MODULE-1

Introduction to Control System

Control:-

Control means to regulate, to direct or to command. Hence a control system is an arrangement of different physical eleme... connected in such a manner so as to regulate, direct or command itself or some other system.

System :-

A system is a combination or arrangement of different physical components which acts as an entire unit to achieve certain objectives.

Input :-

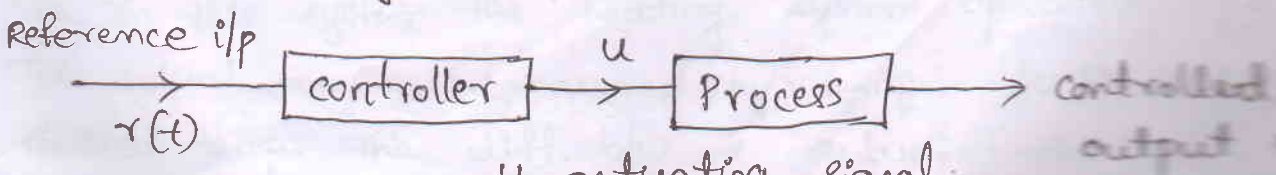
It is an applied signal or an excitation signal applied to a control system from an external energy source in order to produce a specified outp...

Output :-

The actual response obtained from a control syste... which input is applied to it.

open loop system :-

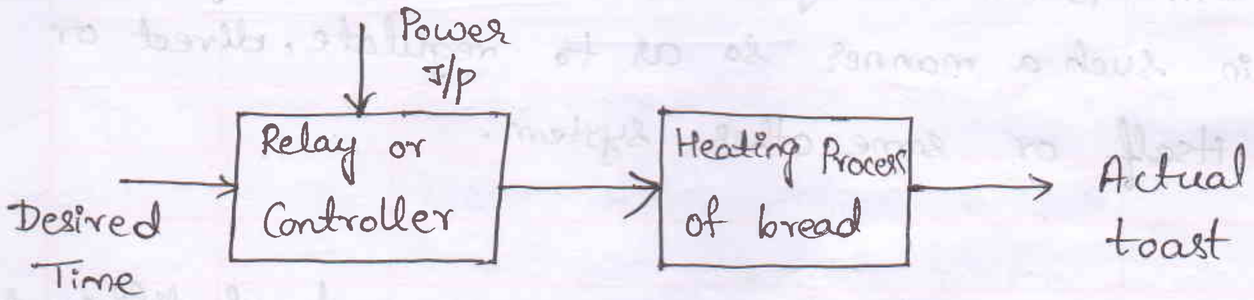
A system in which output is dependent on i... or input is totally independent of output is called open loop system.



u - actuating signal

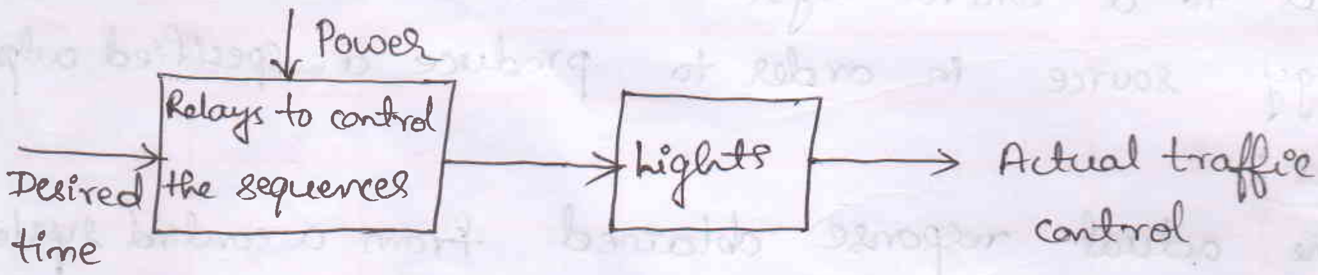
Reference i/p r(t) is applied to a controller which generates a actuating signal 'u' required to control the process which is to be controlled. Process is giving out the necessary desired o/p c(t).

Eg 1:- Automatic Toaster system.



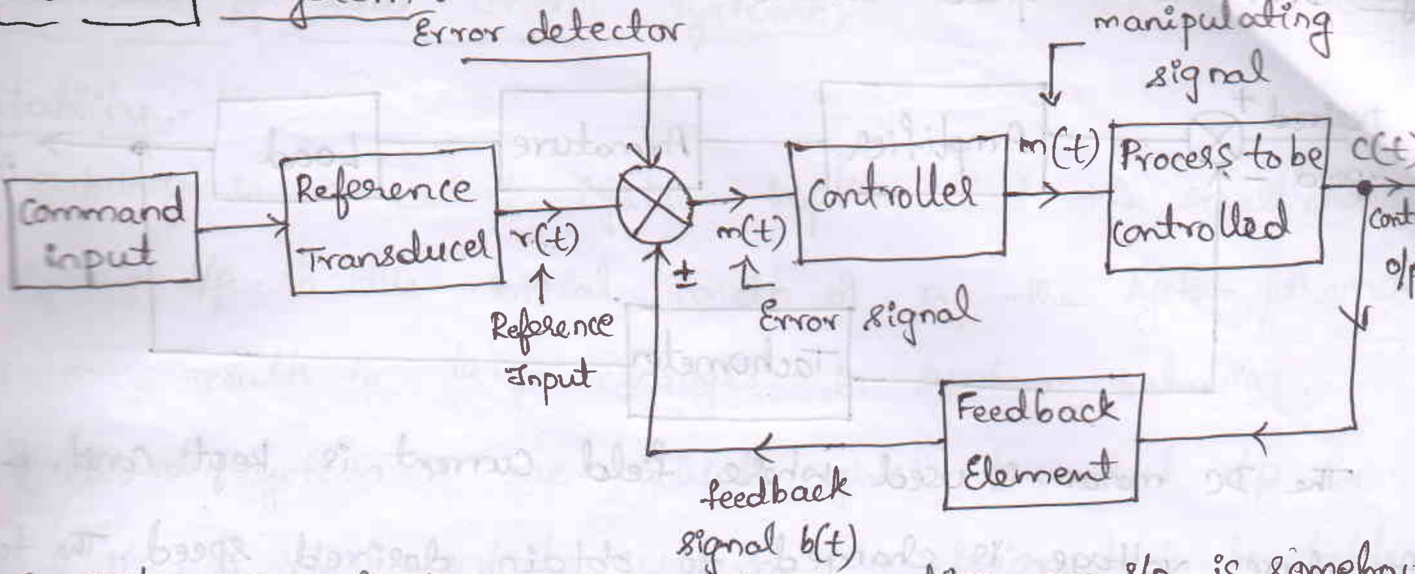
In this system the quality of toast depends upon the time for which toast is heated. Depending on the time setting bread is heated in this system. The toast quality is to be judged by the user & has no effect on input.

Eg 2:- Traffic light controller



A traffic flow control system used on roads is time dependent. The sequence & duration are controlled by relays which are pre determined & not dependent on the rush on the road.

Closed loop system :-



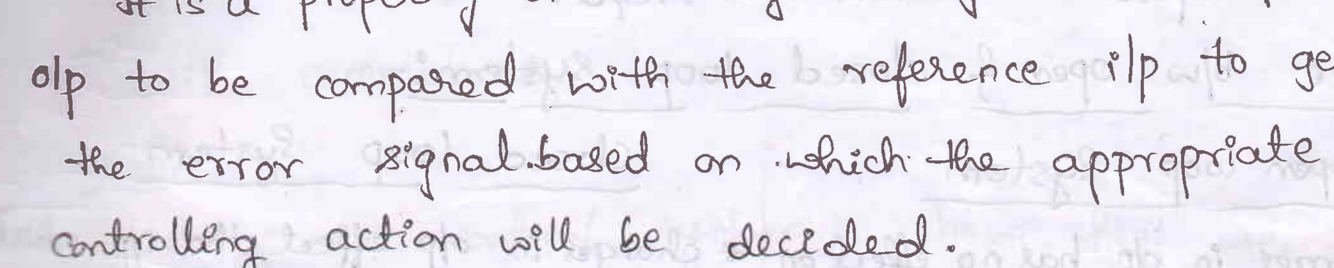
A system in which the controlling action or i/p is somehow dependent on the o/p or changes in o/p is called as closed loop system.

Feedback:-

It is a property of the system by which it permits the o/p to be compared with the reference i/p to generate the error signal based on which the appropriate controlling action will be decided.

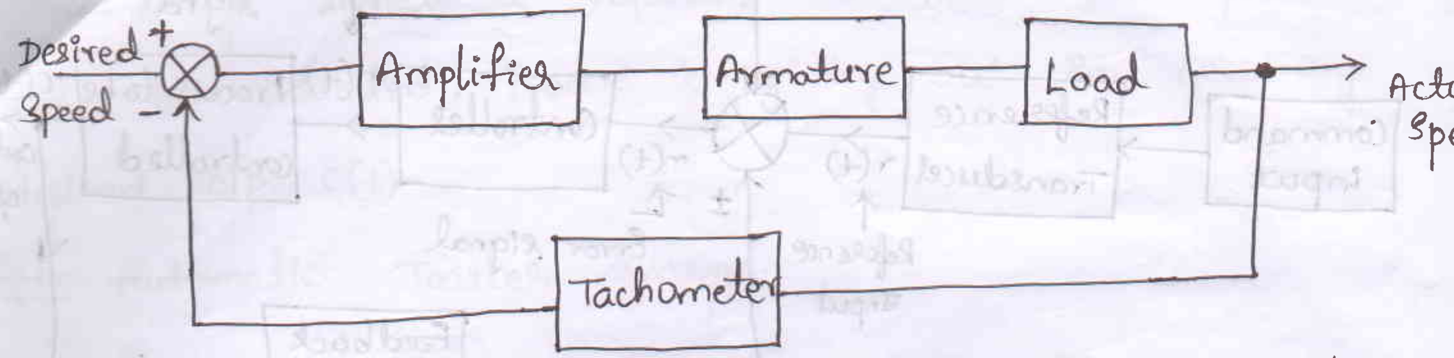
When the feedback signal is +ve systems are called +ve feed back system & if it is -ve systemes are called -ve feedback system.

Eg 1:- Home heating system:-



In this system the heating system operated by a valve. The actual temp. is measured by temp. sensor & compare with desired temp. The diff. b/w 2 actuate the valve mechanism to change the temp. as per the requirement.

Eg2:- D.C. motor speed control



The DC motor is used where field current is kept const. & Armature voltage is changed to obtain desired speed. The feedb is taken by Tachometer. This generates voltage proportional to sp which is compared with the voltage required to the desired sp This diff. is used to change the i/p to control which cumilati changes the speed of motor as reqd.

Difference b/w open & closed loop system:-

Open loop System	Closed loop System
* Any changes in o/p has no effect on i/p i.e. feed back does not exist.	* changes in o/p effect the i/p is possible by the use of feedb
* Feedback element is absent.	* Feedback element is presen
* Error detector is absent.	* Error detector is present
* It is inaccurate & unreliable.	* Highly accurate & reliab
* Highly sensitive to disturbances.	* Less sensitive to disturba
* Highly sensitive to enviornmental changes.	* Less sensitive to enviornme changes.
* Simple to construct & cheap.	* Complicated to design & costly.
* Highly effected by non linearity.	* Reduced effect of non linearity.

Requirements of ideal control systems :-

1 Stability :-

Stability in a control system implise that the small change in system i/p in the initial cond'n or in the system paramete does not results in large changes in system behavior.

A control system is one which gives bounded o/p for bounded i/p. A ideal control system are designed to be stable.

2 Sensitivity :-

An ideal control system should be insensitive to variation in parameter of the system but it should be sensitive to i/p commands.

3 Speed :

Speed of the control system is how fast the o/p of the system approaches to the i/p or the desired value. This is measured interms of settling time & rise time.

An ideal control system should have good speed.

4 Accuracy :-

Accuracy of a control system is how much the o/p of the system is nearer to the i/p of the desired value.

An ideal control system must be highly accurate.

5 Disturbance :-

All control system are subjected to some type of unwanted signal or noise during operation. External disturbance such as wind, thermal noise, voltage are quite common.

6] **Band width:**

The bandwidth of a control system means for a range of i/p the o/p of the control system should be constant.

Controllers:-

Controller is a basic element in a control system which compares the actual value of the plant o/p with the reference i/p or the desired value to determine the error or deviation & produces proper corrective action that will reduce the error to a smaller value or to zero.

The measurement of error is possible due to feedback. The feedback allows to compare the plant o/p with its reference i/p to generate the error. Thus the i/p to the controller is the deviation of the o/p from the desired value is known as error. & o/p from the controller is corrective action is known as manipulated signals.

Types of controllers:-

I] **Proportional (P) controllers:-**

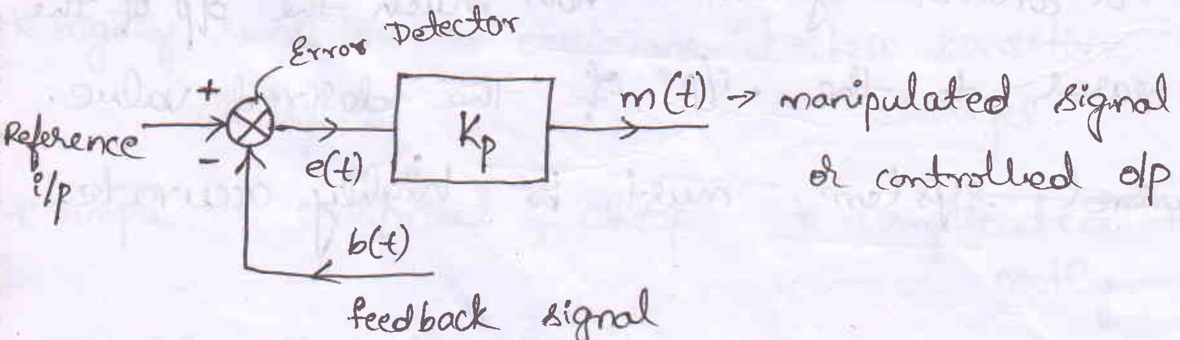


Fig. shows the block diagram of ~~cont~~ proportional controller system. In this the o/p of the ~~system~~ controller i.e.

manipulated signal m(t) is proportional to the i/p of the controller e(t) i.e. error (~~system~~) signal.

Mathmatically represented as

$$m(t) \propto e(t)$$

$$m(t) = K_p \, e(t)$$

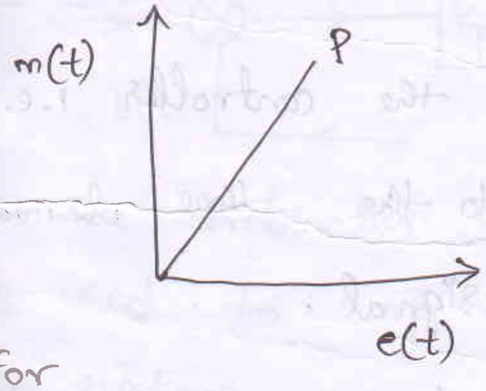
where, K_p - proportional gain

Taking Laplace Transform,

$$M(s) = K_p \; E(s)$$

$$\boxed{K_p = \frac{M(s)}{E(s)}}$$

The relation b/w the o/p of the controller m(t) & error signal e(t) for a unit step i/p as shown in fig.



For a zero error the controller o/p should be zero otherwise the process will come to stop.

Hence mathmatically it is represented as,

$$\boxed{m(t) = K_p \, e(t) + m_0}$$

where, m_0 is the controller o/p for zero error.

Characteristics of proportional Controller :

1) The system is stable.

2) It provides fast response.

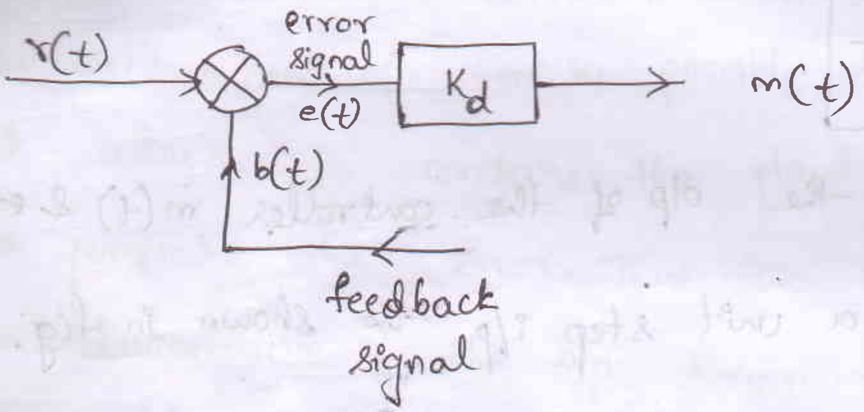
3) Improves steady state errors & rise time.

Disadvantages of proportional controller :-

* Provides heavily damped response.

3)

Differential or Derivative (D) controller :-



In this controller mode the o/p of the controller i.e. manipu
signal is directly proportional to the time derivative of i/p to the controller i.e. error signal.

Mathematically represented as,

$$m(t) \propto \frac{d}{dt} e(t)$$

$$m(t) = K_d \frac{d}{dt} e(t)$$

$K_d \rightarrow$ derivative gain const.

Taking Laplace transform,

$$M(s) = K_d \, s \, E(s)$$

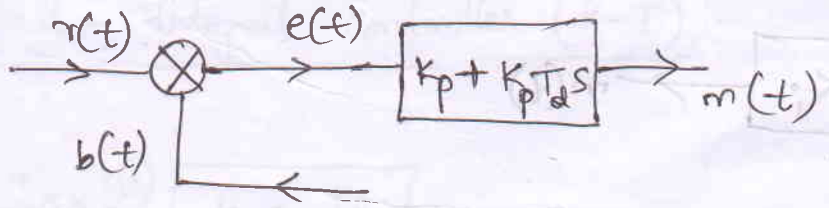
$$\boxed{K_d = \frac{M(s)}{sE(s)}}$$

The main advantages of this control is that it response to the rate of change of error & can produce necessary corrective action before the magnitude of error become too large.

characteristics of derivative

* It tends to increase the stability of system.
* It acts as damping to the system & hence large value of gain which will increase the accuracy.

3. Proportional plus derivative controller: (P-D) controller:



It is the combination of proportional & derivative controller which is used to improve the steady state behaviour of the system.

In this control mode manipulated signal ~~(system)~~ consists of proportional error signal added with derivative error signal.

Mathmatically represented as,

$$m(t) = K_p e(t) + K_d \frac{de(t)}{dt}$$

But $K_d = \cancel{(T_p\ T_d)}\ K_p T_d$

where, T_d - ~~proportional~~ derivative time.

$$m(t) = K_p\, e(t) + \cancel{K_d}\ K_p T_d \frac{de(t)}{dt}.$$

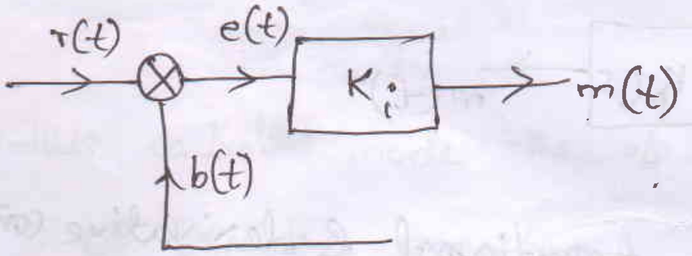
Taking Laplace transform,

$$M(S) = K_p\, E(S) + K_p T_d\, S\, E(S)$$

$$M(S) = E(S)\left[K_p + K_p T_d S\right]$$

$$\boxed{\frac{M(s)}{E(S)} = K_p + K_p T_d S}$$

4] <u>Integral (I) controller</u>:-



In this control mode the o/p of the controller i.e manipulated signal is changed at a rate proportional to the i/p of the controller i.e. error signal

Mathmatically,

$$\frac{d}{dt}(m(t)) \propto e(t)$$

$$\frac{d}{dt}(m(t)) = K_i\, e(t)$$

where K_i - Integral gain.

