

$M=N=7$ use freq sampling approach.

(i) Desired freq response:

$N=7$ $\quad H_d(e^{j\omega}) = \begin{cases} e^{-j3\omega} & ; 0 \le |\omega| \le \frac{\pi}{2} \\ 0 & ; \frac{\pi}{2} \le |\omega| < \pi \end{cases}$

(ii) Sample $H_d(e^{j\omega})$: $\quad \omega = \frac{2\pi K}{N}$; $K = 0, 1, 2, \dots N-1$

$N=7 \Rightarrow \omega = \frac{2\pi}{7}K$; $K = 0, 1, 2 \dots 6$

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\frac{6\pi K}{7}} & ; 0 \le \frac{2\pi K}{7} \le \frac{\pi}{2} \\ 0 & ; \frac{\pi}{2} \le \frac{2\pi K}{7} < \pi \end{cases}$$

$\frac{2\pi K}{7} = \frac{\pi}{2} \Rightarrow \boxed{K = \frac{7}{4}}$

$\frac{2\pi K}{7} = \pi \Rightarrow \boxed{K = \frac{7}{2}}$

$$\therefore H_d(K) = \begin{cases} e^{-j\frac{6\pi K}{7}} & ; 0 \le K \le \frac{7}{4} \\ 0 & ; \frac{7}{4} \le K \le \frac{7}{2} \end{cases}$$

(iii) Find $h(n)$:

$M = N = 7 \rightarrow$ odd

$$h(n) = \frac{1}{M}\left[H(0) + 2\sum_{K=1}^{\frac{M-1}{2}} \mathrm{Re}\left\{H(K)\, e^{\frac{j2\pi K n}{N}}\right\}\right]$$

$$\therefore h(n) = \frac{1}{7}\left[1 + 2\sum_{K=1}^{3} \mathrm{Re}\left\{e^{-j\frac{6\pi K}{7}} \cdot e^{\frac{j2\pi K n}{7}}\right\}\right]$$

$$\therefore h(n) = \frac{1}{7}\left[1 + 2\sum_{K=1}^{3} \mathrm{Re}\left\{e^{-j2\pi K(3-n)/7}\right\}\right]$$

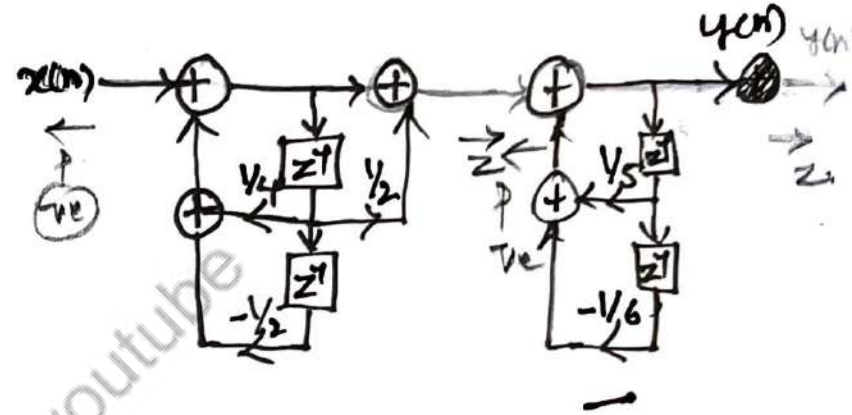
$e^{-j\theta} = \underbrace{\cos\theta}_{Re} - \underbrace{j\sin\theta}_{im.}$ $\quad \mathrm{Re}\left[e^{-j\theta}\right] = \cos\theta$

$$h(n) = \frac{1}{7}\left[1 + 2\sum_{K=1}^{3} \cos\left[\frac{2\pi K(3-n)}{7}\right]\right]$$

; $n = 0, 1, 2 \dots 6$

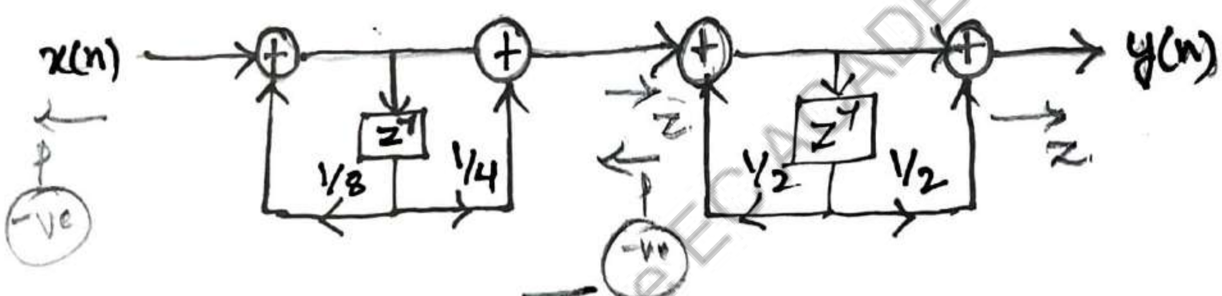
Digital Filter Structures:

II. Cascade form Structure: $H(z) = \underbrace{H_1(z)}_{DF-II} \cdot \underbrace{H_2(z)}_{DF-II}$



① $$H(z) = \frac{1 + \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}{1 - \frac{5}{8}z^{-1} + \frac{1}{16}z^{-2}}$$ $\rightarrow -\frac{1}{4}, -\frac{1}{2}$; $\rightarrow \frac{1}{8}, \frac{1}{2}$

$$H(z) = \frac{(1+\frac{1}{4}z^{-1})(1+\frac{1}{2}z^{-1})}{(1-\frac{1}{8}z^{-1})(1-\frac{1}{2}z^{-1})} = \underbrace{\frac{(1+\frac{1}{4}z^{-1})}{(1-\frac{1}{8}z^{-1})}}_{H_1(z)} \cdot \underbrace{\frac{(1+\frac{1}{2}z^{-1})}{(1-\frac{1}{2}z^{-1})}}_{H_2(z)}$$



