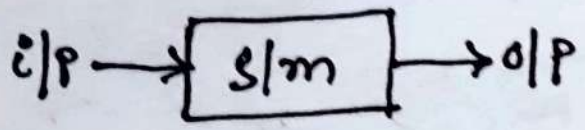
Introduction to Control system

Control System:- The combination of components that are connected together to perform a specific operation. Here the output of the s/m is controlled by the input

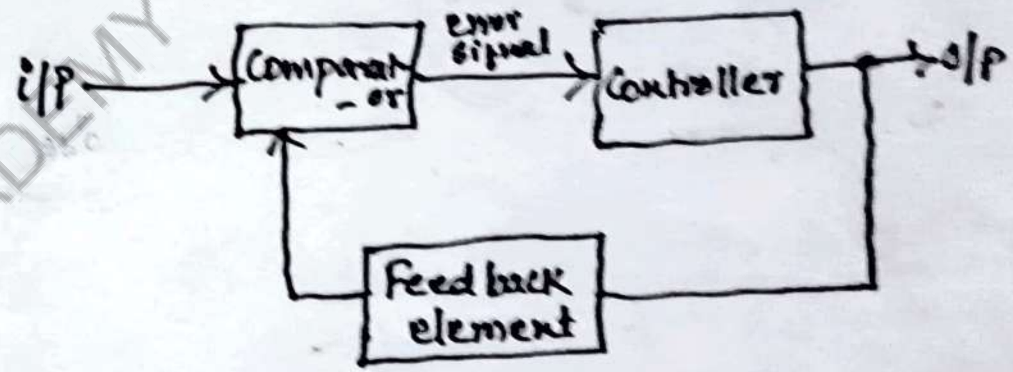
(i) open loop Control System



→ Simple
→ Cheap
→ Stable
→ less accurate

Ex:- Traffic signal
Electric Hand drier
Automatic washing machine

(ii) Closed loop Control System



→ More accurate
→ Complex
→ costly
→ less Stable

Types of Control Systems

① Linear system

Superposition

Homogenity

$x(t) \longrightarrow y(t)$

$A x(t) \longrightarrow A \cdot y(t)$

Additivity

$x_1(t) \rightarrow y_1(t)$

$x_2(t) \rightarrow y_2(t)$

$x_1(t)+x_2(t) \longrightarrow y_1(t)+y_2(t)$

$$\boxed{a x_1(t) + b x_2(t) \rightarrow a y_1(t) + b y_2(t)}$$

② Non linear system

does not follow → superposition.

③ Time invariant s/m

$x(t) \longrightarrow y(t)$

$x(t-t_0) \longrightarrow y(t-t_0)$

Time shift at input causes time shift at o/p

④ Time variant s/m

Time shift at i/p does not cause corresponding time shift at o/p

⑤ Linear time variant s/m

s/m is linear & time variant

LTV s/m

⑥ Linear time invariant s/m

s/m is linear & time invariant

LTI s/m

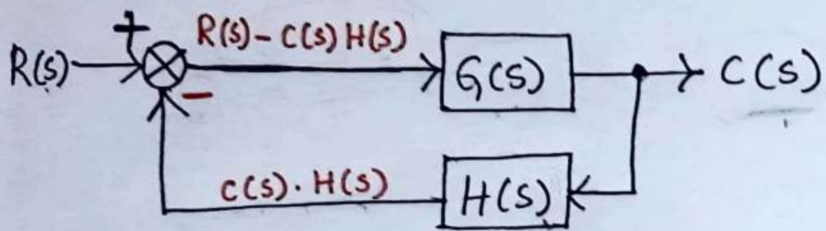
EC Academy

Comparission b/w Open-loop & Closed-loop CS

Open-loop Control system	Closed-loop Control System
1. Simple, cheap	1. Complicated, Costly.
2. less power	2. more power.
3. Feedback doesnt exist.	3. Feedback exist.
4. Highly Sensitive to disturbances	4. less Sensitive to disturbances.
5. inaccurate, unrelible.	5. Highly accurate, relible.
6. Stable	6. less stable
7. Highly sensitive to environmental changes	7. less sensitive to the environ changes.