~~(Taking Laplace transform,)~~

Integrating,

$$m(t) = K_i \int e(t)$$

Taking Laplace transform,

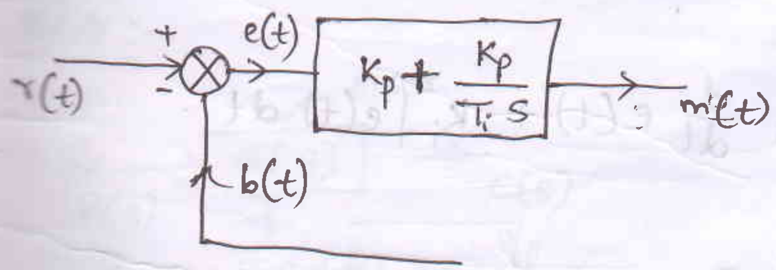
$$M(s) = K_i \frac{E(s)}{s}.$$

$$\frac{M(s)}{E(s)} = \frac{K_i}{s} \Rightarrow \boxed{K_i = \frac{sM(s)}{E(s)}}$$

Characteristics of integral controller :-

* It slows down system response.
* Increases settling & rise time.

5] Proportional - Integral Controller (P-I) :-



This is the combination of integral & proportional controller which is used to increase the performance of the system. In this manipulated signal consists proportional error signal added with integral error.

$$m(t) = K_p\, e(t) + K_i \int e(t)\, dt$$

K_i - Integral gain. $= \frac{K_p}{T_i}$

where, T_i - Integral time.

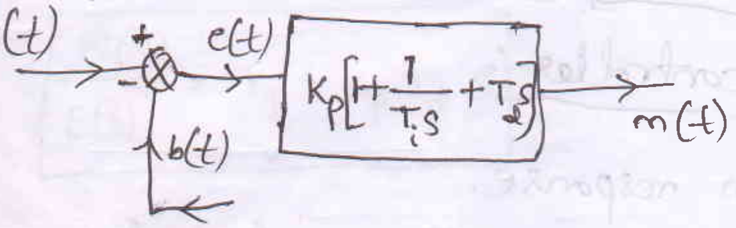
$$m(t) = K_p\, e(t) + \frac{K_p}{T_i} \int e(t)\, dt$$

Taking Laplace,

$$M(s) = K_p\, E(s) + \frac{K_p}{T_i} \frac{E(s)}{s}$$

$$\boxed{M(s) = E(s) \left[K_p + \frac{K_p}{T_i s} \right]}$$

6 Proportional - derivative integral controller (PID) :-



It is a combination of proportional, differential & integral control action so as to derive the advantages of all the control action. Generally it is known as P-I-D controller. The eqn. is given by,

$$m(t) = K_p\, e(t) + K_d \frac{d}{dt} e(t) + K_i \int e(t)\, dt$$

$$K_d = K_p T_d \quad , \quad K_i = \frac{K_p}{T_i}$$

$$\Rightarrow m(t) = K_p\, e(t) + K_p T_d \frac{d}{dt} e(t) + \frac{K_p}{T_i} \int e(t)\, dt$$

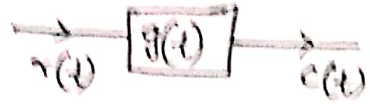
Taking Laplace transform,

$$M(s) = K_p E(s) + K_p T_d\, s E(s) + \frac{K_p}{T_i} \frac{E(s)}{s}$$

$$M(s) = E(s) \left[K_p + K_p T_d s + \frac{K_p}{T_i s} \right] \Rightarrow \boxed{M(s) = E(s) K_p \left[1 + T_d s + \frac{1}{T_i s} \right]}$$

MODULE-2:

Block Diagrams

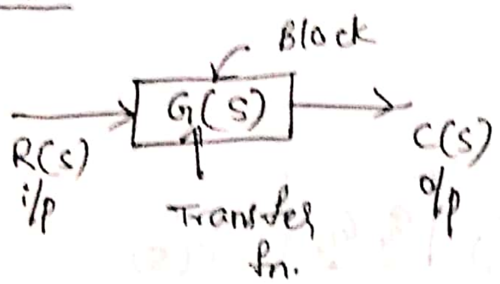


Transfer function $= \dfrac{C(s)}{R(s)}$ initial cond'ns to be zero

Transfer fn. of a linear time invariant system is defined as the ratio of laplace transform of system o/p to the laplace transform of system i/p ~~ss~~ ~~ab~~ with all the initial cond'ns assumed to be zero.

Basic Elements of block diagram :-

1] **Block :-**

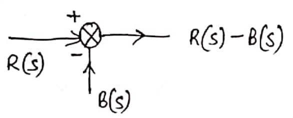


It is a rectangular box or a symbol that explains the mathmatical operation on the i/p to produce corresponding o/p.

$\therefore \quad G(s) = \dfrac{C(s)}{R(s)}$

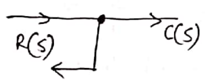
Scanned by CamScanner

2] Summing point :-



It is a symbol that shows algebraic sum of two or more signals. The possitive or negative sign at each arrow head indicates whether the signal to be added or substracted

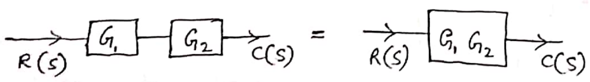
3] Take off point :-



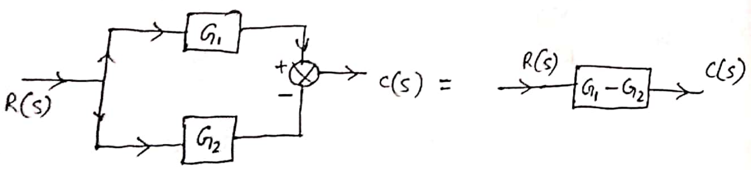
It is a point from which the signal is taken for feedba purpose or distribution to other blocks.

* Imp
Block diagram reduction rules :-

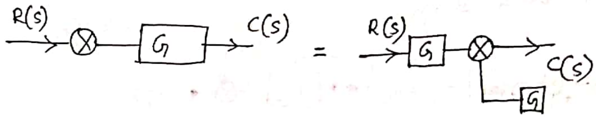
1] When two blocks are in series :-



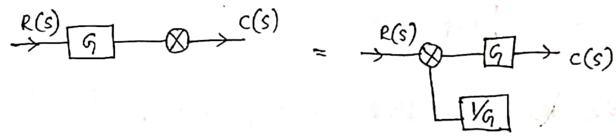
2] When two blocks are in parallel :-



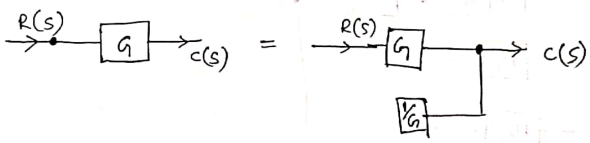
3] Moving a summing point after the block :-



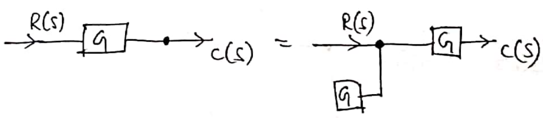
4] Moving a summing point before the block :-



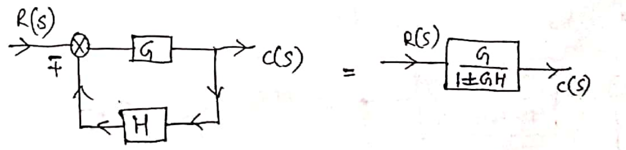
5] Moving a take off point after the block :-



6] Moving a take off point before the block :-

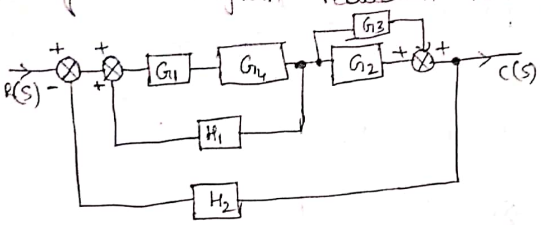


7] Eleminating a feedback loop :



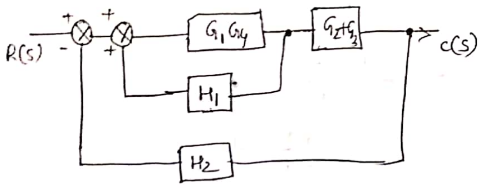
Scanned by CamScanner

1] Determine the transfer fn. $\frac{C(S)}{R(S)}$ for the system shown in fig. using block diagram reduction technique.

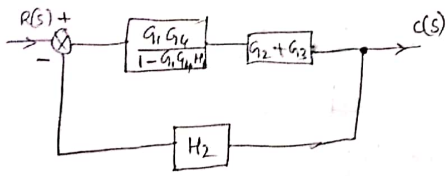


Soln:-

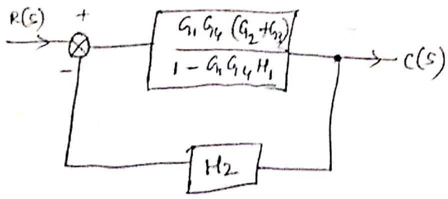
Combine the series blocks G_1 & G_4 and parallel blocks G_2 & G_3



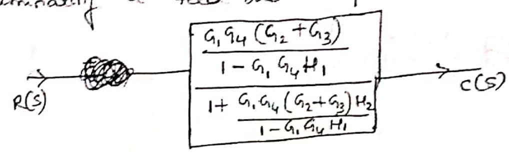
Eliminating a feedback loop,



Combine the series blocks



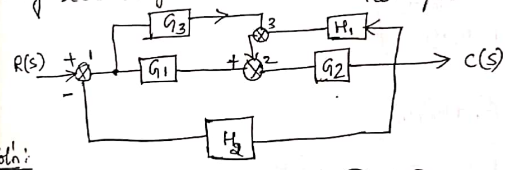
Eliminating a feed back loop.



$$\frac{\frac{G_1G_4(G_2+G_3)}{1-G_1G_4H_1}}{1+\frac{G_1G_4(G_2+G_3)H_2}{1-G_1G_4H_1}}$$

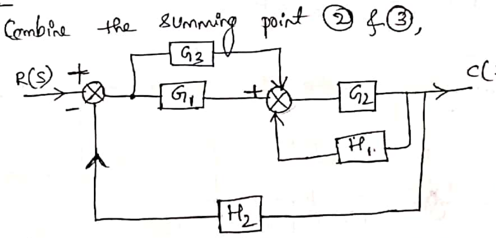
∴ Transfer fn, $$\frac{C(S)}{R(S)} = \frac{G_1G_4(G_2+G_3)}{1-G_1G_4H_1+G_1G_4(G_2+G_3)H_2}$$

2] Obtain the overall transfer fn. of the block diagram shown in fig. Using block diagram reduction technique.

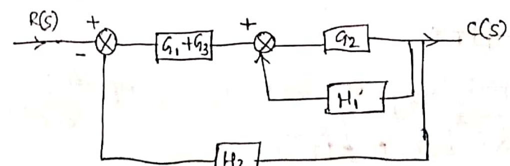


Soln:

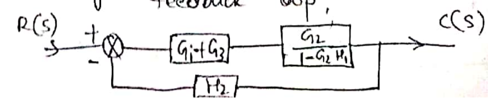
Combine the summing point ② & ③,



Combining the parallel blocks G_1 & G_3

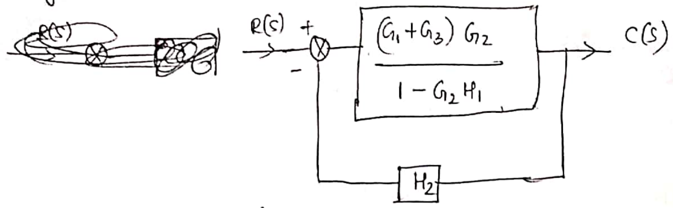


Eliminating feedback loop,

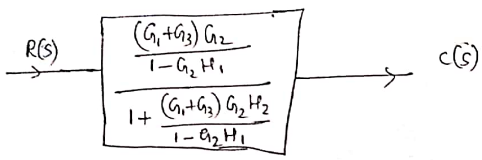


Scanned by CamScanner

Combining the series,

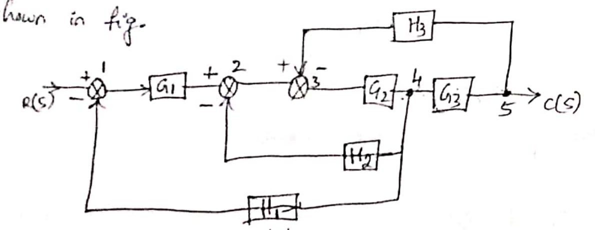


Eliminating the feedback loop:-

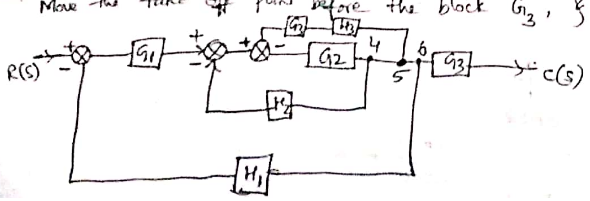


$\therefore$ Transfer function, $\frac{C(s)}{R(s)} = \frac{(G_1+G_3)G_2}{1-G_2H_1+(G_1+G_3)G_2H_2}$

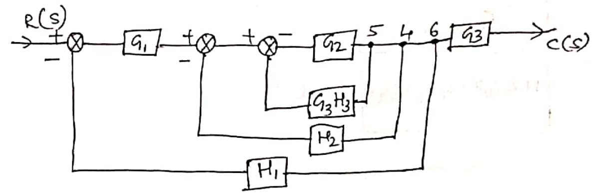
3] Reduce the given block diagram & write the overall transfer fn. shown in fig.



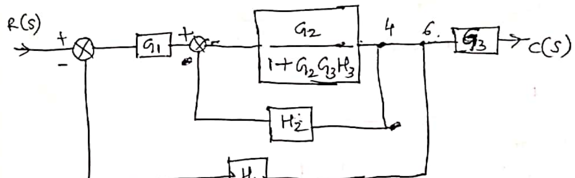
Soln: Move the take off point before the block G_3, & repeating take off point '4'



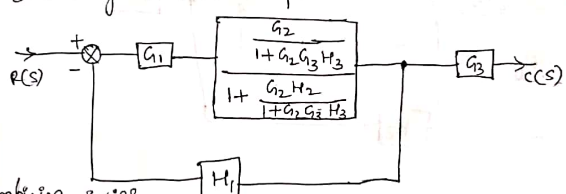
Interchanging take off point 4 & 5



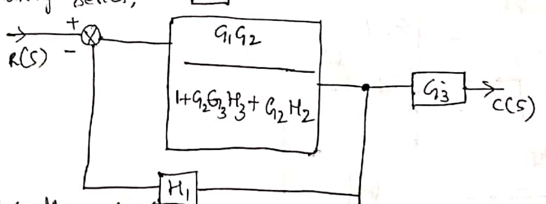
Eliminating the feed back loop



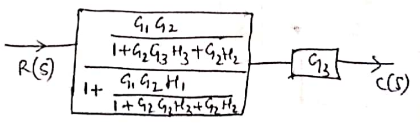
Eliminating feed back loop,



Combining series,

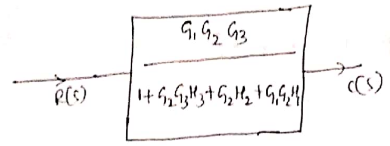


Eliminating feed back loop,



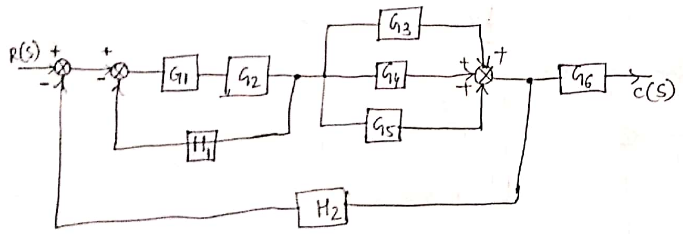
Scanned by CamScanner

combining series,

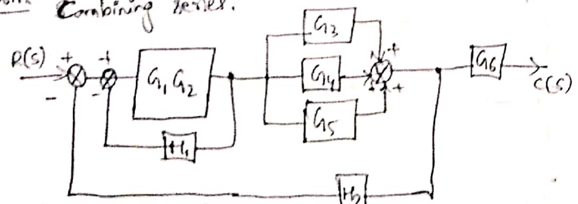


$\therefore$ Transfer fn. $\frac{C(s)}{R(s)} = \frac{G_1G_2G_3}{1+G_2G_3H_3+G_2H_2+G_1G_2H_1}$

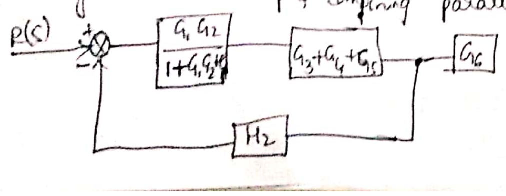
4] Obtain the transfer fn.

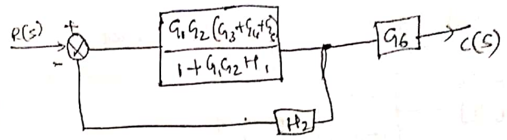


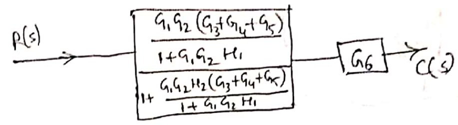
Soln: Combining series.

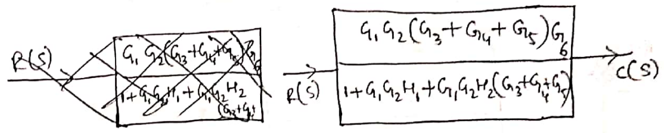


Eliminating feed back loop & combining parallel.



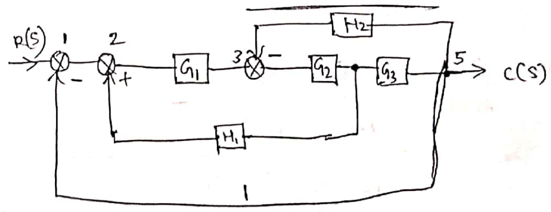




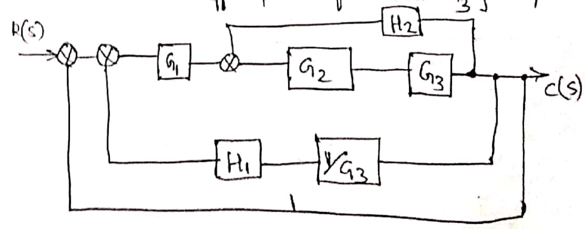


$\Rightarrow$ $\therefore$ Transfer fn. $\frac{C(s)}{R(s)} = \frac{G_1G_2(G_3+G_4+G_5)G_6}{1+G_1G_2H_1+G_1G_2H_2(G_3+G_4+G_5)}$

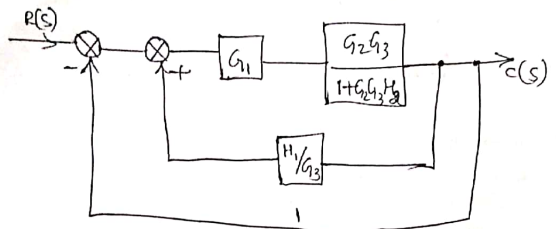
5]



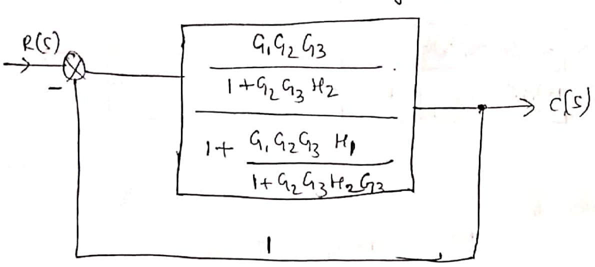
Soln: Shift the takeoff point after block G_3 & seperate the paths,



Combining series blocks & Eliminating feed back.