② $$H(z) = \frac{(1+\frac{1}{2}z^{-1})}{(1-\frac{1}{4}z^{-1}+\frac{1}{2}z^{-2})(1-\frac{1}{5}z^{-1}+\frac{1}{6}z^{-2})}$$

$$H(z) = \underbrace{\frac{(1+\frac{1}{2}z^{-1})}{(1-\frac{1}{4}z^{-1}+\frac{1}{2}z^{-2})}}_{H_1(z)} \cdot \underbrace{\frac{1}{(1-\frac{1}{5}z^{-1}+\frac{1}{6}z^{-2})}}_{H_2(z)}$$

Digital Filter Structure:

III. Lattice Structure:

① $$H(z) = \frac{1}{1+\frac{2}{5}z^{-1}+\frac{3}{4}z^{-2}+\frac{1}{3}z^{-3}}$$

$a_3(0)$, $a_3(1)$, $a_3(2)$, $a_3(3)$

$\therefore m=3$ $\quad K_0 = a_3(0) = 1$

$a_3(1) = 2/5$

$a_3(2) = 3/4$

$K_3 = a_3(3) = 1/3$

$m=3$, $i=1$

$$a_2(1) = \frac{a_3(1) - K_3 a_3(2)}{1-K_3^2} = \frac{\frac{2}{5}-\frac{1}{3}\cdot\frac{3}{4}}{1-\frac{1}{9}}$$

$a_2(1) = 0.16875$

$i=2$

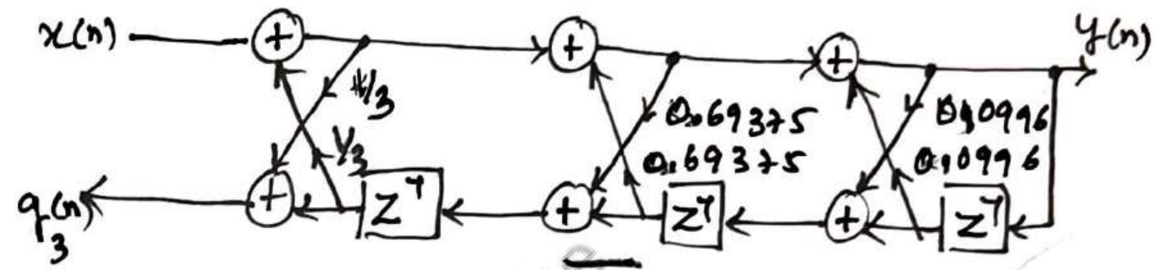
$$a_2(2) = \frac{a_3(2) - K_3 a_3(1)}{1-K_3^2} = \frac{\frac{3}{4}-\frac{1}{3}\cdot\frac{2}{5}}{1-\frac{1}{9}}$$

$a_2(2) = K_2 = 0.69375$

$m=2$, $i=1$

$$a_1(1) = \frac{a_2(1) - K_2\cdot a_2(1)}{1-K_2^2} = \frac{0.16875-(0.69375)(0.16875)}{1-(0.69375)^2}$$

$K_1 = a_1(1) = 0.0996$



② $$H(z) = \frac{1+2z^{-1}}{1+\frac{3}{4}z^{-1}+\frac{1}{4}z^{-2}}$$

$a_2(0)$, $a_2(1)$, $a_2(2)$

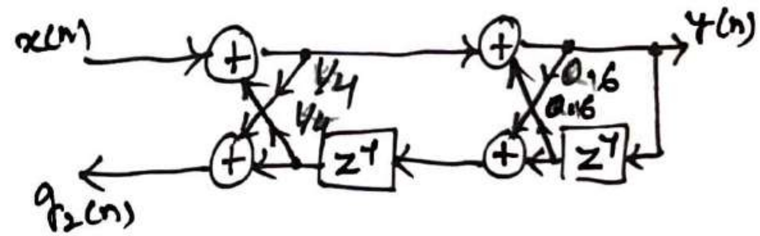
$a_2(0) = 1$ $\qquad a_2(2) = \frac{1}{4} = K_2$

$a_2(1) = \frac{3}{4}$

$m=2$, $i=1$

$$a_1(1) = \frac{a_2(1) - K_2 a_2(1)}{1-K_2^2} = \frac{3/4 - 1/4\cdot 3/4}{1-(1/4)^2}$$

$K_1 = a_1(1) = 0.6$



Infinate Impulse Response: [IIR] Filter

→ $y(n)$ depends → $x(n)$ & $x(n-1)$ also on $y(n-1)$

→ Differen[redacted] eqn

$$y(n) = b_0 x(n) + b_1 x(n-1) + \cdots + b_M x(n-M) - a_1 y(n-1) - \cdots - a_N y(n-N)$$

→ Transfer fun

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + \cdots + b_M z^{-M}}{1 + a_1 z^{-1} + \cdots + a_N z^{-N}}$$

b_i → $(M+1)$ numerator

a_i → N denominator.

$Y(z)$ & $X(z)$ → Z-Transform of $x(n)$ & $y(n)$

→ Ploes (s) → inside the unit circle → STABLE

→ smaller filter size.

→ linear phase is not easy

→ Objective → determine the filter numerator & denominator → satisfy filter specifications

Advantages

→ Easy to design & easy to implement

DisadVantages

→ Non Linear

→ non-stable.

→ Infinate Impulse response.

Bilinear Transformation Design

IIR

Method:

Digital Filter Specifications

↓ 1. Transformation with frq warping.

Analog filter Specifications

↓ 2. Transformation by lowpass prototype filter.

Analog filter Transfer function

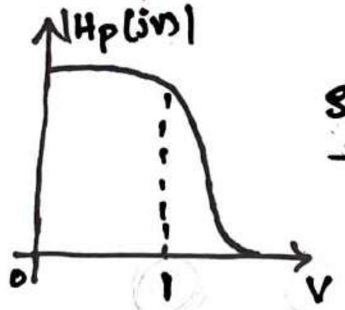
↓ 3. Bilinear Transformation.

Digital filter transfer function & Frequency response Verification

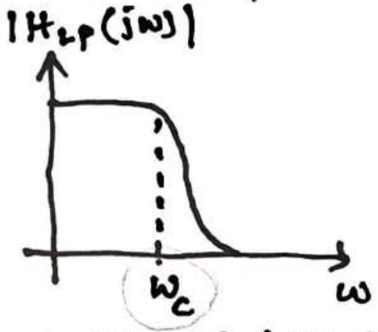
General procedure for IIR Filter design using bilinear transformation.

Analog Filters using Lowpass Prototype Transformation:

① Lowpass prototype into a Lowpass Filter:



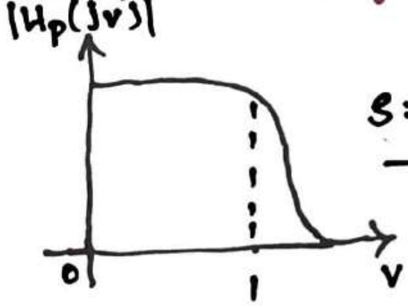
$$s = \frac{s}{\omega_c} \longrightarrow$$



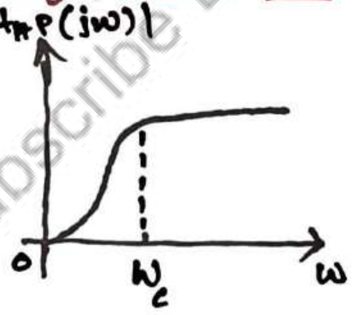
$\omega_c \rightarrow$ Cutoff frq

$$\boxed{H_{LP}(s) = H_p(s)\Big|_{s=\frac{s}{\omega_c}}}$$

② Lowpass Prototype to High pass Filter:

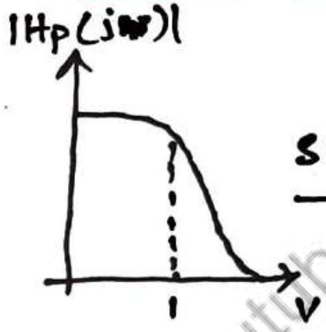


$$s = \frac{\omega_c}{s} \longrightarrow$$

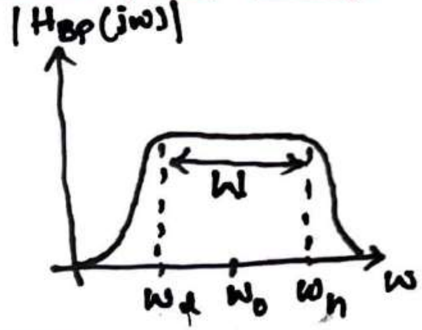


$$\boxed{H_{HP}(s) = H_p(s)\Big|_{s=\frac{\omega_c}{s}}}$$

③ Lowpass Prototype to bandpass Filter:



$$s = \frac{(s^2+\omega_0^2)}{sW} \longrightarrow$$



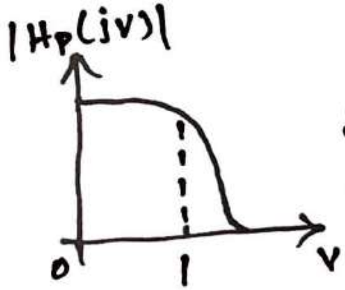
$$\boxed{H_{BP}(s) = H_p(s)\Big|_{s=\frac{(s^2+\omega_0^2)}{sW}}}$$

$\omega_0 = \sqrt{\omega_l \cdot \omega_h}$

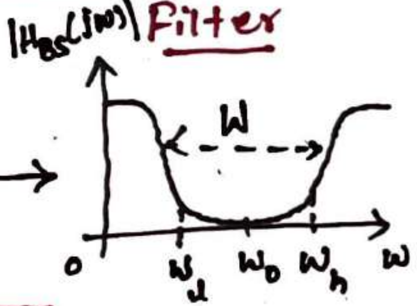
↳ geometric center frq

$W = \omega_h - \omega_l \rightarrow$ Passband bandwidth.

④ Lowpass Prototype to bandstop [band reject] Filter



$$s = \frac{sW}{(s^2+\omega_0^2)} \longrightarrow$$



$$\boxed{H_{BS}(s) = H_p(s)\Big|_{s=\frac{sW}{(s^2+\omega_0^2)}}}$$

Given, a lowpass prototype $H_p(s) = \frac{1}{s+1}$.
Determine each of the following analog filters & plot their magnitude response from 0 to 200 rad/sec.

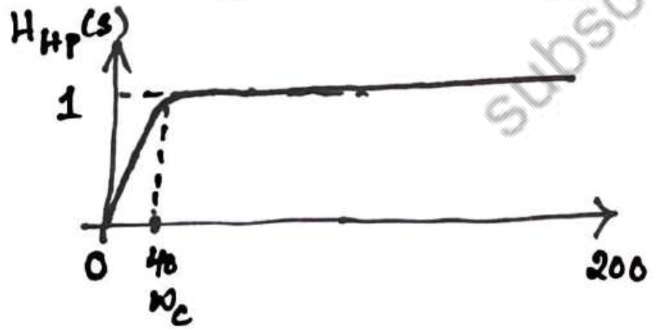
(i) A highpass filter with a cutoff frq of 40 rad/sec.

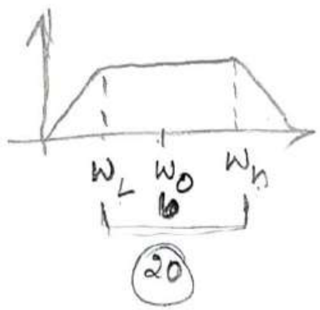
(ii) A bandpass filter with a cutoff frq of 100 rad/sec. & Bandwidth of 20 rad/sec.