Combining series blocks & Eliminating feed back,



$\therefore$ Transfer fn.
$$\frac{C(s)}{R(s)} = \frac{\dfrac{G_1G_2G_3}{1+G_2G_3H_2}}{1+\dfrac{G_1G_2G_3H_1}{1+G_2G_3H_2G_3}}$$

Mason's Gain Formula :-

This formula used for determination of overall transfer fn. of a system. It is a convinient & easy way of finding the relation b/w i/p & o/p variable of a system.

It is given by,

$$T.F = \frac{1}{\Delta}\sum_{K=1}^{n} \Delta_K P_K$$

Where K: no. of forward path

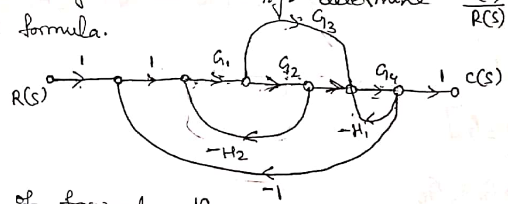
P_K - path gain of K^{th} forward path.

Δ - determinant of the Graph.

= 1 - (sum of Individual loop gain) + (sum of products of all combination of 2 non-touching loops) - (sum of gain products of all combination of 3 non-touching loops) + ...

Δ_K - value of Δ by eliminating all loop gains & associated products which are touching the K^{th} forward path.

1) For the system shown in fig. determine $\frac{C(s)}{R(s)}$ using Mason's gain formula.



Soln:

No. of forward paths,

$P_1 = 1\times1\times G_1\times G_2\times G_4\times 1 = G_1G_2G_4$

$P_2 = 1\times1\times G_1\times G_3\times G_4\times 1 = G_1G_3G_4$

No. of loop gains,

$L_1 = \cancel{G_1 G_2 H_2}\ - G_1 G_2 H_2$

$L_2 = -G_4 H_1$

$L_3 = -G_1 G_2 G_4$

$L_4 = -G_1 G_3 G_4$

Combination of 2 non touching loops = $L_1 L_2$

Combination of 3 non touching loops = 0

$\left.\begin{array}{l} \Delta_1 = 1-(0+0+0+0) = 1 \\ \Delta_2 = 1-(0+0+0+0) = 1 \end{array}\right\}$ compare P & L, i.e. $P_1 L_1, P_1 L_2, P_1 L_3, P_1 L_4$

$\Delta = 1 -$ (Sum of individual loop ~~graph~~ gain) + (sum of product of all combination of 2 non touching loops) − (sum of gain of product of all combination of 3 non touching loops) + ...

$= 1 - (L_1 + L_2 + L_3 + L_4) + (L_1 L_2) - (0) + \ldots$

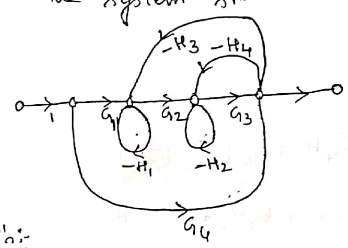
$= 1 + G_1 G_2 H_2 + G_4 H_1 + G_1 G_2 G_4 + G_1 G_3 G_4 + (G_1 G_2 G_4 H_1 H_2)$

$TF = \frac{1}{\Delta} \sum_{K=1} \Delta_K P_K$

$= \frac{1}{\Delta} [\Delta_1 P_1 + \Delta_2 P_2]$

$$= \frac{G_1 G_2 G_4 + G_1 G_3 G_4}{1 + G_1 G_2 H_2 + G_4 H_1 + G_1 G_2 G_4 + G_1 G_3 G_4 + G_1 G_2 G_4 H_1 H_2}$$

8] Using Mason's gain formula determine overall transfer fn. for the system shown in fig.



Sol'n:-

No. of forward paths,

$P_1 = G_1 G_2 G_3$

$P_2 = G_4$

Loop gains,

$L_1 = -G_2 G_3 H_3$

$L_2 = -G_3 H_4$

$L_3 = -H_1$

$L_4 = -H_2$

Combination of 2 non touching loops = $L_3 L_4,\ L_3 L_2$

Combination of 3 non touching loops = 0

$\Delta_1 = 1 - (0+0+0+0) = 1$

$\Delta_2 = 1 - (0+0+L_3+L_4)$

$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (L_3 L_4 + L_3 L_2) - 0 + \ldots$

$= 1 + G_2 G_3 H_3 + G_3 H_4 + H_1 + H_2 + H_1 H_2 + G_3 H_1 H_4$

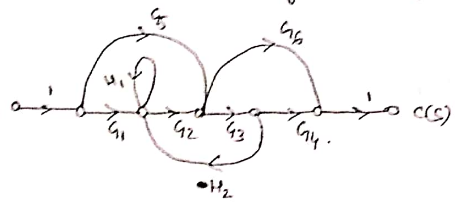
Scanned by CamScanner

$$TF = \frac{1}{\Delta} \sum_{k=1}^{n} \Delta_k P_k$$

$$= \frac{1}{\Delta}\left[\Delta_1 P_1 + \Delta_2 P_2\right]$$

$$= \frac{G_1 G_2 G_3 + \left[1 + G_4 H_1 + G_4 H_2\right]}{1 + G_2 G_3 H_3 + G_3 H_4 + H_1 + H_2 + H_1 H_2 + G_3 H_1 H_4}$$

3] Determine the transfer fn. using Mason's gain formula.



Soln:

No. of forward paths,

$P_1 = G_1 G_2 G_3 G_4$

$P_2 = G_5 G_6$

$P_3 = G_5 G_3 G_4$

$P_4 = G_1 G_2 G_6$

Loop gain,

$L_1 = G_2 G_3 H_2$

$L_2 = H_1$

combination of 2 non touching loops = 0

" " 3 " " " = 0.

$\Delta_1 = 1 - [0 + 0] = 1$

$\Delta_2 = 1 - [L_2 + 0] = 1 - L_2$

$\Delta_3 = 1 - [0 + 0] = 1$

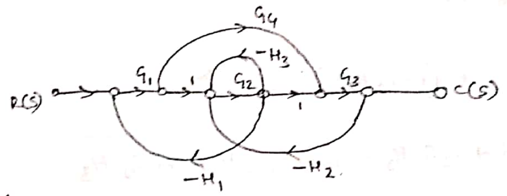
$\Delta_4 = 1 - [L_1 + L_2] + 0 - 0 = 1 - [H_1]$

$TF = \frac{1}{\Delta} \sum \Delta_k P_k \qquad \Delta = 1 - [G_2 G_3 H_2 + H_1] + 0 - 0$

$$\Rightarrow TF = \frac{1}{\Delta} \sum_{k=1}^{n} \Delta_k P_k$$

$$= \frac{G_1 G_2 G_3 G_4 + (1 - H_1) G_5 G_6 + G_5 G_3 G_4 + (1 - H_1)(G_1 G_2 G_6)}{1 - [G_2 G_3 H_2 + H_1]}$$

4]



Soln:

No. of forward paths,

$P_1 = G_1 G_2 G_3$

$P_2 = G_1 G_4 G_3$

Loop gain,

$L_1 = -G_1 G_2 H_1$

$L_2 = -G_2 G_3 H_2$

$L_3 = G_1 G_4 G_3 H_2 G_2 H_1, \quad L_4 = -G_2 H_3$

Scanned by CamScanner

combination of 2 non touching loops = 0
" " 3 " " " = 0.

$\Delta_1 = 1 - [0+0+0+0] = 1$

$\Delta_2 = 1 - [L_4] = 1 + G_2 H_3$

$\Delta = 1 - [-G_1 G_2 H_1 - G_2 G_3 H_2 + G_1 G_4 G_3 G_2 H_1 H_2 - G_2 H_3] + 0 - 0$

$= 1 + G_1 G_2 H_1 + G_2 G_3 H_2 - G_1 G_2 G_3 G_4 H_1 H_2 + G_2 H_3$.

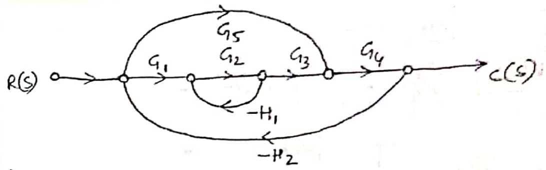
3]

$TF = \frac{1}{\Delta} \sum_{k=1}^{n} \Delta_k P_k$.

$= \frac{1}{\Delta} [\Delta_1 P_1 + \Delta_2 P_2]$

$$= \frac{G_1 G_2 G_3 + (1+G_2 H_3)(G_1 G_3 G_4)}{1 + G_1 G_2 H_1 + G_2 G_3 H_2 - G_1 G_2 G_3 G_4 H_1 H_2 + G_2 H_3}$$

5]



Sol'n:-

No. of forward paths.

$P_1 = G_1 G_2 G_3 G_4$

$P_2 = G_5 G_4$

Loop gains,

$L_1 = G_2 H_1$

$L_2 = -G_1 G_2 G_3 G_4 H_2$

$L_3 = -G_5 G_4 H_2$

Combination of 2 non-touching Loops = $L_1 L_3$
combination of 3 " " " = 0

$\Delta_1 = 1 - (0+0+0) = 1$

$\Delta_2 = 1 - (L_1 + 0 + 0) = 1 - L_1 = 1 + G_2 H_1$

$\Delta = 1 - (L_1 + L_2 + L_3) + (L_1 L_3) - 0$

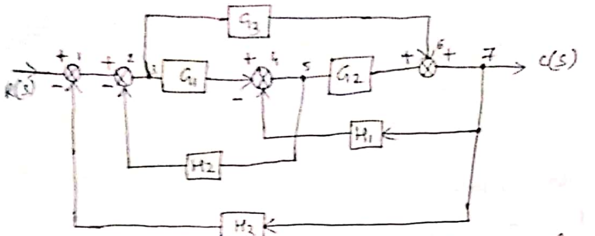
$= 1 + G_2 H_1 + G_1 G_2 G_3 G_4 H_2 + G_5 G_4 H_2 + G_2 G_4 G_5 H_1 H_2$

$TF = \frac{1}{\Delta} \sum_{k=1}^{n} \Delta_k P_k$

$= \frac{1}{\Delta} [\Delta_1 P_1 + \Delta_2 P_2]$

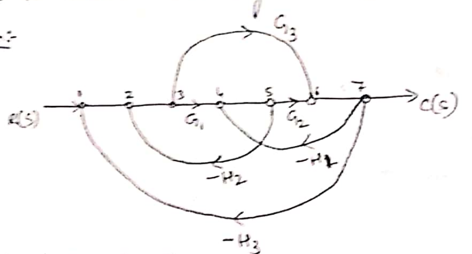
$$= \frac{G_1 G_2 G_3 G_4 + G_5 G_4 + G_2 G_4 G_5 H_1}{1 + G_2 H_1 + G_1 G_2 G_3 G_4 H_2 + G_5 G_4 H_2 + G_2 G_4 G_5 H_1 H_2}$$

Scanned by CamScanner



5] Draw the signal flow graph for the block diagram shown in fig. 5 find its control ratio using mason's gain formula.

Sol'n:-



No. of forward paths.

$P_1 = G_1 G_2$

$P_2 = G_3$

Loop gains.

$L_1 = -G_1 H_2$

$L_2 = -G_2 H_1$

$L_3 = -G_1 G_2 H_3$

$L_4 = G_3 H_1 H_2$

$L_5 = -G_3 H_3$

$L_6 = G_1 G_2 H_1 H_2$

Combination of 2 non touching loops = 0

" " 3 " " " = 0

$\Delta_1 = 1-(0)$, $\Delta_2 = 1$

$\Delta = 1-(L_1+L_2+L_3+L_4+L_5+L_6)$

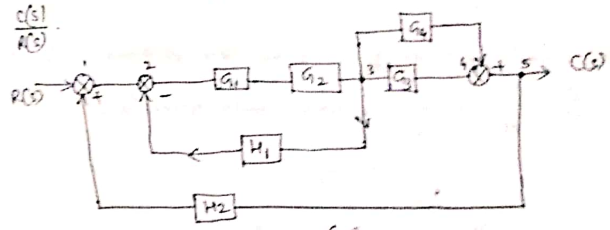
$= 1+G_1H_2+G_2H_1+G_1G_2H_3+G_3H_3-G_3H_1H_2-G_1G_2H_1H_2$

$TF = \frac{1}{\Delta}\sum_{k=1}^{n} \Delta_k P_k$

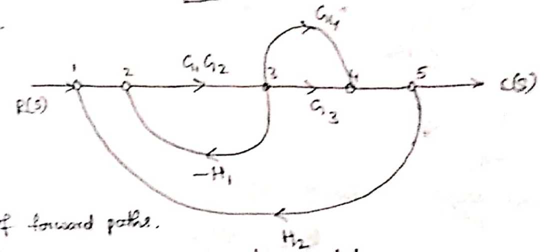
$= \frac{1}{\Delta}[\Delta_1 P_1 + \Delta_2 P_2]$

$= \frac{G_1 G_2 + G_3}{1+G_1H_2+G_2H_1+G_1G_2H_3+G_3H_3-G_3H_1H_2-G_1G_2H_1H_2}$

7] Draw the corresponding signal ~~program~~ flow graph of the given block diagram. find $\frac{C(s)}{R(s)}$



Sol'n:-



No. of forward paths.

$P_1 = G_1 G_2 G_3$

$P_2 = G_1 G_2 G_4$

Loop gains.

$L_1 = -G_1 G_2 H_1$

$L_2 = +G_1 G_2 G_3 H_2$

$L_3 = G_1 G_2 G_4 H_2$

Combination of 2 non touching loops = 0

" " 3 " " " = 0

$\Delta_1 = 1-(0)$, $\Delta_2 = 1-0$

$\Delta = 1-(L_1+L_2+L_3) - 0 + \dots$

$= 1+G_1G_2H_1 - G_1G_2G_3H_2 - G_1G_2G_4H_2$

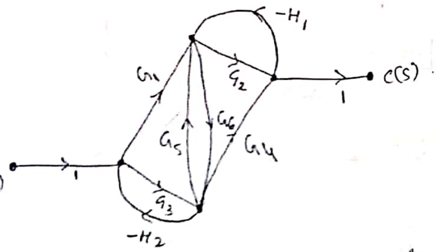
Scanned by CamScanner

$$TF = \frac{1}{\Delta}\sum_{k=1}^{2} \Delta_k P_k$$

$$= \frac{1}{\Delta}\left[\Delta_1 P_1 + \Delta_2 P_2\right]$$

$$= \frac{G_1G_2G_3 + G_1G_2G_4}{1 + G_1G_2H_1 - G_1G_2G_3H_2 - G_1G_2G_4H_2}$$

8] For the signal program of a given block diagram, determine transfer fn. $\frac{C(s)}{R(s)}$ using mason's gain formula.



Sol^n: No. of forward paths,

$P_1 = G_1G_2$

$P_2 = G_3G_4$

$P_3 = G_3G_5G_2$

$P_4 = G_1G_6G_4$

loop gain,

$L_1 = -G_3H_2$

$L_2 = -G_2H_1$

$L_3 = -G_4H_1G_6$

$L_4 = G_5G_6$

$L_5 = -G_1G_6H_2$

combination of 2 non touching loops = L_1L_2,

" 3 " " = 0

$\Delta_1 = 1$, $\Delta_2 = 1$, $\Delta_3 = 1$, $\Delta_4 = 1$

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5) + (L_1L_2)$$

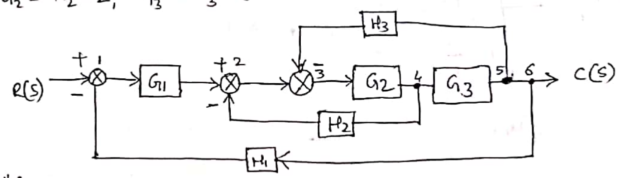
$$= 1 + G_3H_2 + G_2H_1 + G_4G_6H_1 - G_5G_6 + G_1G_6H_2 + G_3G_2H_1H_2$$

$$TF = \frac{1}{\Delta}\sum_{k=1}^{n} \Delta_k P_k$$

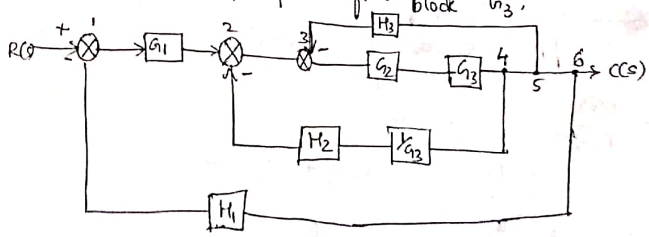
$$= \frac{1}{\Delta}\left[\Delta_1 P_1 + \Delta_2 P_2 + \Delta_3 P_3 + \Delta_4 P_4\right]$$

$$= \frac{G_1G_2 + G_3G_4 + G_2G_3G_5 + G_1G_4G_6}{1 + G_3H_2 + G_2H_1 + G_4G_6H_1 - G_5G_6 + G_1G_6H_2 + G_2G_3H_1H_2}$$

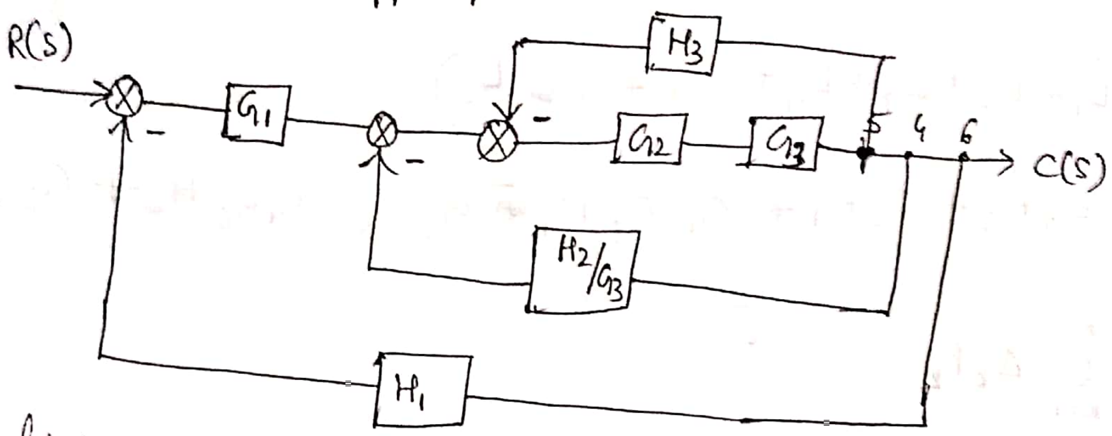
9] Reduce the block diagram shown in fig. & determine the transfer fn. using blockdiagram reduction technique. If $G_1 = H_1 = 1$, $G_2 = H_2 = 2$, $G_3 = H_3 = 3$



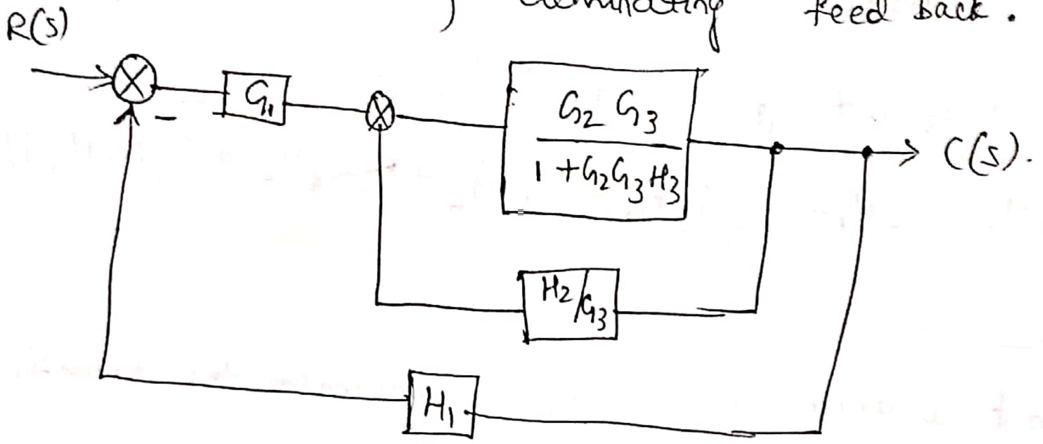
Sol^n:- Move the take off point after block G_3.