(i) High-pass Filters

$$H_p(s) = \frac{1}{s+1}\Bigg|_{s=\frac{w_c}{s}} \Rightarrow s = \frac{40}{s}$$

$$H_{HP}(s) = \frac{1}{\frac{40}{s}+1} = \frac{s}{40+s}$$





(ii) band pass Filter

$$W_0 = \sqrt{W_L W_h} = 100$$

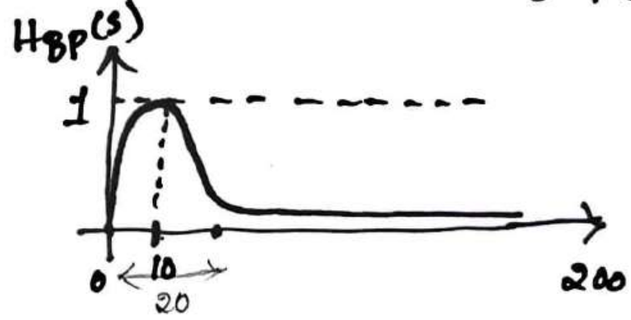
$$W_0^2 = 100 \Rightarrow W_0 = 10 \text{ rad/sec}$$

$$W = W_h - W_L = 20 \text{ rad/sec}.$$

$$H_p(s) = \frac{1}{s+1}\Bigg|_{s = \frac{s^2+w_0^2}{sW}}$$

$$s = \frac{s^2+100}{s \cdot 20}$$

$$H_{BP}(s) = \frac{1}{\frac{s^2+100}{20s}+1} = \frac{20s}{s^2+20s+100}$$



Bilinear Transformation & Frequency Warping

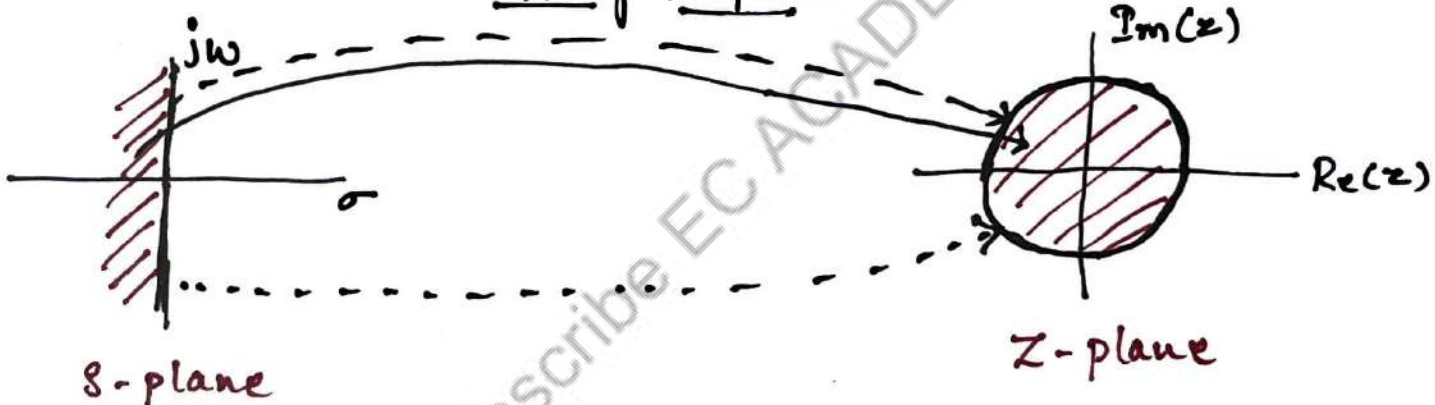
Analog Filter transfer fun → Digital filter transfer fun

$$H(s) \longrightarrow H(z)$$

$$\boxed{H(z) = H(s)\Big|_{s = \frac{2}{T}\cdot\frac{(z-1)}{(z+1)}}}$$

$T \rightarrow$ Sampling period.

Mapping Properties



Given an analog filter whose transfer fun is $H(s) = \frac{10}{s+10}$. Convert it to the digital filter transfer fun & difference eqn. when the sampling period is given as $T = 0.01$ sec.

Bilinear Transformation,

$$H(z) = H(s)\Big|_{s = \frac{2}{T}\cdot\frac{(z-1)}{(z+1)}} = \frac{10}{s+10}\Big|_{s = \frac{2}{0.01}\cdot\frac{(z-1)}{(z+1)}}$$

$$H(z) = \frac{10}{\frac{2}{0.01}\frac{(z-1)}{(z+1)}+10} = \frac{10}{\frac{200(z-1)}{z+1}+10} = \frac{0.05}{\frac{(z-1)}{(z+1)}+0.05}$$

$$= \frac{0.05(z+1)}{(z-1)+0.05(z+1)} = \frac{0.05z+0.05}{1.05z-0.95} = \frac{(0.05z+0.05)/1.05z}{(1.05z-0.95)/1.05z}$$

$$\frac{Y(z)}{X(z)} = H(z) = \frac{0.0476+0.0476z^{-1}}{1-0.9048z^{-1}}$$

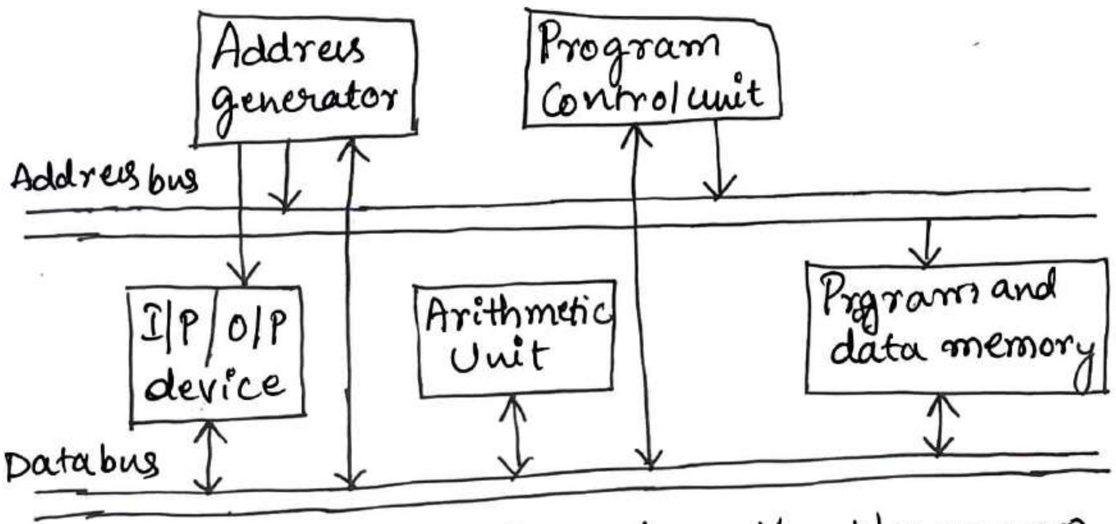
$$Y(z) - 0.9048z^{-1}Y(z) = 0.0476X(z) + 0.0476z^{-1}X(z)$$

$$\therefore Y(z) = 0.0476X(z) + 0.0476z^{-1}X(z) + 0.9048z^{-1}Y(z)$$

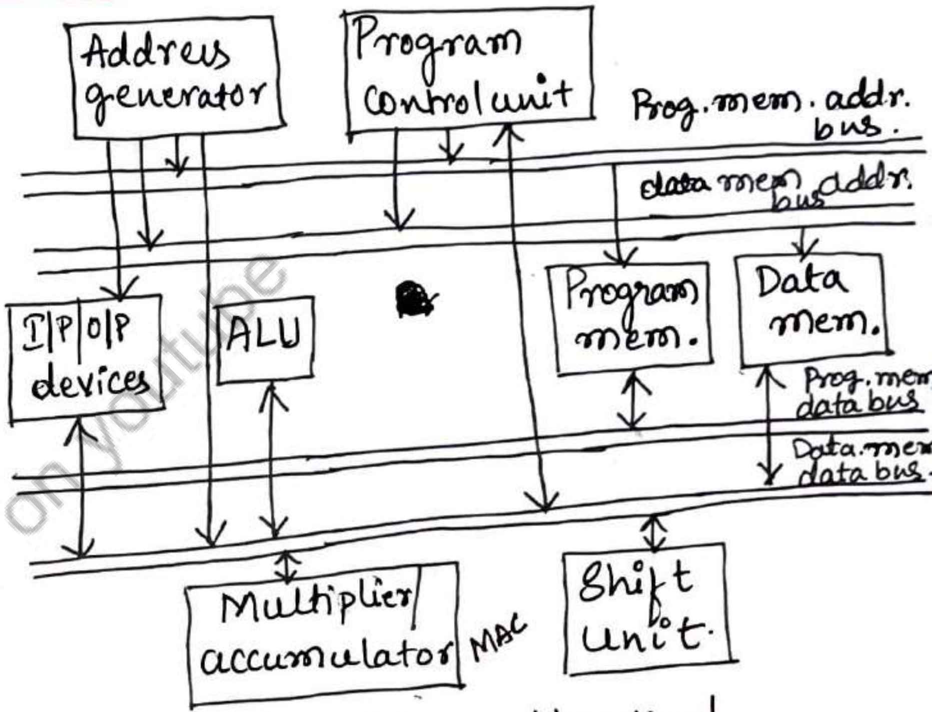
IZT

$$y(n) = 0.0476x(n) + 0.0476x(n-1) + 0.9048y(n-1)$$

Digital Signal Processor Architecture:

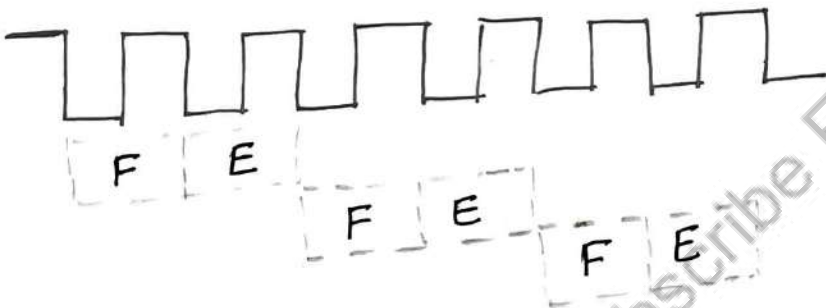


ⓐ Microprocessor based on Von Neumann Architecture.

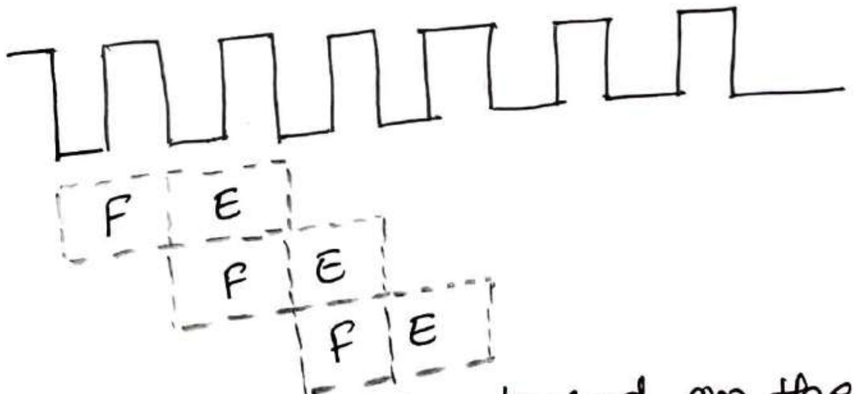


ⓑ DSP based on Harvard Architecture.

Opcode ✓
Operand ✓



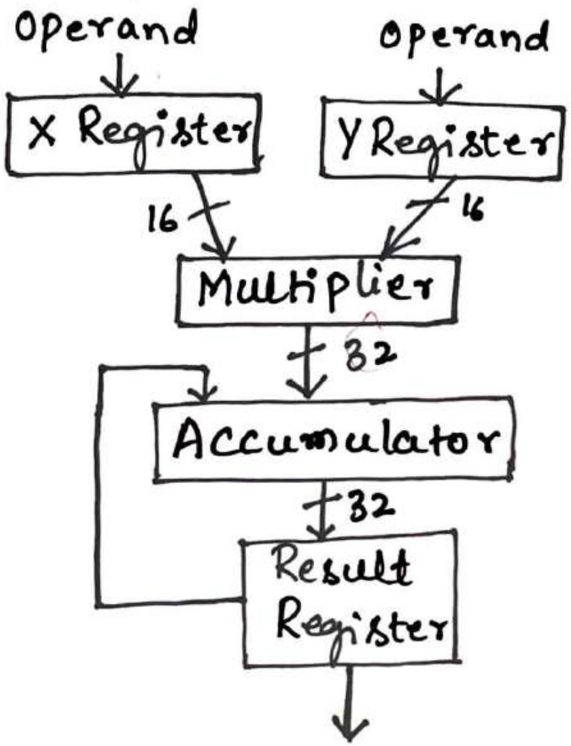
ⓒ Execution cycle based on the VonNeumann Architecture