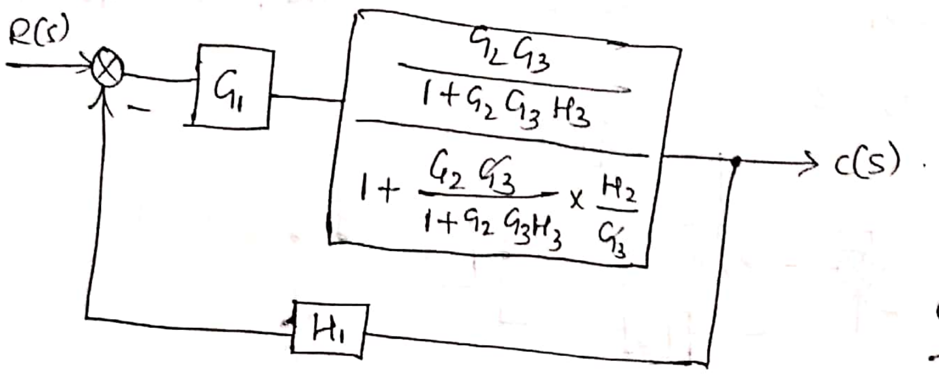


Interchanging take off points 4 & 5.



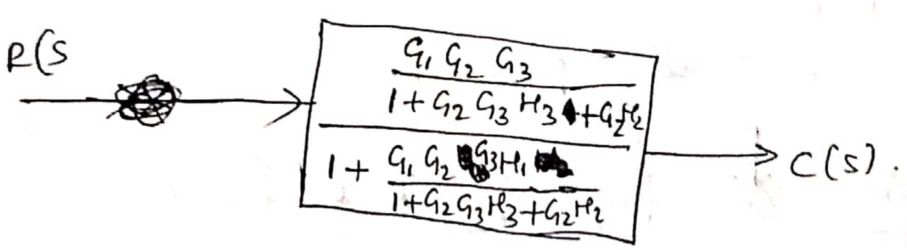
Combining series blocks & eleminating feed back.





$$\frac{\frac{G_1G_2G_3}{1+G_2G_3H_3}}{1+\frac{G_2H_2}{1+G_2G_3H_3}}$$

$$\Rightarrow \frac{G_1G_2G_3}{1+G_2G_3H_3+G_2H_2}$$



$$TF = \frac{C(S)}{R(S)} = \frac{G_1G_2G_3}{1+G_2G_3H_3+G_2H_2+G_1G_2G_3H_1}$$

$$= \frac{6}{1+18+4+6} = \frac{6}{29}$$

MODULE - 3 :

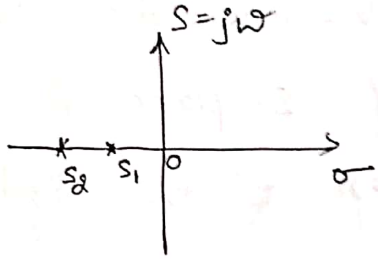
System Stability using R-H (Routh's Harwitz) criterion :-

For a control system designer main concern is whether the system is stable or unstable. Becoz stability is a very imp. characteristics of any control system. Almost every working system designed to be stable & unstable system is generally considered to be useless. Thus study of the system is stable or unstable is known as stability analysis.

Cond'n for stability :-

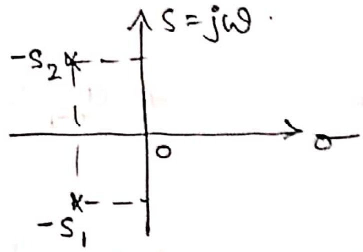
Nature of Roots — **Location of Roots** — **Stability Cond'n.**

1] Real, negative i.e. all the roots are in the left of s-plane.



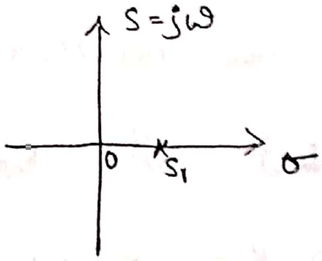
Stable.

2] Complex conjugate with -ve real part i.e. all the roots are in the left of the s-plane.



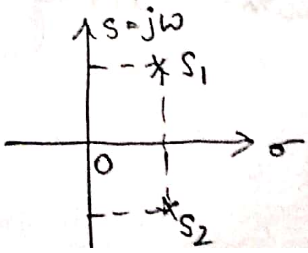
Stable

3] Real, positive i.e. any one root will lie on the right half of s-plane.



unstable.

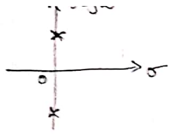
4] Complex conjugate with +ve real part, i.e. all the roots are in right of the s-plane.



unstable.

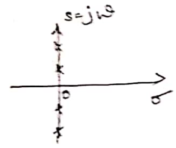
Scanned by CamScanner

5] Repeated pair roots on imaginary axis



unstable.

5] Non repeated pair root on imaginary axis.



marginally stable

Routh's Harwitz (R-H) criterion :-

R-H criterion is a algebraic method of determining the linear time invariant system. This criterion provides information provide whether the system is stable or not depending upon position of root of characteristic Eqn. whether they lies in the left half of s-plane or right half of s-plane.

The characteristic eqn. of the system can be written as,

$$a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_{n-1} s + a_n = 0$$

Note :-

R-H array also known as R-H stability criterion or R-H method.

Thus it states that the necessary & sufficient cond'n for system stability is that all the terms in the first column of the Routh's array must have same sign.

1] Determine the stability of the system for the characteristic eqn. given by $s^3 + 9s^2 + 26s + 24 = 0$.

Sol'n:-

s^3	1	26
s^2	9	24
s^1	$\frac{(9\times26)-(24\times1)}{9}$ $=23.33$	0
s^0	$\frac{(23.33\times24)-(9\times0)}{23.33}$ $=24$	0

Since there are no sign changes in first column of Routh's array & are +ve. Hence system is stable.

2] Investigate the stability of the system using Routh's array having following characteristic Eqn. $s^5 + 4s^4 + 12s^3 + 20s^2 + 30s + 100 = 0$

Sol'n:-

s^5	1	12	30
s^4	4	20	100
s^3	$\frac{48-20}{4} = 7$	$\frac{600-1200}{20} = -30$	0
s^2	$\frac{140+120}{7} = 37.14$	$\frac{-3000}{-30} = 100$	0
s^1	-48.84	0	0
s^0	100	0	0

Since from Routh's array it is found that in first column there are 2 sign changes i.e. +ve to −ve & −ve to +ve & hence system is unstable.

Scanned by CamScanner

Determine stability of system $s^6+3s^5+4s^4+6s^3+5s^2+3s+2=0$

Soln:

s^6	1	4	5	2
s^5	3	6	3	0
s^4	$\frac{12-6}{3}=2$	$\frac{15-3}{3}=4$	$\frac{6}{3}=2$	0
s^3	0	0	0	0
s^2				
s^1				
s^0				

s^6	1	4	5	2
s^5	3	6	3	0
s^4	2	4	2	0
s^3	2	8	0	0
s^2	8	8	0	0
s^1	2	2	0	
s^0				

$A(s) = 2s^4 + 4s^2 + 2$

$2s^4+4s^2+2=0$

$8s^3+8s$

Diff. w.r.t s.

$$\frac{d\,A(s)}{ds} = 8s^3 + 8s.$$

s^6	1	4	5	2
s^5	3	6	3	0
s^4	2	4	2	0
s^3	8	8	0	0
s^2	2	2	0	0
s^1	0	0	0	0
s^0				

$A'(s) = 2s^2+2$

Dif. w.r.t s,

$$\frac{d\,A'(s)}{ds} = 4s$$

s^6	1	4	5	2
s^5	3	6	3	0
s^4	2	4	2	0
s^3	8	8	0	0
s^2	2	2	0	0
s^1	4	0	0	0
s^0	2	0	0	0

Scanned by CamScanner

Let $A(s) = 0$.

$2s^4 + 4s^2 + 2 = 0$

put $s^2 = x$

$2x^2 + 4x + 2 = 0$

$2x^2 + 2x + 2x + 2 = 0$

$2x(x+1) + 2(x+1) = 0$

$2x + 2 = 0$, $x + 1 = 0$

$2x = -2$ $x = -1$

$x = -1$

$s^2 = -1$, $s^2 = -1$

$s = \pm i$, $s = \pm i$

∴ As there are repeated roots on imaginary axis, the system is unstable.

Determine stability of system for characteristic eqn.

$$s^5 + s^4 + 2s^3 + 2s^2 + 3s + 15 = 0$$

Soln:

s^5	1	2	3
s^4	1	2	15
s^3	~~0~~ 4	-24	0
s^2	1	15	0
s^1	-56	0	
s^0	15		

$s^4 + 2s^2 + 15 = 0$

$4s^3 + 4s = 0$

Replace S by $\frac{1}{x}$

$$\left(\frac{1}{x}\right)^5 + \left(\frac{1}{x}\right)^4 + 2\left(\frac{1}{x}\right)^3 + 2\left(\frac{1}{x}\right)^2 + 3\left(\frac{1}{x}\right) + 15 = 0$$

Taking LCM,

$$\frac{1 + x + 2x^2 + 2x^3 + 3x^4 + 15x^5}{x^5} = 0$$

$$15x^5 + 3x^4 + 2x^3 + 2x^2 + x + 1 = 0$$

x^5	15	2	1
x^4	3	2	1
x^3	-8	-4	0
x^2	0.5	+1	0
x^1	12	0	0
x^0	+1	0	0

Since there are 2 sign changes +ve to -ve & -ve to +ve.

Hence system unstable.

Determine the value of the K so that the system is stable for characteristic eqn. $s^4 + 5s^3 + 5s^2 + 4s + K = 0$

Soln:

s^4	1	5	K
s^3	5	4	0
s^2	$\frac{21}{5} = 4.25$	K	0
s^1	$\frac{16.8 - 5K}{4.25}$	0	0
s^0	K	0	0

$\boxed{K > 0}$

For stability, $K > 0$

$$\frac{16.8 - 5K}{4.25} > 0$$

$16.8 - 5K > 0$

$16.8 < 5K$

$K \leq \frac{16.8}{5}$

$K < 3.36$

Range of K will be $0 < K < 3.36$

Scanned by CamScanner

A system has a following characteristic eqn. Determine the value of K so that system is stable by R-H criterion.

$$2s^4+3s^3+4s^2+s+K=0$$

Sol'n: Given,

s^4	2	4	K
s^3	3	1	0
s^2	$\frac{10}{3}=3.33$	K	0
s^1	$\frac{3.33-3K}{3.33}$	0	0
s^0	K	0	0

For stability $K>0$

$$\frac{3.33-3K}{3.33}>0$$

$$3.33-3K>0$$

$$3.33<3K$$

$$K<1.11$$

Range of K will be $0<K<1.11$

Transient response Analysis:

Transient response is defined as the part of tot. time response which decays & dies out & goes to zero, over a period of time & as time becomes very large & after large interval of time. It is denoted by $y_i(t)$.

Mathematically it is defined as,

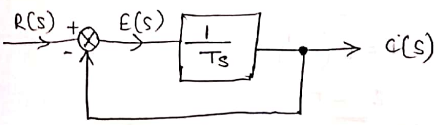
$$\lim_{t\to\infty} y_i(t)=0$$

Std. test i/p:-

Input	R(s)
1. Step	A/s
2. Ramp	A/s^2
3. Parabolic	A/s^3
4. Impulse	1

It is represented in laplace transform.

Transient response of 1st order system subjected to unit step i/p.



Consider the 1st order linear unity feedback control system whose block diagram is shown in fig.

The transfer fn. of system is given by,

$$\frac{C(s)}{R(s)}=\frac{G(s)}{1+G(s)H(s)}$$

Scanned by CamScanner

$$\frac{C(s)}{R(s)} = \frac{1/Ts}{1+\frac{1}{Ts}\times 1}$$

$$= \frac{1}{1+Ts}$$

O/p response, $C(s) = \frac{R(s)}{1+Ts}$

For unit step i/p, $R(s) = \frac{1}{s}$

$$C(s) = \frac{1}{s}\,\frac{1}{1+Ts}$$

Using partial frs,

$$\frac{1}{s}\,\frac{1}{1+Ts} = \frac{A}{s} + \frac{B}{1+Ts}$$

$$\frac{1}{s(1+Ts)} = \frac{A(1+Ts)+B(s)}{s(1+Ts)}$$

$$1 = A(1+Ts) + B(s)$$

put $s=0$,

$1 = A(1)$

$\Rightarrow A = 1$

$1+Ts = 0$

$s = -\frac{1}{T}$

$1 = B\left(-\frac{1}{T}\right)$

$B = -T$

$$\therefore\ C(s) = \frac{1}{s} - \frac{T}{1+Ts}$$

Taking Inverse Laplace transform,

$$L^{-1}[C(s)] = L^{-1}\left(\frac{1}{s}\right) - L^{-1}\left(\frac{T}{1+Ts}\right)$$

$$\boxed{c(t) = 1 - e^{-\frac{t}{T}}}$$

Error response, $e(t) = r(t) - c(t)$

$$= 1 - \left(1 - e^{-t/T}\right)$$

$$\boxed{e(t) = e^{-t/T}}$$

Steady state error, $e_{ss} = \lim_{t\to\infty} e(t)$

$$= \lim_{t\to\infty} e^{-t/T}$$

$$= 0$$

$$c(t) = 1 - e^{-t/T}$$

for $t = T$,

$$c(t) = 1 - e^{-T/T} = 1 - 0.3678$$

$$= 0.632 = 63.2\%$$

for $t = 2T$

$$c(t) = 1 - e^{-2T/T} = 0.8646$$

$$= 86.46\%$$

for $t = 3T$,

$$c(t) = 1 - e^{-3T/T} = 0.9502$$

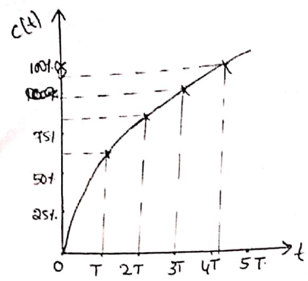
$$= 95.02\%$$

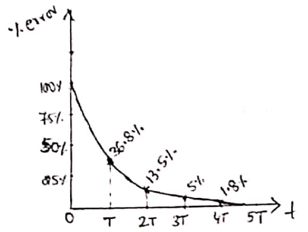
for $t = 4T$,

$$c(t) = 1 - e^{-4T/T} = 0.9816$$

$$= 98.16\%$$

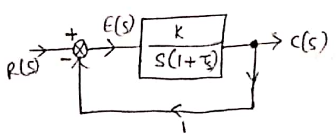
Scanned by CamScanner





Imp

Transient response of 2nd order system subjected to unit step input:



Consider a 2nd order unity feed back control system whose block diagram is shown in fig.

Transfer transfer, $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$

$$= \frac{\frac{K}{s(1+Ts)}}{1+\frac{K}{s(1+Ts)}\times 1}$$

$$= \frac{K}{s(1+Ts)+K} = \frac{K}{s+Ts^2+K}$$

$$\frac{C(s)}{R(s)} = \frac{K}{T\left(s^2+\frac{s}{T}+\frac{K}{T}\right)}$$

$$\frac{C(s)}{R(s)} = \frac{K/T}{s^2+\frac{s}{T}+\frac{K}{T}} \quad \text{---- (1)}$$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} \quad \text{---- (2)}$$

Eqn. (2) is called the transfer fn. of single degree of 2nd order damped free vibration system.

Comparing (1) & (2)

$\omega_n^2 = \frac{K}{T}$, $\omega_n = \sqrt{\frac{K}{T}}$

$2\xi\omega_n = \frac{1}{T} \Rightarrow 2\xi\sqrt{\frac{K}{T}} = \frac{1}{\sqrt{T}\sqrt{T}}$

$2\xi\sqrt{K} = \frac{1}{\sqrt{T}}$

$\Rightarrow \boxed{\xi = \frac{1}{2\sqrt{KT}}}$

The o/p response c(s) of a 2nd order system depends upon the value of ξ.

i.e. when, $\xi<1$ it is called under damped system.
when, $\xi>1$ it is called over damped system.
when, $\xi=1$ it is called critically damped system.
when, $\xi=0$ it is called undamped system.

Scanned by CamScanner

When the system is under damped ($\xi < 1$)

Let unit step i/p is applied to the 2nd order system. then

$R(S) = \frac{1}{S}$.

$$\frac{C(S)}{R(S)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$C(S) = \frac{R(S)\,\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$= \frac{1}{S}\,\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

Using partial fn's,

$$\frac{1}{S}\,\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} = \frac{A}{S} + \frac{Bs+D}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$\frac{\omega_n^2}{s(s^2+2\xi\omega_n s+\omega_n^2)} = \frac{A(s^2+2\xi\omega_n s+\omega_n^2)+(Bs+D)s}{s(s^2+2\xi\omega_n s+\omega_n^2)}$$

$$\omega_n^2 = A(s^2+2\xi\omega_n s+\omega_n^2)+(Bs^2+Ds)$$

put $s=0$,

$\omega_n^2 = A\omega_n^2+0$

$\boxed{A=1}$

comparing the co-eff. of s^2.

$0 = A+B$

$\Rightarrow \boxed{B=-1}$

Compare co-eff. of s.

$0 = A\,2\xi\omega_n + D$

$\Rightarrow \boxed{D = -2\xi\omega_n}$

$$C(s) = \frac{1}{s} + \frac{-s-2\xi\omega_n}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$= \frac{1}{s} + \frac{-s-\xi\omega_n-\xi\omega_n}{s^2+2\xi\omega_n s+\omega_n^2} = \frac{1}{s} - \frac{(s+\omega_n\xi)-\xi\omega_n}{s^2+2\xi\omega_n s+\omega_n^2+(\xi\omega_n)^2-(\xi\omega_n)^2}$$

$$C(S) = \frac{1}{S} - \frac{S+\xi\omega_n}{(S+2\xi\omega_n)^2+\omega_n^2(1-\xi^2)} - \frac{\xi\omega_n}{(S+2\xi\omega_n)^2+\omega_n^2(1-\xi^2)}$$

$$= \frac{1}{S} - \frac{S+\xi\omega_n}{(S+2\xi\omega_n)^2+\omega_d^2} - \frac{\xi\omega_n}{(S+2\xi\omega_n)^2+\omega_d^2}$$

ω_d – damped freq.