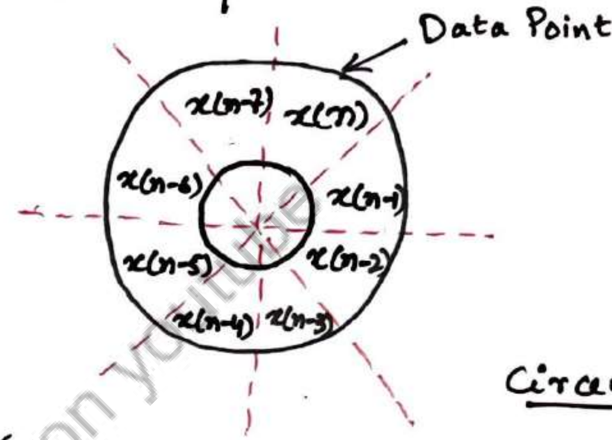
ⓓ Execution cycle based on the Harvard Architecture

Digital Signal processor Hardware Units:

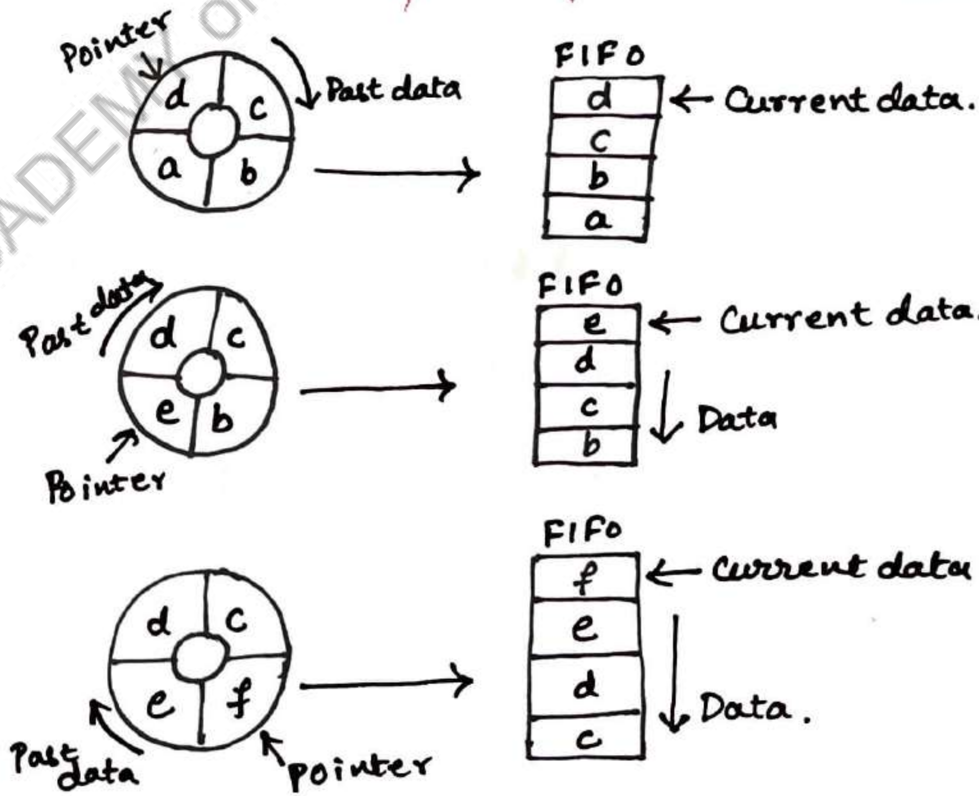
1. Multiplier & Accumulator: MAC operation



3. Address Generator:



Circular buffering



Circular buffer & eqe. FIFO

2. Shifters:

Ex:- $(011)_2 = (3)_{10}$

Shift right $\frac{3}{2} = 1.5$

$(001)_2 = (1)_{10}$

shift left $3 \times 2 = 6$

$(110)_2 = (6)_{10}$

Fixed-Point Formats:

Decimal no		2's Complement
3	→	0 1 1
2	→	0 1 0
1	→	0 0 1
0	→	0 0 0
−1	→	1 1 1
−2	→	1 1 0
−3	→	1 0 1
−4	→	1 0 0

3 bit 2's complement

STEPS:

1. magnitude to its binary number.
2. If no → +ve → binary no is it's 2's Comp. representation.

   If no → −ve → Perform 2's Comp. operation
   - a. negate ⇒ 0→1 & 1→0
   - b. Add logic 1 to data

Ex:- 3 → 011

−3 → 011 → 100 + 1 = $\boxed{101}$ ✓

Ex:- 2 × −1

$$
\begin{array}{r}
010 \times 001 \\
\hline
010 \\
000\times \\
000\times\times \\
\hline
00010
\end{array}
\xrightarrow{2's}
\begin{array}{r}
11101 \\
+1 \\
\hline
\cancel{11}110
\end{array}
\Rightarrow \underline{110} \Rightarrow -\underline{2}
$$

2 × −3 ⇒

$$
\begin{array}{r}
010 \times 011 \\
\hline
010 \\
010\times \\
000\times\times \\
\hline
00110
\end{array}
\xrightarrow{2's}
\begin{array}{r}
11001 \\
+1 \\
\hline
\cancel{11}010
\end{array}
\Rightarrow \underline{2}
$$

Fixed Point Format

Decimal no	Decimal Fraction	2's comp.
3 →	3/4 →	0.11
2 →	2/4 →	0.10
1 →	1/4 →	0.01
0 →	0 →	0.00
−1 →	−1/4 →	1.11
−2 →	−2/4 →	1.10
−3 →	−3/4 →	1.01
−4 →	−4/4 = −1 →	1.00

3bit 2's complement s/m using fractional representation.