$\omega_d = \omega_n\sqrt{1-\xi^2}$

Taking Inverse laplace transform,

$$c(t) = 1 - e^{-\xi\omega_n t}\cos\omega_d t - \frac{e^{-\xi\omega_n t}\,\xi\omega_n \sin\omega_d t}{\omega_d}$$

$$= 1 - e^{-\xi\omega_n t}\left[\cos\omega_d t + \frac{\xi\omega_n}{\omega_d}\sin\omega_d t\right]$$

$$= 1 - e^{-\xi\omega_n t}\left[\cos\omega_d t + \frac{\xi\omega_n}{\omega_n\sqrt{1-\xi^2}}\sin\omega_d t\right]$$

$$= 1 - e^{-\xi\omega_n t}\left[\cos\omega_d t + \frac{\xi}{\sqrt{1-\xi^2}}\sin\omega_d t\right]$$

$$= 1 - \frac{e^{-\xi\omega_n t}}{\sin\theta}\left[\cos\omega_d t\,\sin\theta + \cos\theta\,\sin\omega_d t\right]$$

$$\boxed{c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sin(\omega_d t+\theta)\right]}$$

$$L^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at$$

$S \to S+b$

$$L^{-1}\left[\frac{s+b}{s^2+a^2}\right] = e^{-bt}\cos at$$

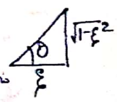
$$L^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{\sin at}{a}$$

$$L^{-1}\left[\frac{b}{(s+b)^2+a^2}\right] = \frac{e^{-bt}\,b\sin at}{a}$$

$\sin\theta = \sqrt{1-\xi^2}$

$\cos\theta = \xi$

ξ = damping factor/ratio



$$\tan\theta = \frac{\sqrt{1-\xi^2}}{\xi}$$

Expression for peak time (t_p) :-

It is time reqd. for the response (output) to reach the first peak.

The transient response of a 2^{nd} order system subjected to unit step input, is given by,

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$$

To obtain peak time,

$$c(t) = \frac{1}{\sqrt{1-\xi^2}} - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$$

$$\left.\frac{d\,c(t)}{dt}\right|_{t=t_p} = 0$$

Product rule (uv)

$$0 - \frac{1}{\sqrt{1-\xi^2}}\left[e^{-\xi\omega_n t_p}\cos(\omega_d t_p+\theta)\,\omega_d + \sin(\omega_d t_p+\theta)\, e^{-\xi\omega_n t_p} \cdot (-\xi\omega_n)\right] = 0$$

$$\frac{-e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}}\left[\cos(\omega_d t_p+\theta)\,\omega_n\sqrt{1-\xi^2} - \xi\omega_n \sin(\omega_d t_p+\theta)\right] = 0$$

$$\frac{-\omega_n e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}}\left[\cos(\omega_d t_p+\theta)\sin\theta - \cos\theta \sin(\omega_d t_p+\theta)\right] = 0$$

$$\frac{-\omega_n e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}}\left[\sin(\theta - \omega_d t_p - \theta)\right] = 0$$

$$\frac{\omega_n e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}} \sin\omega_d t_p = 0$$

$$\frac{\omega_n e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}} \neq 0, \quad \therefore \sin\omega_d t_p = 0$$

$$\omega_d t_p = \sin^{-1}(0)$$

$$\omega_d t_p = \pi, 2\pi, 3\pi, \quad \omega_d t_p = n\pi,$$

where, $n = 1, 2, 3 \ldots$

$$t_p = \frac{n\pi}{\omega_d}$$

$$\boxed{t_p = \frac{n\pi}{\omega_n\sqrt{1-\xi^2}}}$$

Expression for rise time (t_r) :-

It is the time reqd. for the response (o/p) to reach 100% of its final value in a very first time. The transient response of a 2^{nd} order system.

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$$

for rise time, $c(t)\big|_{t=t_r} = 1$

$$1 - \frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \theta) = 1.$$

$$-\frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \theta) = 0$$

$$\frac{-e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}} \neq 0, \quad \therefore \sin(\omega_d t_r + \theta) = 0$$

$$\Rightarrow \omega_d t_r + \theta = \sin^{-1}(0)$$

$$\omega_d t_r + \theta = \pi, 2\pi, 3\pi \ldots n\pi$$

$$\omega_d t_r + \theta = n\pi$$

$$t_r = \frac{n\pi - \theta}{\omega_d} \quad \text{where, } n = 1, 2, 3$$

$$\boxed{t_r = \frac{n\pi - \theta}{\omega_n\sqrt{1-\xi^2}}} \text{ secs}$$

Scanned by CamScanner

Expression for max. over shoot (M_p):-

It is the max. value of the response curve measured from the unique value. It is defined interms of %.

The transient response of a 2nd order system subjected to unit step i/p is given by.

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$$

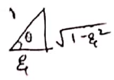
For max. ~~value~~ (or) over shoot $M_p = c(t)\big|_{t=t_p} - 1$

$$M_p = \cancel{1} - \frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}} \sin(\omega_d t_p + \theta) - \cancel{1} \qquad \boxed{t_p = \frac{\pi}{\omega_d}}$$

$$= \frac{-e^{-\xi\omega_n \times \frac{\pi}{\omega_n\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}} \sin\left(\omega_d \times \frac{\pi}{\omega_d} + \theta\right)$$

$$= \frac{-e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}} \sin(\pi + \theta)$$

$$= \frac{-e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}} (-\sin\theta)$$



$$= \frac{e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}} \sin\theta$$

$$= \frac{e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}} \sqrt{1-\xi^2}}{\sqrt{1-\xi^2}} \Rightarrow \boxed{M_p = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}} \times 100}$$

Expression for settling time (t_s):-

It is the time reqd. for the response (o/p) to reach & stay with in a range of ±2% to ±5% specified by a tollerance band of its final value.

The transient response of 2nd order system subjected to unit step i/p is given by,

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta) \quad \text{—①}$$

Eqn ① comprises of two parts.

1- exponential decaying. i.e. $\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}$

2- transient oscillator. i.e. $\sin(\omega_d t + \theta)$

The settling time can be obtained by considering the exponential time & neglecting all other term.

i) For 2% tolerance band.

$$\frac{e^{-\xi\omega_n t_s}}{\sqrt{1-\xi^2}} = 2\% = 0.02$$

For lower value of ξ, $\sqrt{1-\xi^2} \approx 1$

$$e^{-\xi\omega_n t_s} = 0.02$$

Taking ln on both sides.

$$-\xi\omega_n t_s = \ln(0.02)$$

$$-\xi\omega_n t_s = -4.$$

$$\Rightarrow \boxed{t_s = \frac{4}{\xi\omega_n}}$$

Scanned by CamScanner

2) For 5%. tollerance band,

$$\frac{e^{-\xi\omega_n t_s}}{\sqrt{1-\xi^2}} = 5\% = 0.05$$

For lower value of ξ, $\sqrt{1-\xi^2} \approx 1$

$$e^{-\xi\omega_n t_s} = 0.05$$

Taking ln on both sides,

$$-\xi\omega_n t_s = \ln(0.05)$$

$$-\xi\omega_n t_s = -3$$

$$\Rightarrow \boxed{t_s = \frac{3}{\xi\omega_n}} \text{ — sec}$$

1] A unity feedback system is characterized by an open loop tr. fn. $G(s) = \frac{10}{s^2+2s+6}$. Determine the following when the system is subjected to unit step i/p.

1] Undamped natural freq.

2] damping ratio

3] Peak over shoot

4] Peak time

5] Settling time.

$M_p = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}} \times 100$.

$t_p = \frac{n\pi}{\omega_n\sqrt{1-\xi^2}}$

$t_s = \frac{4}{\xi\omega_n}, \frac{3}{\xi\omega_n}$

$t_r = \frac{\pi-\theta}{\omega_n\sqrt{1-\xi^2}}$

Soln:- Given, $H(s) = 1$

characteristic eqn. $1 + G(s)H(s) = 0$

$$1 + \frac{10 \times 1}{s^2+2s+6} = 0$$

$$s^2+2s+16 = 0 \quad \text{— (1)}$$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0 \quad \text{— (2)}$$

Comparing (1) & (2),

$\omega_n^2 = 16 \Rightarrow \omega_n = 4$ rad/sec

$2\xi\omega_n = 2 \Rightarrow 2\xi \times 4 = 2$

$\xi = 0.25$

$$M_p = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}} \times 100$$

$= 44\%$.

$t_p = \frac{n\pi}{\omega_n\sqrt{1-\xi^2}}$, $n = 1$.

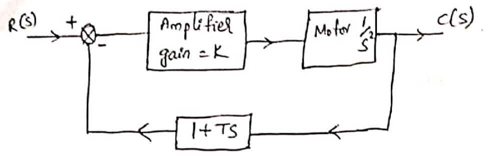
$t_p = 0.81$ sec

$t_s = \frac{4}{\xi\omega_n} = 4$ sec for 2%. tollerance

$t_s = \frac{3}{\xi\omega_n} = 3$ sec for 5%. tollerance.

Scanned by CamScanner

3) The block diagram of a simple servo mechanism is shown in fig. determine the value of K & T to gives an overshoot of 15% & a peak time of 3 sec for a unit step I/p. Also find the settling time for 2% & 5% tollerance band.



Soln:-

$G(s) = K \times \frac{1}{s^2} = \frac{K}{s^2}$

$H(s) = 1 + Ts$

$1 + G(s)H(s) = 0$

$1 + \frac{K}{s^2}(1+Ts) = 0$

$s^2 + KTs + K = 0$ —①

$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$ —②

Comparing ① & ②.

$\omega_n^2 = K \Rightarrow \boxed{\omega_n = \sqrt{K}}$

$2\xi\omega_n = KT \Rightarrow 2\xi\sqrt{K} = KT$

$\Rightarrow \boxed{\xi = \frac{\sqrt{K}\,T}{2}}$

$M_p = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}}$

$\xi =$

$t_p = \frac{n\pi}{\omega_n\sqrt{1-\xi^2}}$

$\omega_n =$

$M_p = 15\% = 0.15$

$M_p = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}}$

$\Rightarrow 0.15 = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}}$

$\Rightarrow \xi = 0.5169$

$t_p = \frac{n\pi}{\omega_n\sqrt{1-\xi^2}}$ $(\because n = 1)$.

$3 = \frac{1 \times \pi}{\omega_n\sqrt{1-0.5169^2}}$

$\Rightarrow \omega_n = 1.22$ rad/sec

W.K.T. $\omega_n = \sqrt{K} \Rightarrow K = 1.48$

Also, $\xi = \frac{\sqrt{K}\,T}{2} \Rightarrow T = 0.849$

$t_s = \frac{4}{\xi\omega_n} = 6.34$ sec.

$t_s = \frac{3}{\xi\omega_n} = 4.75$ sec

Scanned by CamScanner

3] The open loop transfer fn. of a unity feedback control system is given by. $G(s) = \frac{25}{s(s+5)}$.

1] Obtain max. over stroke

2] Peak time.

3] Rise time.

4] Settling time.

Sol'n:- Characteristic eqn.

$$1+G(s)\,H(s) = 0$$

$$1+\frac{25}{s(s+5)}\times 1 = 0$$

$$s^2+5s+25=0 \quad \text{—} \ ①$$

$$s^2+2\xi\omega_n s+\omega_n^2=0 \quad \text{—} \ ②$$

Comparing ① & ②,

$2\xi\omega_n=5$, $\omega_n^2=25 \Rightarrow \omega_n=5$.

$2\xi\times 5=5$

$\Rightarrow \xi=\frac{1}{2}=0.5$

1] Max. overshoot,

$$M_p = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}}\times 100$$

$$= e^{-\frac{\pi\times 0.5}{\sqrt{1-0.5^2}}}\times 100$$

$$= 16.30\,\%$$

2] Peak time,

$$t_p = \frac{n\pi}{\omega_n\sqrt{1-\xi^2}} = \frac{1\times\pi}{5\sqrt{1-0.5^2}}$$

$$= 0.7255 \text{ sec}$$

3] Rise time,

$$t_r = \frac{n\pi-\theta}{\omega_n\sqrt{1-\xi^2}}$$

$$= \frac{1\times\pi - 60\times\pi/180}{5\times\sqrt{1-0.5^2}}$$

$$= 0.4836 \text{ sec}$$

$\sin\theta = \sqrt{1-\xi^2}$

$\cos\theta = \xi$

$\tan\theta = \frac{\sqrt{1-\xi^2}}{\xi}$

$\theta = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)$

$= \tan^{-1}\frac{\sqrt{1-0.5^2}}{0.5}$

$= 60^\circ$

4] Settling time,

2%.

$$t_s = \frac{4}{\xi\omega_n} = 1.6$$

5%.

$$t_s = \frac{3}{\xi\omega_n} = 1.2$$

Root Locus plots:-

Introduction:-

The plot representing the root locus is always drawn is s-plane. s is a complex co-ordinate which is expressed as $\sigma+j\omega$, where σ is real axis (x-axis) & $j\omega$ represents imaginary axis. (y-axis). To construct the root locus open loop poles & a open loop zero's are necessary. These are obtained as follows.

Let $G(s)\,H(s)$ be the open loop transfer fn. in the control system. It can be expressed in pole-zero form.

Scanned by CamScanner

$$G(s)\,H(s) = \frac{K(s+z_1)(s+z_2)}{s^n(s+P_1)(s+P_2)}$$

where K is the loop gain or system gain,

z_1, z_2 are the open loop zero's.

P_1, P_2 are the open loop poles.

General Rules for construction of root locus plots :-

1] Obtain the open loop poles & open loop zero's in the given open loop transfer fn.

2] Identify the starting points & terminating point.

3] Determine the no. of asymptotes i.e. $q = p - z$

where, p - no. of poles.

z - no. of zero's.

4] Determine the angle of asymptotes i.e.

$$\theta = \frac{(2q+1)\,180^\circ}{p-z} \qquad \text{where } q = 0, 1, 2, \ldots$$

5] Locate the centroid i.e. $\sigma = \frac{\Sigma(\text{Real part of poles}) - (\Sigma \text{ Real part of zeros})}{p-z}$

6] Determine the break away point. The break away point can be found using characteristic eqn. $1 + G(s)\,H(s) = 0$

7] Determine the Intersection point with imaginary axis.

The intersection with imaginary axis can be found out using R-H criterion.

8] Determine the angle of departure from complex conjugate poles & angle of arrival from the complex conjugate zero's

$\phi_d = 180° - \phi$ where, $\phi = \Sigma\phi_p - \Sigma\phi_z$

$\phi_a = 180° + \phi$ where, $\phi = \Sigma\phi_p - \Sigma\phi_z$

1] Sketch the root locus for the -ve feedback system whose open loop transfer fn. is given by, $G(s)\,H(s) = \frac{K}{s(s+3)(s^2+3s+4.5)}$

Soln:

1] open loop poles = $0, -3, -1.5+j1.5, -1.5-j1.5$

open loop zero's = 0

2] Starting points = $0, -3, -1.5+j1.5, -1.5-j1.5$

Terminating points = $\infty, \infty, \infty, \infty$

3] No. of poles, $p = 4$

No. of zero, $z = 0$

No. of asymptotes, $q = p - z$

$= 4 - 0 = 4$

4] Angle of asymptotes,

$$\theta = \frac{(2q+1)\,180°}{p-z}, \quad q = 0, 1, 2, 3$$

$\theta_1 = 45°$

$\theta_2 = 135°$

$\theta_3 = 225°$

$\theta_4 = 315°$

Scanned by CamScanner

5]

centroid, $\sigma = \frac{(\Sigma \text{ Real part of poles}) - (\Sigma \text{ Real part zero's})}{P-Z}$

$= \frac{(0-3-1.5-1.5)-(0)}{4}$

$= \frac{-6}{4} = -\frac{3}{2} = -1.5$

6] Break away point.

$1+G(s)H(s) = 0$

$1+\frac{K}{S(S+3)(S^2+3S+4.5)} = 0$

$S(S+3)(S^2+3S+4.5)+K = 0$

$(S^2+3S)(S^2+3S+4.5)+K = 0$

$S^4+3S^3+4.5S^2+3S^3+9S^2+13.5S+K = 0$

$S^4+6S^3+13.5S^2+13.5S+K = 0 - ①$

$\Rightarrow K = -S^4-6S^3-13.5S^2-13.5S. - ②$

Diff. ② w.r.t K & equate to zero,

i.e. $\frac{dK}{ds} = 0$

$+4S^3+18S^2+27S+13.5 = 0$

$\Rightarrow S = -1.5, -1.5, -1.5$

Since -1.5 lies b/w 0 to -3. Hence it is a valid break away point.

7] Intersection with imaginary axis,

$S^4+6S^3+13.5S^2+13.5S+K = 0$

S^4	1	13.5	K
S^3	6	13.5	0
S^2	11.25	~~[illegible]~~ 6K	0
S^1	$\frac{151.875-6K}{11.25}$	0	0
S^0	6K.		

The roots of ~~imaginary~~ characteristic eqn may lies on imaginary axis. Hence element in S^1 row $= 0$

$\frac{151.875-6K}{11.25} = 0 \Rightarrow 6K = 25.3125$

Auxiliary eqn.

$11.25S^2+K = 0$

$\Rightarrow 11.25S^2 = -25.3125$

$\Rightarrow S = \pm 1.5j$

Since complex pole is present we need to find angle of departure.

$-1.5 \pm j1.5$

$\phi_{p1} = 135$, $\phi_{p2} = 90°$, $\phi_{p3} = 45$

Scanned by CamScanner

2] Construct a root locus for the open loop transfer fn.

$$G(s)\,H(s) = \frac{K(s+2)}{s(s+1)(s+8)}$$

Soln:-

1] Open loop poles $= 0, -1, -8$

Open loop zero's $= -2$

2] Starting points $= 0, -1, -8$

Terminating points $= -2, \infty, \infty$

3] No. of poles $= 3$

No. of zero's $= 1$

No. of ~~assymptotes~~ assymptotes, $q = P - Z$

$= 3 - 1$

$= 2$

4] Angle of assymptotes, $\theta = \dfrac{(2q+1)\,180^\circ}{P-Z}$, $q = 0, 1$.

$\theta_1 = 90^\circ$

$\theta_2 = 270^\circ$

5] Centroid, $\sigma = \dfrac{(\Sigma \text{ Real part of poles}) - (\Sigma \text{ real part zero's})}{P-Z}$

$= \dfrac{(0-1-8) - (-2)}{3-1}$

$= -3.5$

6] Breakaway point,

$$1 + G(s)\,H(s) = 0$$

$$1 + \frac{K(s+2)}{s(s+1)(s+8)} = 0$$

$$s(s+1)(s+8) + K(s+2) = 0$$

$$(s^2+s)(s+8) + k(s+2) = 0$$

$$s^3 + 8s^2 + s^2 + 8s + ks + 2k = 0$$

$$s^3 + 9s^2 + 8s + k(s+2) = 0 \quad \text{—(1)}$$

$$\Rightarrow \quad K = \frac{-s^3 - 9s^2 - 8s}{(s+2)} \quad \text{—(2)}$$

Diff. (2) w.r.t s & equate to zero,

i.e. $\dfrac{dk}{ds} = 0$

$$\frac{u}{v} = \frac{v\frac{du}{ds} - u\frac{dv}{ds}}{v^2}$$

$$\frac{(-3s^2 - 18s - 8)(s+2) + (s^3 + 9s^2 + 8s)(1)}{(s+2)^2} = 0$$

$$-3s^3 - 18s^2 - 8s - 6s^2 - 36s - 16 + s^3 + 9s^2 + 8s = 0$$

$$+2s^3 + 15s^2 + 36s + 16 = 0$$

$s = -0.569$, $-3.465 + 1.43j$, $-3.465 - 1.43j$

Scanned by CamScanner

Since s = −0.569 lies b/w 0 & −1. It is a valid breakaway point.