Ex:- 2×−3

```
       0.10 × 0.11
      ------------
         010
        010 x
      000 x x
     ---------
     0.0110 --2's--> 1.1001
                         +1
                     ------
                     1.1010
```

$$1.1010 \Rightarrow (-1)\times(0.0110)$$

$$\Rightarrow -(0\times2^{0}+0\times2^{-1}+1\times2^{-2}+1\times2^{-3}+0\times2^{-4})$$

$$\Rightarrow -\frac{3}{8}$$

$$\frac{2}{4}\times-\frac{3}{4} \Rightarrow -\frac{3}{8}$$

Fixed Point Format

Q-format number Representation

Q-15:

Q-15																
	-2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}	2^{-6}	2^{-7}	2^{-8}	2^{-9}	2^{-10}	2^{-11}	2^{-12}	2^{-13}	2^{-14}	2^{-15}

↑ binary point

Ex:- 0.560123

$0.560123\times2 = 1.120246$ 1 [MSB]
$0.120246\times2 = 0.240492$ 0
$0.240492\times2 = 0.480984$ 0
$0.480984\times2 = 0.961968$ 0
$0.961968\times2 = 1.923936$ 1
$0.923936\times2 = 1.847872$ 1
$0.847872\times2 = 1.695744$ 1
$0.695744\times2 = 1.391488$ 1
$0.391488\times2 = 0.782976$ 0
$0.782976\times2 = 1.565952$ 1
$0.565952\times2 = 1.131904$ 1
$0.131904\times2 = 0.263808$ 0
$0.263808\times2 = 0.527616$ 0
$0.527616\times2 = 1.055232$ 1
$0.055232\times2 = 0.110464$ 0 [LSB]

0.100011110110010

$(0.560123)\times2^{15} = 18354.$

Ex:- -0.160123

$0.160123 = 0.001010001111110$

↓ 2's

1.110101110000001
+1

1.110101110000010

$(-0.160123)\times2^{15} = -5246.9$

⇓

-5246

Floating-Point format:

$$\boxed{x = M \cdot 2^{E}}$$

M → mantissa (or) Fractional part. $\boxed{-1 \to +1}$
in Q-format.

E → the exponent.

-2^3	2^2	2^1	2^0	-2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}	2^{-6}	2^{-7}	2^{-8}	2^{-9}	2^{-10}	2^{-11}

4-bit exponent ← → ← 12-bit mantissa. →

Most negative number $= (1.00000000000)_2 \cdot 2^{(0111)_2} = (-1) \times 2^7 = -128$

Most positive number $= (0.11111111111)_2 \cdot 2^{(0111)_2} = (1-2^{-11}) \times 2^7 = 127.9375$

Smallest positive number $= (0.00000000001)_2 \cdot 2^{(1000)_2} = (2^{-11}) \times 2^{-8} = 2^{-19}$

Ex:- 0.1601230

$0.1601230 = 0.640492 \times 2^{-2}$

2's comp $-2 = 1110$.

$0.6404920 \to Q11 \Rightarrow 01010001111$

1110010100011111

IEEE Floating-Point Formats:

1. Single Precision Format

31	30 … 23	22 … 0
S	exponent	fraction

$$X = (-1)^S \times (1.F) \times 2^{E-127}$$

F → mantissa in 2's comp. binary fraction. $\boxed{+1 \text{ to } +2}$

S = 0 → number is possitive

S = 1 → number is negative.

E → Excess 127 form. 8-bit → 0 to 255.

E − 127 → range from −127 to +128.

2. Double Precision Format

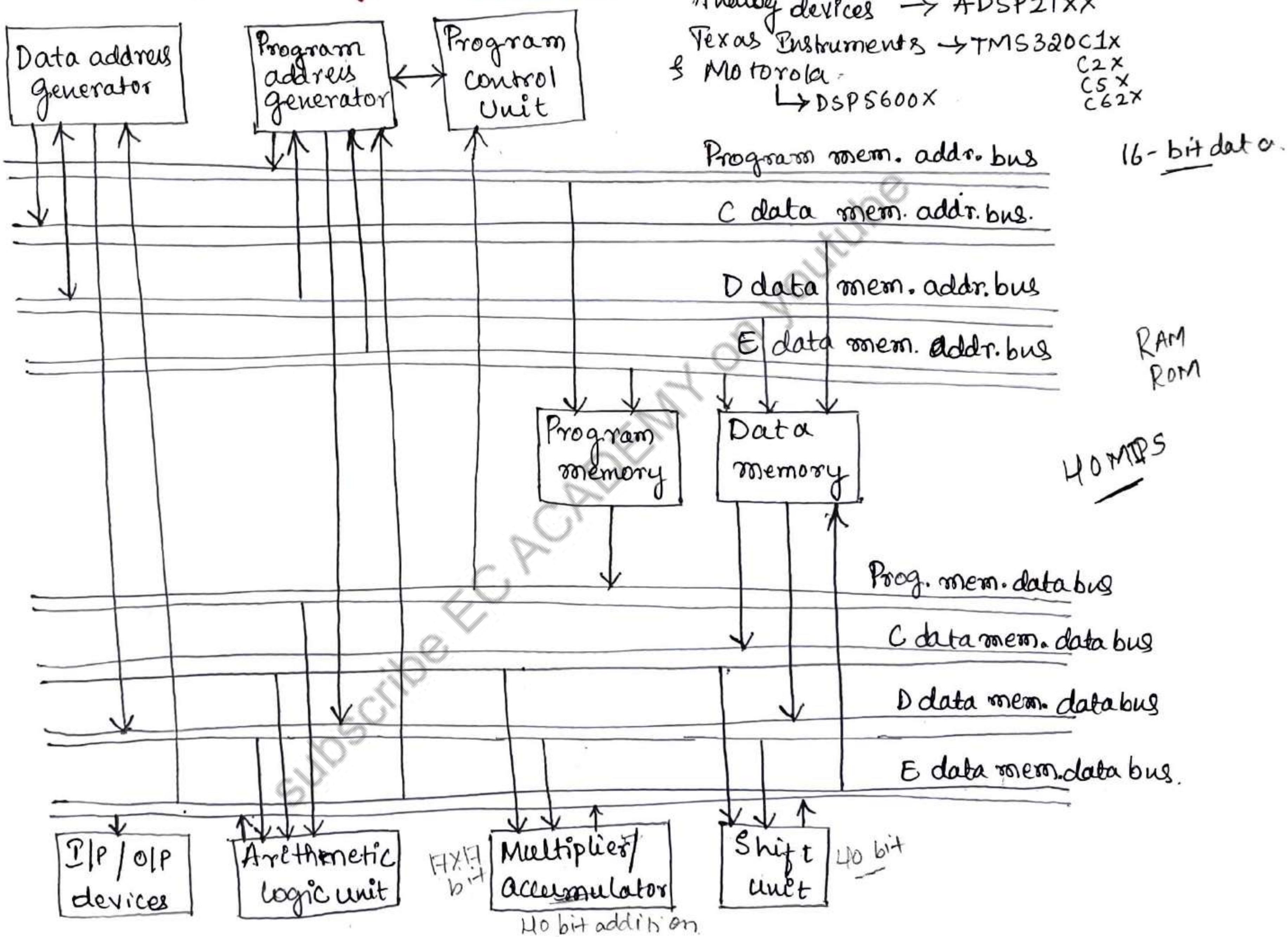
31	30 … 20	19 … 0	31 … 0
S	exponent	fraction	fraction.
odd register			even register.