Angle condn:

Take, $s = -3.46 + j1.43$ ($\angle$ = Angle symbol)

$\angle G(s)H(s) = \pm 180^\circ$

$\angle G(s)H(s) \angle \dfrac{k(s+2)}{s(s+1)(s+8)}$

$\angle \dfrac{k(-3.46 + j1.43 + 2)}{(-3.46+j1.43)(-3.46+j1.43+1)(-3.46+j1.43+8)}$

$\angle \dfrac{k(-1.46+j1.43)}{(-3.46+j1.43)(-2.46+j1.43)(+4.54+j1.43)}$

$G(s)H(s) \angle \dfrac{0^\circ(135.59^\circ)}{(157.54^\circ)(149.83^\circ)(17.48^\circ)}$ $\longrightarrow$ $\boxed{\text{shift POL, POL }(-1.46, 1.43)}$

$\angle \dfrac{0(135.59^\circ)}{324.85^\circ} \Rightarrow G(s)H(s) = -189.26^\circ$

$-3.46+j1.43$ & $-3.46-j1.43$ is invalid breakaway pt.

Intersection with imaginary axis,

$s^3 + 9s^2 + 8s + k(s+2) = 0$

$s^3 + 9s^2 + 8s + ks + 2k = 0$

$s^3 + 9s^2 + (k+8)s + 2k = 0$

s^3	1	$k+8$	0
s^2	9	$2k$	0
s^1	$\frac{7k+72}{9}$	0	0
s^0	$2k$	0	0

The roots of the characteristic eqn. may lie on the imaginary axis. Since element in s^1 row = 0

$s^1 = 0$

$\dfrac{7k+72}{9} = 0$

$7k + 72 = 0 \Rightarrow k = -\dfrac{72}{7}$

$= -10.285$

Auxiliary eqn,

$A(s) = 9s^2 + 2k = 0$

$9s^2 = -2k$

$s^2 = \dfrac{-2k}{9}$

$s = \pm 1.51i$

Since there is no imaginary values the root locus does not intersect the imaginary axis.

Since there a no complex pole the angle of departure is not exist.

... root locus for the open loop transfer fn.

$$G(s)H(s) = \frac{K}{s(s+4)(s^2+4s+20)}$$

Sol:

1) open loop poles = $0, -4, -2+j4, -2-j4$

open loop zero's = Nil

2) Starting point = $0, -4, -2+j4, -2-j4$

Terminating points = ∞, ∞, ∞.

3) No. of poles = 4

No. of zero's = 0

No. of assymptotes, $q = p-z$

$= 4-0 = 4$

4) Angle of assymptotes,

$$\theta = \frac{(2q+1)180^\circ}{p-z}, \quad q = 0, 1, 2, 3$$

$\theta_1 = 45^\circ$

$\theta_2 = 135^\circ$

$\theta_3 = 225^\circ$

$\theta_4 = 315^\circ$

5) Centroid, $\sigma = \frac{(\Sigma \text{ real part of poles}) - (\Sigma \text{ real part of zeros})}{p-z}$

$$= \frac{(0-4-2-2) - 0}{4} = -2$$

6) Breakaway point,

$1 + G(s)H(s) = 0$

$$1 + \frac{k}{s(s+4)(s^2+4s+20)} = 0$$

$s(s+4)(s^2+4s+20) + k = 0$

$(s^2+4s)(s^2+4s+20) + k = 0$

$s^4 + 4s^3 + 20s^2 + 4s^3 + 16s^2 + 80s + k = 0$

$s^4 + 8s^3 + 36s^2 + 80s + k = 0$ — ①

$\Rightarrow k = -s^4 - 8s^3 - 36s^2 - 80s$ — ②

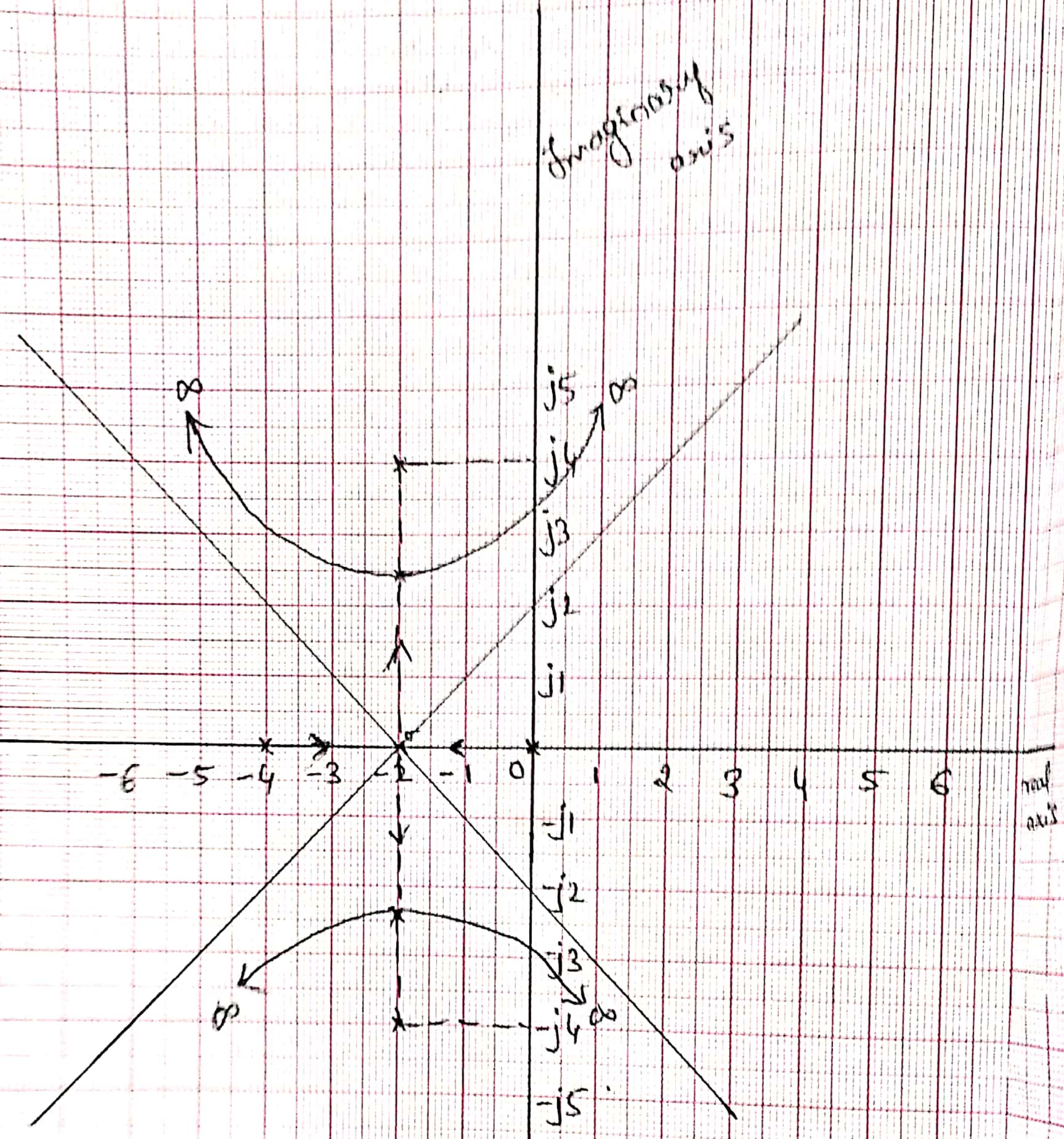
Diff. ② w.r.t s & equate to zero,

i.e. $\frac{dk}{ds} = 0$

$-4s^3 - 24s^2 - 72s - 80 = 0$

$\Rightarrow s = -2, \; -2+j2.44, \; -2-j2.44$

Since -2 lies b/w 0 & -4. It is a valid breakaway point.



Scanned by CamScanner

Angle cond'n :-

$$\angle G(s)H(s) = \pm 180^\circ$$

$$\angle G(s)H(s) = \frac{K}{s(s+4)(s+2-j4)(s+2+j4)}$$

Take, $s = -2+j2.44$

$$\angle G(s)H(s) = \angle \frac{K}{(-2+j2.44)(-2+j2.44+4)(-2+j2.44+2-j4)(-2+j2.44+2+j4)}$$

$$\angle G(s)H(s) = \angle \frac{K}{(-2+j2.44)(2+j2.44)(-1.56j)(6.44j)}$$

$$= \angle \frac{0^\circ}{(129.34^\circ)(50.65^\circ)(-90^\circ)(90^\circ)}$$

$$= \angle \frac{0}{180^\circ}$$

$$= 0 - 180^\circ = \underline{\underline{-180^\circ}}$$

$\therefore$ $-2+j2.44$ & $-2-j2.44$ is a valid breakaway point.

Intersection with imaginary axis,

$$s^4 + 8s^3 + 36s^2 + 80s + k = 0$$

s^4	1	36	k.
s^3	8	80	0
s^2	26	k	0
s^1	$\frac{2080-8k}{26}$	0	0
s^0	k	0	0

Root of characteristic eqn. may lie on imaginary axis, the element in s^1 row is zero,

$$\frac{2080-8k}{26} = 0 \Rightarrow \underline{\underline{k = 260}}.$$

Intersecting point on imaginary axis that corresponds to $k=260$ & determined from auxilliary eqn. by considering s^2 row

$$A(s) = 26s^2 + k = 0.$$

$$26s^2 = -k$$

$$26s^2 = -260$$

$$\Rightarrow 26s^2 + 260 = 0.$$

$$\Rightarrow s = \underline{\underline{\pm j3.16}}$$

Scanned by CamScanner

4] Draw the root locus for the open loop transfer fn.

$$G(s)\,H(s) = \frac{k}{s(s^2+5s+6)}$$

Sol'n:

1] open loop poles = 0, −2, −3

open loop zero's = Nil

2] Starting points = 0, −2, −3

Terminating points = ∞, ∞, ∞

3] No. of poles = 3

No. of zero's = 0

No. of assymptotes, $q = p - z$

$= 3$

4] Angle of assymptotes,

$$\theta = \frac{(2q+1)180^\circ}{p-z}, \quad q = 0, 1, 2$$

$\theta_1 = 60^\circ$

$\theta_2 = 180^\circ$

$\theta_3 = 300^\circ$

5] Centroid,

$$\sigma = \frac{(\Sigma \text{ real parts of poles}) - (\Sigma \text{ real part zero's})}{p-z}$$

$$= \frac{0-2-3}{3} = -1.667$$

6] Break away point.

$$1 + G(s)\,H(s) = 0$$

$$1 + \frac{k}{s(s^2+5s+6)} = 0$$

$$s(s^2+5s+6) + k = 0$$

$$s^3+5s^2+6s+k = 0 \quad —①$$

$$\Rightarrow k = -s^3-5s^2-6s \quad —②$$

diff. ② w.r.t s & equate to zero,

$$\frac{dk}{ds} = 0$$

$$\Rightarrow -3s^2-10s-6 = 0$$

$$\Rightarrow s = -0.784, \; -2.548$$

Since −0.784 ~~−2.548~~ lies b/w 0 & −2. It is valid break away point.

7] Intersection with imaginary.

$$s^3+5s^2+6s+k = 0$$

s^3	1	6	0
s^2	5	k	0
s^1	$\frac{30-k}{5}$	0	0
s^0	k	0	0

Roots of characteristic eqn may lies on imaginary axis. the element in s^1 row is zero.

$$\frac{30-k}{5} = 0 \Rightarrow k = 30$$

Intersecting point on imaginary axis that corresponds k = 30 & determined from auxiliary eqn by considering s^2 row

$$A(s) = 5s^2 + k = 0$$

$$5s^2 = -30$$

$$\Rightarrow s^2 = -6 \Rightarrow s = \pm j\,2.44$$

Scanned by CamScanner

5] Draw the root locus for the open loop transfer fn, $G(s)H(s) = \frac{K}{s(s+3)(s+5)}$ & also obtain the value of K, when $\xi = 0.6$

Soln:-

1] open loop poles = 0, -3, -5

open loop zero's = Nil

2] Starting points - 0, -3, -5

Terminating points - ∞, ∞, ∞

3] No. of poles = 3

No. of zero's = 0

No. of asymptotes, $q = p - z$

$= 3$

4] Angle of asymptotes,

$$\theta = \frac{(2q+1)180^\circ}{p-z}, \quad q = 0, 1, 2$$

$\theta_1 = 60^\circ$

$\theta_2 = 180^\circ$

$\theta_3 = 300^\circ$

5] Centroid,

$$\sigma = \frac{(\Sigma \text{ real parts of poles}) - (\Sigma \text{ real part of zero's})}{P - Z}$$

$$= \frac{0-3-5}{3} = -2.667$$

6] Breakaway point,

$1 + G(s)H(s) = 0$

$$1 + \frac{K}{s(s+3)(s+5)} = 0$$

$s(s+3)(s+5) + k = 0$

$(s^2+3s)(s+5) + k = 0$

$s^3 + 5s^2 + 3s^2 + 15s + k = 0$

$s^3 + 8s^2 + 15s + k = 0$ — ①

$\Rightarrow k = -s^3 - 8s^2 - 15s$ — ②

diff. ② w.r.t s & equate to zero,

$\frac{dk}{ds} = 0$

$-3s^2 - 16s - 15 = 0$

$\Rightarrow s = -1.21, \ -4.11$ (0 to -3 & -5 to ∞)

Since -1.21 lies b/w 0 to -3. It is valid break away point

7] Intersection with imaginary,

$s^3 + 8s^2 + 15s + k = 0$

s^3	1	15	0
s^2	8	k	0
s^1	$\frac{120-k}{8}$	0	0
s^0	k	0	0

Roots of characteristic eqn may lies on imaginary axis. the element in s^1 row is zero.

$\frac{120-k}{8} = 0 \Rightarrow k = 120$

Intersection point on imaginary axis that corresponds k = 120 & determined from auxilary eqn. by considering s^2 row.

$A(s) = 8s^2 + K = 0$

$8s^2 = -120$

$\Rightarrow s^2 = -15 \Rightarrow s = \pm j\,3.87$

Scanned by CamScanner

$\cos\theta = \xi \Rightarrow \theta = \cos^{-1}(0.6)$

$= 53.13^\circ$

K for $\xi = 0.6$ is obtained from magnitude cond'n.

$P = -1 + j1.3$

$|G(s)H(s)|_P = 1$

$$\left|\frac{K}{S(S+3)(S+5)}\right|_{-1+j1.3} = 1$$

$$\frac{K}{|(-1+j1.3)||(-1+j1.3+3)||-1+j1.3+5|} = 1$$

$$\frac{K}{1.64 \times 2.38 \times 4.20} = 1$$

$\Rightarrow K = 16.39$

6] Draw the root locus for the open loop transfer fn. $G(s) = \frac{K}{S(S+2)(S^2+2S+2)}$

Sketch the root locus & determine,

a] Limiting value of 'K' for stability.

b] Value of 'K' so that damping ratio $= 0.707$

Sol'n:-

1] open loop poles $= 0, -2, -1+j, -1-j$

open loop zero's = Nil

2] Starting points $= 0, -2, -1+j, -1-j$

Terminating points $= \infty, \infty, \infty, \infty$

3] No. of poles $= 4$

No. of zero's $= 0$

No. of assymptotes, $q = P - Z$

$= 4$

4] Angle of assymptotes.

$$\theta = \frac{(2q+1)180}{P-Z}, \quad q = 0, 1, 2, 3$$

$\theta_1 = 45^\circ$

$\theta_2 = 135^\circ$

$\theta_3 = 225^\circ$

$\theta_4 = 315^\circ$

5] Centroid,

$$\sigma = \frac{(\Sigma \text{ real parts of poles}) - (\Sigma \text{ real part zero's})}{P-Z}$$

$$= \frac{(0-2-1-1) - 0}{4}$$

$= -1$

6] Breakaway point,

$1 + G(s)H(s) = 0$

$$1 + \frac{K}{S(S+2)(S^2+2S+2)} = 0$$

$S(S+2)(S^2+2S+2) + K = 0$

$(S^2+2S)(S^2+2S+2) + K = 0$

$S^4 + 2S^3 + 2S^2 + 2S^3 + 4S^2 + 4S + K = 0$

$S^4 + 4S^3 + 6S^2 + 4S + K = 0 \Rightarrow K = -S^4 - 4S^3 - 6S^2 - 4S$ —— ①

Scanned by CamScanner

diff. ① w.r.t s & equate to zero,

$$\frac{dk}{ds} = 0 .$$

$$-4s^3 - 12s^2 - 12s - 4 = 0$$

$$4s^3 + 12s^2 + 12s + 4 = 0$$

$$\Rightarrow s = -1 .$$

Intersection with imaginary axis,

$$s^4 + 4s^3 + 6s^2 + 4s + k = 0$$

s^4	1	6	k
s^3	4	4	0
s^2	5	k	0
s^1	$\frac{20-4k}{5}$	0	
s^0	k	0	0

Roots of characteristic eqn. may lie on imaginary axis, the element in s^1 row is zero,

$$\frac{20-4k}{5} = 0 \Rightarrow 20 - 4k = 0$$

$$\Rightarrow 20 = 4k \Rightarrow k = 5$$

Intersecting point on imaginary axis that corresponds to k = 5 is determined from auxillary eqn. by considering s^2 row,

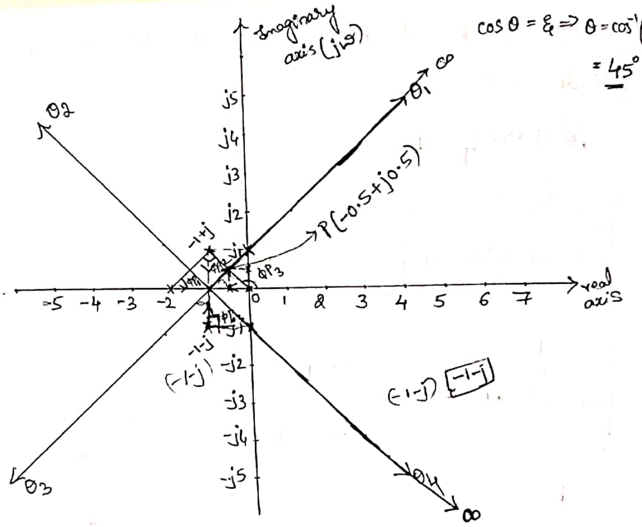
$$A(s) = 5s^2 + k = 0$$

$$\Rightarrow 5s^2 = -5$$

$$\Rightarrow s = \pm j$$

$\cos\theta = \xi \Rightarrow \theta = \cos^{-1}(0.707)$

$= 45^\circ$



(-1-j) $\boxed{-1-j}$

Angle of departure (ϕ_d)

$\phi_{p1} = 45^\circ$

$\phi_{p2} = 90^\circ$

$\phi_{p3} = 135$

$\Sigma\phi_p = \phi_{p1} + \phi_{p2} + \phi_{p3}$

$= 270^\circ$

$\Sigma\phi_z = 0$

$\phi = \Sigma\phi_p - \Sigma\phi_z$

$= 270^\circ$

$\phi_d = 180^\circ - \phi$

$= -90$ at $-1+j$

$= 90$ at $-1-j$

Scanned by CamScanner

The value of k at $p = -0.5 + j0.5$ is obtained from magnitude cond'n.

$$|G(s)\,H(s)|_p = 1$$

$$\left| \frac{k}{s(s+2)(s^2+2s+2)} \right|_{-0.5+j0.5} = 1$$

$$\left| \frac{k}{s(s+2)(s+1-j)(s+1+j)} \right|_{-0.5+j0.5} = 1.$$

$$\frac{k}{(-0.5+j0.5)(-0.5+j0.5+2)(-0.5+j0.5+1-j)(-0.5+j0.5+1+j)} = 1$$

$$\frac{k}{(-0.5+j0.5)(1.5+j0.5)(0.5-0.5j)(0.5+1.5j)} = 1.$$

$$\frac{k}{0.707 \times 1.581 \times 0.707 \times 1.581} = 1$$

$$\underline{\underline{k = 1.25}}$$

Scanned by CamScanner

MODULE-4:-

Bode plots

Logarithmic plot or bode plot :-

Representation of variation in the magnitude & phase angle against a i/p frequency in the logarithmic scale is known as ~~logarithmic or scale~~ bode plot.