$$X = (-1)^S \times (1.F) \times 2^{E-1023}$$

Fixed-Point Digital Signal Processor

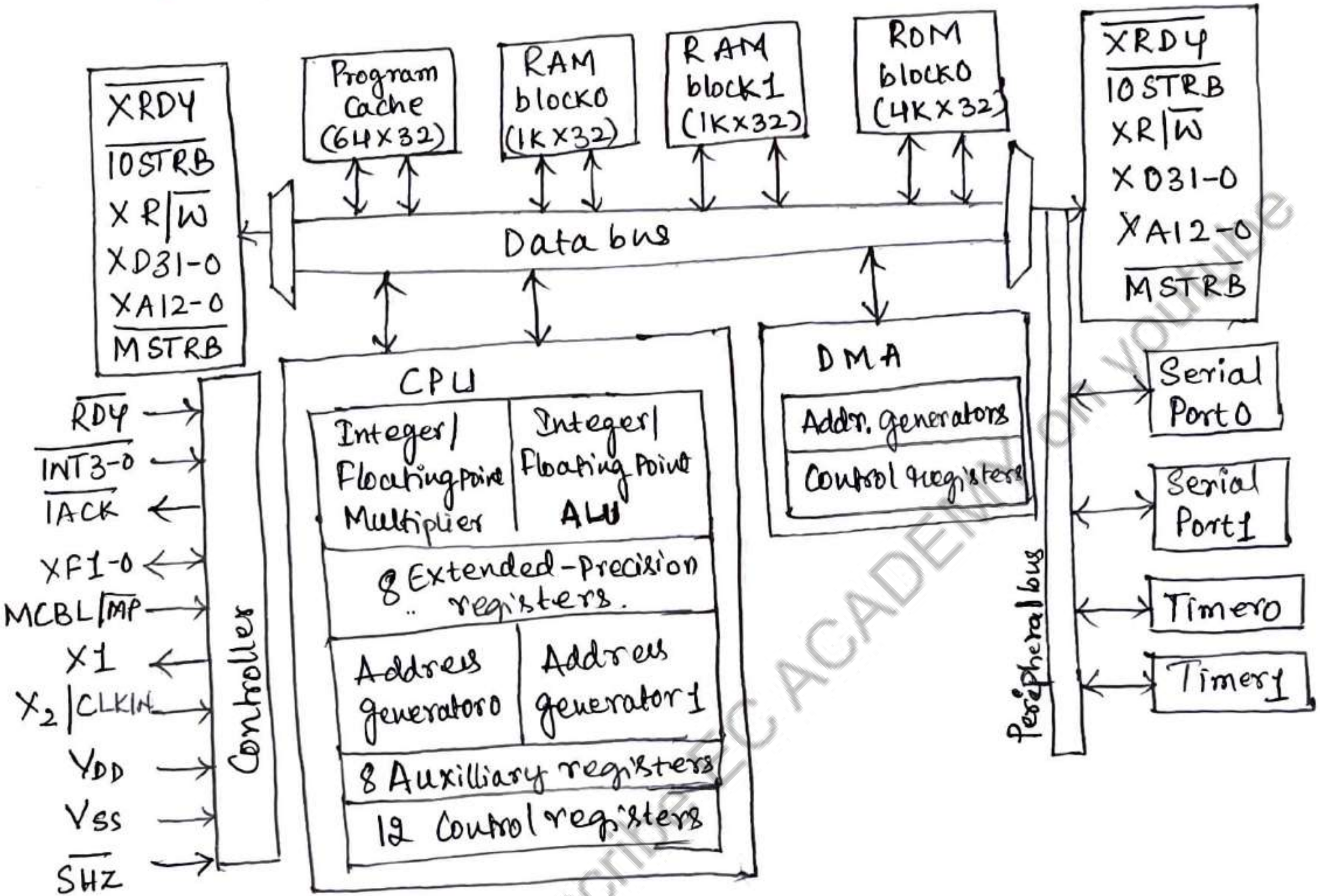
Analog devices → ADSP21XX
Texas Instruments → TMS320C1x, C2x, C5x, C62x
& Motorola → DSP5600X

16-bit data.



Basic Architecture Of TMS320C54X Family

Floating Point Processors



32 bit

3-types

16 bit 4-E 1-S 11-M

32 bit 8-E 1-S 23-M

40-bit 8-E 1-S 31-M.

The Typical TMS320C3x floating-point DS Processor