The logarithmic plot are of two types.

1] Magnitude plot :-

It is a plot in which magnitude of $G(j\omega)H(j\omega)$ is expressed in decibells plotted against frequency 'ω' in logarithmic scale.

Magnitude in $dB = 20\log_{10}|G(j\omega)H(j\omega)|$

2] Phase angle plot :-

It is a plot in which phase angle ϕ is expressed in degrees against freq. 'ω' in logarithmic scale.

phase angle, $\phi = \angle G(j\omega)H(j\omega)$

General rules for constructing bode plot :-

Factor	Nature of bode plot	Phase angle
1] constant 'K'	Straight line of $20\log_{10}K$	0° if K is +ve 180° if K is -ve.
2] Pole at the origin ($1/s$)	Straight line of -20dB/decade intersecting ($\omega = 1$ rad/sec)	-90°

Scanned by CamScanner

3] Zero at the origin (S)	Straight line of 20dB/decade intersecting ($\omega = 1$ rad/sec).	90°
4] Simple pole $\left[\frac{1}{1+ST}\right]$	Straight line of 0dB till corner freq. (ω_c) & after that straight line of slope. -20dB/decade	$-\tan^{-1}(\omega T)$
5] Simple zero $(1+ST)$	Straight line of 0dB till corner freq. (ω_c) & after that straight line of slope $+20$ dB/decade.	$\tan^{-1}(\omega T)$
6] Quadratic pole $\dfrac{1}{1+\frac{2\xi s}{\omega_n}+\frac{s^2}{\omega_n^2}}$	Straight line of 0dB till corner freq. (ω_c) & after that straight line of slope -40 dB/decade	$-\tan^{-1}\left[\dfrac{\frac{2\xi\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right]$
7] Quadratic zero, $1+\frac{2\xi s}{\omega_n}+\frac{s^2}{\omega_n^2}$	Straight line of 0 dB till ω_c & after that straight line of slope 40dB/decade	$\tan^{-1}\left[\dfrac{\frac{2\xi\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right]$

1] A unity feedback system has $G(s) = \dfrac{80}{S(S+2)(S+20)}$. i] Draw the bode plot. ii] If the phase cross over occurs at $\omega = 6.35$ rad/sec find the gain margin.

Sol^n:- Given,

$$G(s) = \frac{80}{s(s+2)(s+20)}$$

$$= \frac{80}{S\times 2\left(1+\frac{S}{2}\right)\times 20\left(1+\frac{S}{20}\right)} \quad \rightarrow \text{in the form, } 1+ST.$$

$$= \frac{2}{S(1+0.5S)(1+0.05S)}$$

Simple pole $= \dfrac{1}{1+0.5S} = \dfrac{1}{1+ST}$

$T = 0.5$

$\omega_{c1} = \dfrac{1}{T} = \dfrac{1}{0.5} = \underline{2 \text{ rad/sec}}$

Simple pole $= \dfrac{1}{1+0.05S} = \dfrac{1}{1+ST}$

$T = 0.05$

$\omega_{c2} = \dfrac{1}{T} = \dfrac{1}{0.05} = \underline{20 \text{ rad/sec}}$

Factor	Nature of Bode plot	Resultant (dB/decade)
1] Constant $K=2$	Straight line of $20\log_{10} K = 6$dB	—
] Pole at the origin. $\left(\frac{1}{S}\right)$	Straight line of -20dB/decade intersecting. {$\omega = 1$ rad/sec}	-20. $0 < \omega < 2$.

3] Simple pole $= \frac{1}{1+0.5S}$ — Straight line of 0dB till $\omega_{c_1} = 2$ rad/sec & after that straightline of slope -20 dB/decade — $-20-20 = -40$, $2 < \omega < 20$

5] Simple pole $= \frac{1}{1+0.05S}$ — Straight line 0dB till $\omega_c = 20$ rad/sec & other that straight line of slope -20 dB/decade — $-40-20 = -60$, $20 < \omega < \infty$.

Frequency ω (rad/sec)	const. $K=2$	Pole at the origin $= \frac{1}{S}$	Simple pole $\frac{1}{1+0.5S}$ $-\tan^{-1}\omega T$, $T=0.5$	Simple pole $\frac{1}{1+0.05S}$ $-\tan^{-1}\omega T$, $T=0.05$	Resultant phase angle ϕ (deg)
0.1	0°	-90	-2.862	-0.286	-93.148
0.2	0	-90	-5.71	-0.572	-96.282
0.5	0	-90	-14.03	-1.43	-105.46
1	0	-90	-26.56	-2.86	-119.42
2	0	-90	-45	-5.71	-140.71
5	0	-90	-68.19	-14.03	-172.22
10	0	-90	-78.69	-26.56	-195.25
20	0	-90	-84.28	-45	-219.28
50	0	-90	-87.70	-68.19	-245.89
100	0	-90	-88.85	-78.69	-257.54

] Sketch a bode plot for transfer fn. $G(S) = \frac{KS^2}{(1+0.02S)(1+0.2S)}$ & determine value of K for the phase crossover freq. of 5 rad/sec.

Soln:

Simple pole $= \frac{1}{1+0.02S}$

$T = 0.02$

$\omega_{c_1} = \frac{1}{T} = \frac{1}{0.02} = 50$ rad/sec

$\frac{+5(1+0.2S)}{S(S^2+15S+100)}$

Simple pole $= \frac{1}{1+0.2S}$

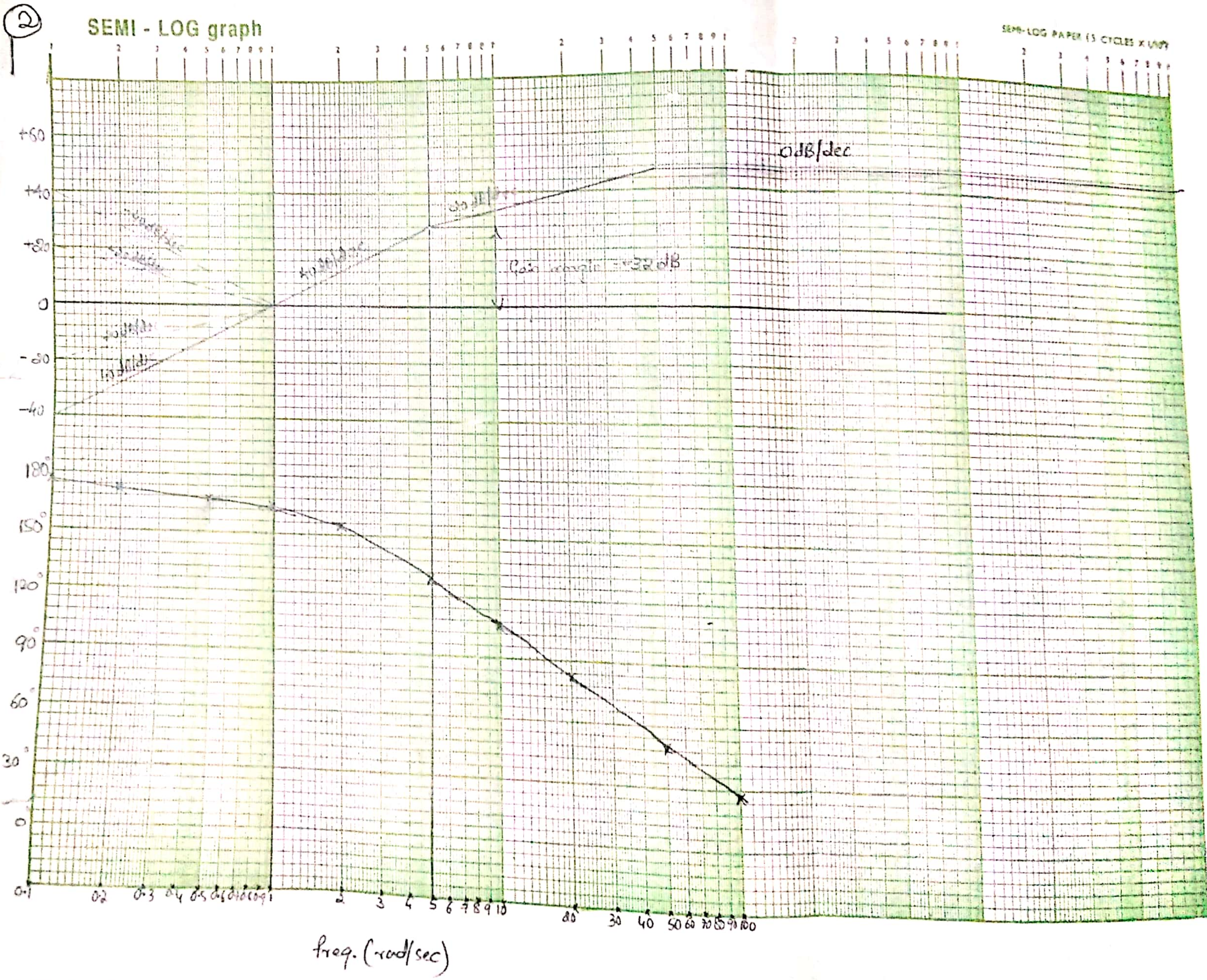
$T = 0.2$

$\omega_{c_2} = \frac{1}{T} = \frac{1}{0.2} = 5$ rad/sec

Factor	Nature of bode plot	Resultant state (dB/decade)
Const. K	straight line of $20\log_e K$	—
Two zero's at the origin S^2	Straight line of $2\times 20 = 40$dB intersecting $\omega = 1$ rad/sec	40, $0<\omega<5$
Simple pole $\frac{1}{1+0.2S}$	Straight line of 0 dB till $\omega_{c_2} = 5$ rad/sec & after that st. lines of slope -20 dB/decade	$40-20 = 20$, $5<\omega<50$
Simple pole $\frac{1}{1+0.02S}$	St. line of 0 dB till $\omega_{c_1} = 50$ rad/sec & after that st. lines of slope -20 dB/decade	$20-20 = 0$, $50<\omega<\infty$

Scanned by CamScanner

②



Scanned by CamScanner

Frequency ω (rad/sec)	Two zero's at the origin, s^2	Simple pole $\frac{1}{1+0.02s}$ $-\tan^{-1}(\omega T)$	Simple pole $\frac{1}{1+0.02s}$	Resultant pole. (ϕ) deg
0.1	180°	-1.145	-0.114	178.741
0.2	180°	-2.29	-0.22	177.49
0.5	180	-5.71	-0.572	173.718
1	180	-11.30	-1.145	167.55
2	180	-21.80	-2.29	155.91
5	180	-45	-5.71	129.29
10	180	-63.43	-11.30	105.27
20	180	-75.96	-21.80	82.24
50	180	-84.28	-45	50.72
100	180	-87.13	-63.43	29.44

$20\log_{10} K$ = Gain margin,

$20\log_{10} K = -32$

$\log_{10} K = \frac{-32}{20}$

$\Rightarrow K = 0.025$

3] Sketch the bode plot for transfer fn $G(s)H(s) = \frac{300(s^2+2s+4)}{s(s+10)(s+20)}$ & also find phase margin.

Soln:

Given. $G(s)H(s) = \frac{300(s^2+2s+4)}{s(s+10)(s+20)}$

$\Rightarrow 1 + \frac{2\xi}{\omega_n}s + \frac{s^2}{\omega_n^2}$

$\Rightarrow 1 + sT.$

$= \frac{300\times 4(1+0.5s+0.25s^2)}{s\times 10(1+0.1s)\times 20(1+0.05s)}$

$= \frac{1200(1+0.5s+0.25s^2)}{200s(1+0.1s)(1+0.05s)}$

$= \frac{6(1+0.5s+0.25s^2)}{s(1+0.1s)(1+0.05s)}$

Quadratic zero $= 1+0.5s+0.25s^2$

$= 1 + \frac{2\xi}{\omega_n}s + \frac{s^2}{\omega_n^2}$

$\frac{1}{\omega_n^2} = 0.25 \Rightarrow \omega_n^2 = 4$

$\omega_n = 2 = \omega_{c1}$

$\omega_{c1} = 2$ rad/sec

$\frac{2\xi}{\omega_n} = 0.5$

$2\xi = 0.5\times 2$

$\xi = 0.5$

Simple pole $= \frac{1}{1+0.1s}$.

$T = 0.1$

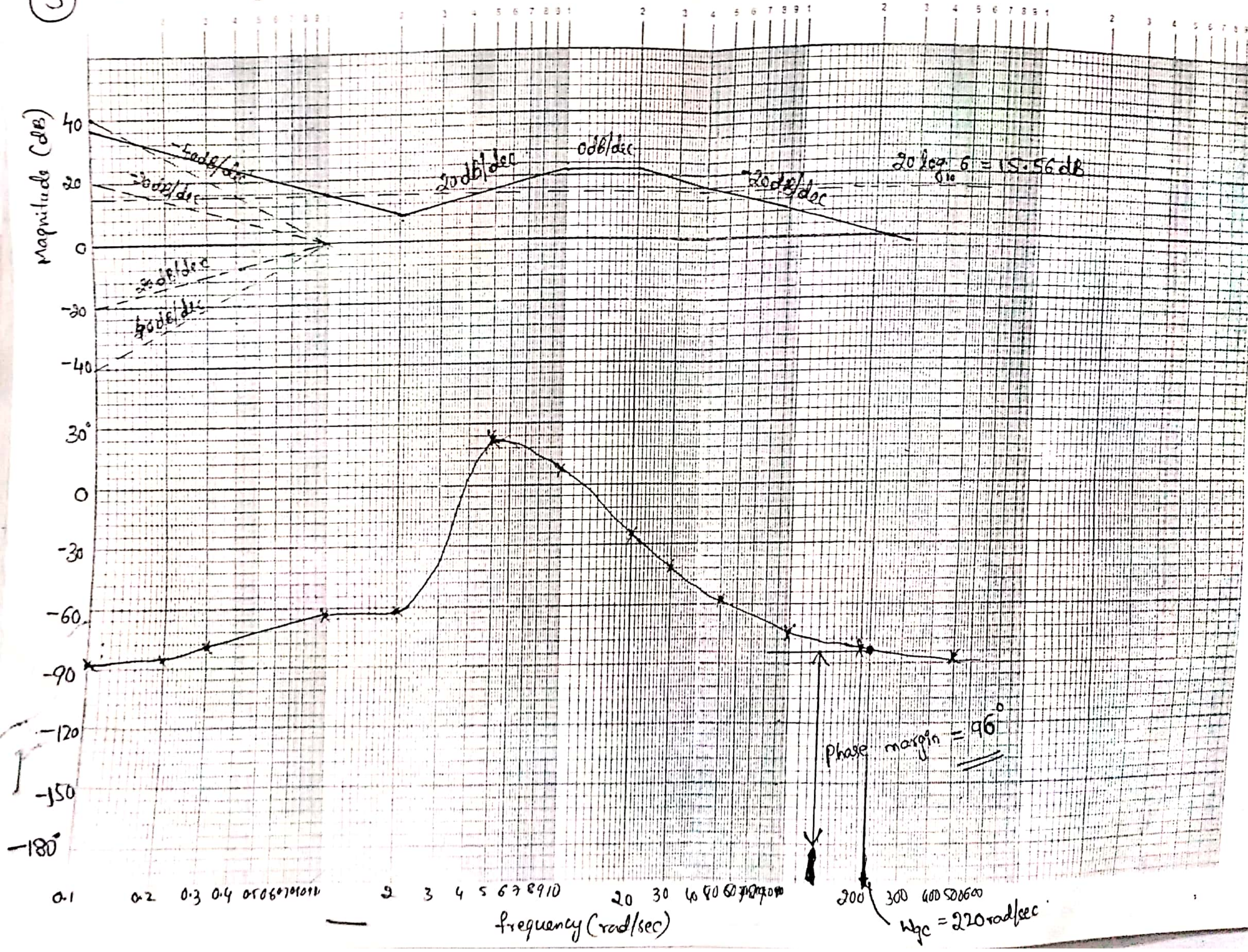
$\omega_{c2} = \frac{1}{T} = \frac{1}{0.1} = 10$ rad/sec

Simple pole $= \frac{1}{1+0.05s}$

$\omega_{c3} = \frac{1}{T} = \frac{1}{0.05} = 20$ rad/sec

Scanned by CamScanner

(3) SEMI - LOG graph



Scanned by CamScanner

Factor	Nature of bode plot	Resultant Slope (dB/dec)
1] const. $K=6$	St. line of $20\log_{10} K$ $\Rightarrow$ 15.563	—
2] Pole at origin $=\frac{1}{S}$	St. line of -20 dB/dec intersecting $\omega = 1$ rad/sec	-20 $0<\omega<2$
3] Quadratic zero $(1+0.5s+0.25s^2)$	St. line of 0 dB/dec till $\omega_q = 2$ rad/sec & after that st. line of slope 40 dB/dec	$-20+40=20$ $2<\omega<10$
4] Simple pole. $\frac{1}{1+0.1s}$	St. line of 0 dB till $\omega_{c_1} = 10$ rad/sec & after that st. line of slope -20 dB/dec	$20-20=0$ $10<\omega<20$
5] Simple pole. $\frac{1}{1+0.05s}$	St. line of 0 dB till $\omega_{c_2} = 20$ rad/sec & after that st. line of slope -20 dB/dec	$0-20=-20$ $20<\omega<\infty$

Frequency (rad/sec)	constant $k=6$	Pole at the origin $\frac{1}{s}$	Quadratic zero $(1+0.5s+0.25s^2)$ $\tan^{-1}\left[\frac{2\xi\frac{\omega}{\omega_q}}{1-\left(\frac{\omega}{\omega_q}\right)^2}\right]$	Simple pole $\frac{1}{1+0.1s}$ $-\tan^{-1}0.1\omega$	Simple pole $\frac{1}{1+0.05s}$ $-\tan^{-1}0.05\omega$	Resultant phase angle (ϕ) deg.
0.1	0°	$-90°$	2.869	-0.572	-0.286	-87.989
0.2	"	"	5.76	-1.1457	-0.572	-85.95
0.5	"	"	14.93	-2.86	-1.43	-79.36
1	"	"	33.69	-5.71	-2.86	-64.88
2	"	"	90°	-11.30	-5.71	-62.01
5	"	"	$-25.46+180 = 154.53$	-26.56	-14.03	23.94
10	"	"	168.24	-45	-26.56	6.68
20	"	"	174.24	-63.43	-45	-24.19
30	"	"	176.17	-71.56	-56.30	-41.69
50	"	"	177.71	-78.69	-68.19	-59.17
100	"	"	178.86	-84.28	-78.69	-74.11
200	"	"	179.43	-87.13	-84.28	-81.98
500	"	"	179.78	-88.85	-87.70	-86.77

$\tan 90 = \frac{\sin 90}{\cos 90} = \frac{1}{0} = \infty$

$\tan^{-1}\infty = 90°$

4]

5] Draw the bode plot for following T.F & determine gain margin, phase margin, gain cross over freq & phase cross over freq.

Soln:-

$$G(s) = \frac{10.5}{(s+0.2)(s+0.8)(s+10)}$$

$$= \frac{10.5}{0.2\left(1+\frac{s}{0.2}\right) \times 0.8\left(1+\frac{s}{0.8}\right) \times 10\left(1+\frac{s}{10}\right)}$$

$$= \frac{6.5625}{(1+5s)(1+1.25s)(1+0.1s)}$$

Simple pole $= \frac{1}{1+5s}$, $T=5$

$$\omega_{c_1} = \frac{1}{T} = \frac{1}{5} = 0.2 \text{ rad/sec}$$

Simple pole $= \frac{1}{1+1.25s}$, $T=1.25$

$$\omega_{c_2} = \frac{1}{T} = \frac{1}{1.25} = 0.8 \text{ rad/sec}$$

Simple pole $= \frac{1}{1+0.1s}$, $T=0.1$

$$\omega_{c_3} = \frac{1}{T} = \frac{1}{0.1} = 10 \text{ rad/sec}$$

Scanned by CamScanner

Factor	Nature of bode plot	Resultant slope (dB/dec)
1] Const. $k = 6.5625$	St. line of $20\log_{10} K$ $\Rightarrow$ ~~[illegible]~~ 16.3413	—
2] Simple pole, $\frac{1}{1+5S}$	St. line of 0dB till $W_{c1} = 0.2$ rad/sec & after that st. line of slope −20dB/dec.	−20 $0.2 < \omega < 0.8$.
3] Simple pole, $\frac{1}{1+1.25S}$	St. line of 0dB till $\omega_{c2} = 0.8$ rad/sec & after that st. line of slope −20 dB/dec	$-20-20=-40$ $0.8 < \omega < 10$.
4] Simple pole. $\frac{1}{1+0.1S}$	St. line of 0dB till $\omega_{c3} = 10$ rad/sec & after that st. line of slope −20dB/dec	$-20-40=-60$ $10 < \omega < \infty$

Frequency	const $k = 6.5625$	Simple pole $\frac{1}{1+5S}$	Simple pole $\frac{1}{1+1.25S}$	Simple pole $\frac{1}{1+0.1S}$	Resultant phase angle (ϕ) deg.
		$-\tan^{-1} 5\omega$	$-\tan^{-1} 1.25\omega$	$-\tan^{-1} 0.1\omega$	
0.1	0°	−26.565	−7.125	−0.572	−34.262
0.2	0°	−45	−14.036	−1.145	−60.187
0.5	0°	−68.1985	−32.005	−2.86	−103.06
0.8	0°	−75.963	−45	−4.57	−125.53
1	0°	−78.69	−51.34	−5.71	−135.74
2	0°	−84.28	−68.19	−11.30	−163.77
5	0°	−87.70	−80.90	−26.56	−195.16
10	0°	−88.85	−85.42	−45	−219.27

5] The open loop transfer fn. of unity feedback system,

$G(s) = \frac{k}{s(1+0.1s)(1+s)}$. Determine a) the value of 'k' so that the gain margin is 30dB. what is the corresponding phase margin.

b] The value of 'k' so that face margin is 30° what is the corresponding gain margin.

Sol'n:-

$$G(s) = \frac{k}{s(1+0.1s)(1+s)}$$

Simple pole $= \frac{1}{1+0.1S}$, $T = 0.1$

$$\omega_{c_1} = \frac{1}{T} = \frac{1}{0.1} = 10 \text{ rad/sec}$$

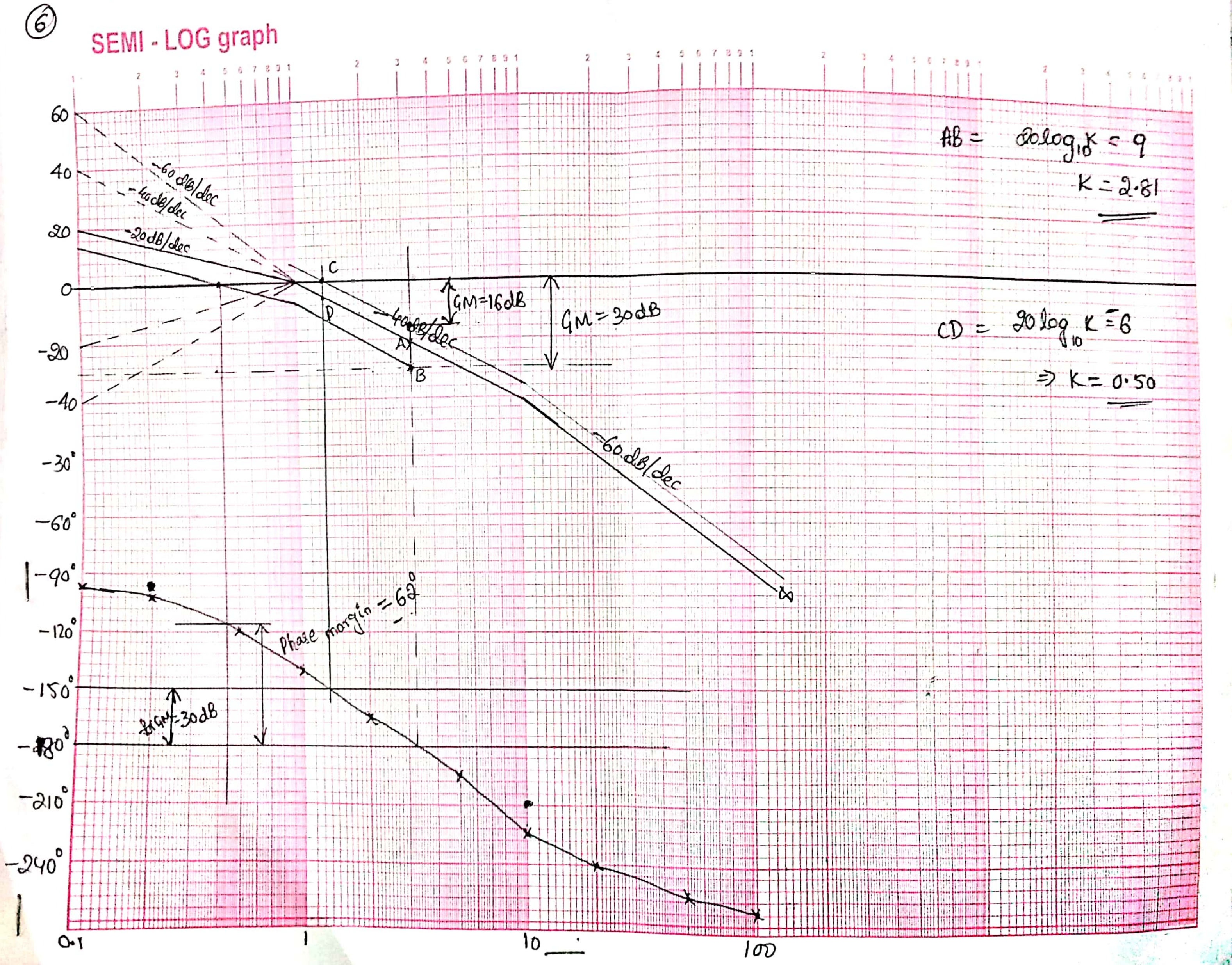
Simple pole $= \frac{1}{1+S}$, $T = 1$

$$\omega_{c2} = 1 \text{ rad/sec}$$

Factor	Nature of bode plot	Resultant slope (dB/dec)
1) Pole at the origin	St. line of −20 dB/dec. intersecting $\omega = 1$ rad/sec	−20 $0 < \omega < 1$
2] Simple pole	St. line of 0dB till $\omega_{c1} = 1$ rad/sec & after that st. line of slope −20 dB/dec	$-20-20=-40$ $1 < \omega < 10$
3] Simple pole	St. line of 0dB till $\omega_{c2} =$ 2rad/sec & after that st. line of slope −20 dB/dec	$-40-20=-60$ $10 < \omega < \infty$

Scanned by CamScanner

⑥



Scanned by CamScanner

Frequency	Pole at the origin	Single pole $\frac{1}{1+s}$ $-\tan^{-1}(\omega)$	Single pole $\frac{1}{1+0.1s}$ $-\tan^{-1}(0.1\omega)$	Resultant phase angle (ϕ) deg.
0.1	-90°	-5.71	-0.570	-96.2
0.2	-90°	-11.30	-1.145	-10[illegible] 1.45
0.5	-90°	-26.56	-2.86	-119
1	-90°	-45	-5.71	-140.7
2	-90°	-63.43	-11.30	-164
5	-90°	-78.69	-26.56	$-195.$
10	-90°	-84.28	-45	$-219.$
20	-90°	-87.13	-63.43	$-240.$
50	-90°	-88.85	-78.69	$-257.$
100	-90°	-89.42	-84.28	$-263.$

MODULE-5

System compensation & state variable characteristics of a system

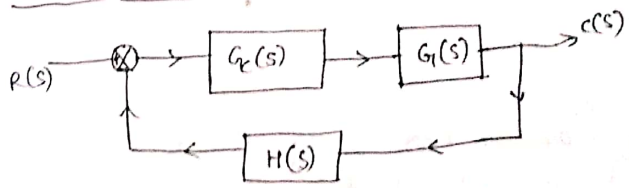
Introduction:

In practice it is necessary to alter the system by adding an external device to it, so as to meet the reqd. specification. Such a redesign & alteration of the system is called compensation of a control system.

An external device which is used to alter the behavior of the system to so as to achieve given specification is called compensation.

Types of compensation:

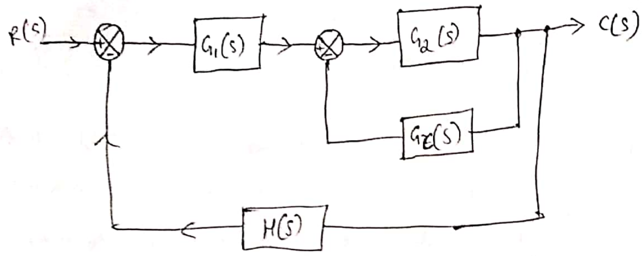
1) Series compensations:



Compensation is a physical device whose T.F. is denoted by $G_c(S)$.

If the compensator is placed in series with the forward path T.F. of the plant it is called series compensation or cascade compensation. The flow of signal is from lower energy level to higher energy level.

3) Parallel compensation :-



In some cases feed back is taken from some internal element of compensator is introduced in ~~such~~ as such a feedback path to provide an addition internal feed back loop. such compensation is called as feedback compensation or parallel compensation.

The energy transfer is from higher energy to lower energy level.

1) State :-

The state of a dynamic system is defined as a min. set of variable such that the knowledge of these variable at $t = t_0$ together with the knowledge of the i/p for $t \geq t_0$ completly determines the behavior of system for $t > t_0$.

2) State variables :-

The variables involved in determining the state of a dynamic system $X(t)$ are called state variable.

3) State vector :-

The 'n' state variables necessary to describe the complete behavior of the system can be considered as 'n' ~~component~~ component of a vector $X(t)$ called the state vector at time 't'.

4) State space :-

The space whose co-ordinate axis are nothing but the 'n' state variable with time as the implicit variable is called the state space.

5) State tragectory :-

It is the locus of the tips of the state vector with time as the implicit variable.

Controllability & obserability :-

Controllability :-

A system is controllable if any initial state can be transferred to any desired state in a finite length of time by some control action.

observability :-

A system is completly observable if there exist a finite time 'T' such that initial state $X(0)$ can be determined from the observation history $y(t)$ given the control $u(t)$ $(0 \leq t \leq T)$.

Scanned by CamScanner

Kalman's Test of controllability :-

According to this the system is completly controllable if the determinant of controllability matrix $K(C)$ is non-zero. ($K_c \neq 0$)

If the determinant of controllability matrix $K_c = 0$ then the system is not controllable.

For the system $\dot{X} = AX(t) + BU(t)$

$$K_c = [B : AB : A^2B : A^3B \ldots A^nB]$$

Kalman's test of observability :-

According to kalman's test of observability the single i/p single o/p linear system is observable if & only if the determinant of the nxn observability matrix K_o is non-zero. ($K_o \neq 0$)

If the determinant of observability matrix $K_o = 0$ then the system is not observable.

$\dot{x} = AX(t) + Bu(t)$
$y = CX$

$$K_o = [C^T : (CA)^T : (CA^2)^T \ldots]^T$$

1] Consider the system which state eqn,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

Estimate the state of controlability by Kalman's Test.

Sol'n:

$\dot{x} = AX(t) + BU(t)$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$K_c = [B : AB : A^2B]$$

$$AB = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 1 \\ -6 \end{bmatrix}$$

$$A^2 = A * A$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ -6 & -11 & -6 \\ 36 & 60 & 25 \end{bmatrix}$$

$$A^2B = \begin{bmatrix} 0 & 0 & 1 \\ -6 & -11 & -6 \\ 36 & 60 & 25 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ -6 \\ 25 \end{bmatrix}$$

$$K_c = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ -6 \end{bmatrix} \begin{bmatrix} 1 \\ -6 \\ 25 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -6 \\ 1 & -6 & 25 \end{bmatrix}$$

Scanned by CamScanner

$$|K_c| = \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & -6 \\ 1 & -6 & 25 \end{vmatrix}$$

$$= 0[25-36] - 0(0+6) + 1(0-1)$$

$$= -1$$

Since the value of $|K_c|$ is non zero. hence the system is ~~controllable~~ controllable.

3) Apply Kalman's test of check whether the following system is observable or not.

a) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -5 & 4 \\ -6 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$ & $y = [-2, 3]x$

Soln:

$\dot{X} = AX(t) + BU(t)$

$y = CX$, $C = [-2, 3]$

$A = \begin{bmatrix} -5 & 4 \\ -6 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$K_o = [C^T : (CA)^T]$

$C^T = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

$CA = [-2 \ \ 3] \begin{bmatrix} -5 & 4 \\ -6 & 5 \end{bmatrix}$

$= [-8 \ \ 7]$, $CA^T = \begin{bmatrix} -8 \\ 7 \end{bmatrix}$

$K_o = \begin{bmatrix} -2 & -8 \\ 3 & 7 \end{bmatrix}$

$|K_o| = \begin{vmatrix} -2 & -8 \\ 3 & 7 \end{vmatrix}$

$= [-14 + 24]$

$= 10$

Since value of $|K_c|$ is non zero, hence the system is completely observable.

Gilbert test of controllability :-

For the gilbert test it is necessary that the matrix A must be in cannonical form. Hence the given state model is reqd. to be transformed to the cannonical form first to apply gilbert's test.

Consider single i/p linear time invariant system represented by,

$\dot{X}(t) = AX(t) + BU(t)$

$X(t)$ can be written as $MZ(t)$.

where M is called vander Moder matrix

$$M = \begin{bmatrix} 1 & 1 & 1 \\ \lambda_1 & \lambda_2 & \lambda_3 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 \end{bmatrix}.$$

$\bar{B} = M^{-1}B$. According to gilbert test, the system is controlled if the value of $\bar{B}$ is non zero.

Scanned by CamScanner

Gilbert's test for observability :-

Consider the linear time invariant system,

$$\dot{X}(t) = X \cdot A(t) + BU(t) \quad \text{--- (1)}$$

$$Y(t) = CX$$

$$\bar{B} = CM$$

According to gilbert's test for observability the system is completely observable if the element in the $\bar{B}$ matrix is non zero. If the element in the matrix $\bar{B}$ is zero then the system is not observable.

1] Consider the system with state eqn.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

Estimate the state controllability using gilbert's test.

Soln: According to eqn (1).

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$|\lambda I - A| = \begin{vmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix} - \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{vmatrix}$$

$$= \begin{vmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ +6 & +11 & \lambda+6 \end{vmatrix}$$

$$|\lambda I - A| = 0 \Rightarrow \lambda(\lambda^2 + 6\lambda + 11) + 1(0 + 6) + 0 = 0$$

$$\Rightarrow \lambda^3 + 6\lambda^2 + 11\lambda + 6 = 0$$

$$\Rightarrow \lambda = -3, -1, -2. \quad \lambda_1 = -1, \lambda_2 = -2, \lambda_3 = -3$$

$$\bar{B} = M^{-1}B$$

$$M = \begin{vmatrix} 1 & 1 & 1 \\ \lambda_1 & \lambda_2 & \lambda_3 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ -1 & -2 & -3 \\ 1 & 4 & 9 \end{vmatrix}$$

$$M^{-1} = \frac{[\text{adj } M]^T}{|M|}$$

$$\text{adj } M = \begin{bmatrix} -6 & 6 & -2 \\ -5 & 8 & -3 \\ -1 & +2 & -1 \end{bmatrix}^T$$

neglecting first row & column we get.

$(-2\times9) - (4\times-3)$

$-18 + 12 = -6.$

$-9+3=-6$

$-3+2=-1$

$$[\text{adj } M]^T = \begin{bmatrix} -6 & -5 & -1 \\ 6 & 8 & 2 \\ -2 & -3 & -1 \end{bmatrix}$$

$$|M| = 1(-18-(-12)) - 1(-9+3) + 1(-2+4)$$

$$= -6 + 6 - 2$$

$$= -2$$

$$M^{-1} = -\frac{1}{2} \begin{bmatrix} -6 & -5 & -1 \\ 6 & 8 & 2 \\ -2 & -3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 2.5 & 0.5 \\ -3 & -4 & -1 \\ 1 & 1.5 & 0.5 \end{bmatrix}$$

$\bar{B} = M^{-1} B$

$$= \begin{bmatrix} 3 & 2.5 & 0.5 \\ -3 & -4 & -1 \\ 1 & 1.5 & 0.5 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5 \\ -1 \\ 0.5 \end{bmatrix}$$

Since there are all the elements in the $\bar{B}$ matrix are non zero. The system is controllable.

5] Evaluate observability of the system using gilbert test. $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ & $C = \begin{bmatrix} 3 & 4 & 1 \end{bmatrix}$

Sol:

$$|\lambda I - A| = \begin{vmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix} - \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{vmatrix}$$

$$= \begin{vmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ 0 & 2 & \lambda+3 \end{vmatrix}$$

$$|\lambda I - A| = 0$$

$$\begin{vmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ 0 & 2 & \lambda+3 \end{vmatrix} = 0$$

$$\lambda(\lambda^2+3\lambda+2) + 1(0) + 0 = 0$$

$$\lambda^3 + 3\lambda^2 + 2\lambda = 0$$

$$\lambda_1 = 0, \quad \lambda_2 = -1, \quad \lambda_3 = -2$$

$$\bar{C} = CM$$

$$M = \begin{bmatrix} 1 & 1 & 1 \\ \lambda_1 & \lambda_2 & \lambda_3 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -2 \\ 0 & 1 & 4 \end{bmatrix}$$

$$\bar{C} = \begin{bmatrix} 3 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -2 \\ 0 & 1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 & -1 \end{bmatrix}$$

Since there are one zero element in $\bar{C}$ the system is not observable

Scanned by CamScanner