Types of Feedback S/m.

1. Negative Feedback:



$$C(s) = [R(s) - C(s)H(s)]\,G(s)$$

$$C(s) = [G(s)R(s) - G(s)C(s)H(s)]$$

$$C(s) + G(s)C(s)H(s) = G(s) \cdot R(s)$$

$$C(s)[1 + G(s)H(s)] = G(s) \cdot R(s)$$

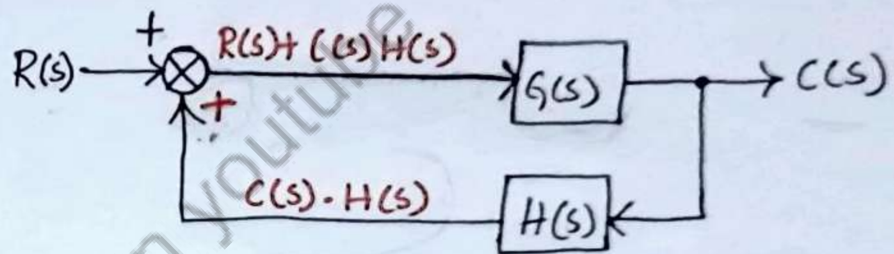
$$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}}$$

if $H(s) = 1$

$$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}}$$

Gain ↓

2. Positive Feedback:



$$C(s) = [R(s) + C(s)H(s)]\,G(s)$$

$$C(s) = [R(s)G(s) + G(s)C(s)H(s)]$$

$$C(s) - G(s)C(s)H(s) = G(s)R(s)$$

$$C(s)[1 - G(s)H(s)] = G(s)R(s)$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1-G(s)H(s)}}$$

if $H(s) = 1$

$$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1-G(s)}}$$ [redacted]

Gain ↑

Effect of Feedback.

1. on Stability:

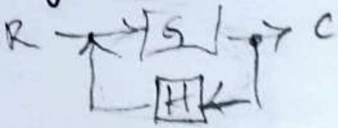
Unstable ⇒ o/p is out of Control

$\dfrac{G}{1+GH}$ $\quad GH = 1 \Rightarrow \boxed{o/p = \infty}$

↳ -ve $\quad$ ↳ "Unstable"

$GH > 1 \Rightarrow$ Stability improve

↳ loop gain



2. on Overall Gain:

G, H → frequency → Practical C.S.

$1 + GH > 1 \Rightarrow$ Gain ↑

$1 + GH < 1 \Rightarrow$ Gain ↓

3. On Sensitivity:

$$S \propto \frac{1}{1+GH}$$

GH ↑ Sensitivity ↓

In-Brief:

→ Gain will reduce → -ve FB s/m.

→ reduction in Stability.

→ improvement in sensitivity to i/p

& reduction in sensitivity ⇒ Variation in s/m parameter

→ reduces the effect of Noise & disturbances.

→ The s/m ⇒ More accurate

Requirement of a Good Control s/m.

① **Accuracy:** → high accuracy.
→ little error
OL → less accurate → Feed back → CL

② **Sensitivity:** → important in design of C.S.
→ physical element → properties changes with change in environment & age.
→ Good C.S. → insensitive to change in Properties.
Sensitive → input commands.

③ **Noise:** [External disturbances]
s/m → subjected to noise. during operation
→ Good C.S. → insensitive to noise
sensitive → i/P commands.

④ **Stability:** Good C.S. → Stable in nature.
→ For bounded i/P → bounded o/P.

⑤ **Bandwidth:** Range of frequencies for which s/m should give satisfactory o/P.

⑥ **Speed:**
C.S. → should have good speed

⑦ **Oscillations:**
Small no of oscillations → of o/P.
tend to s/m to be stable.

Mathematical Modeling → "equivalent representation" of C.S.
↳ Can be obtained from Mathematical eqns

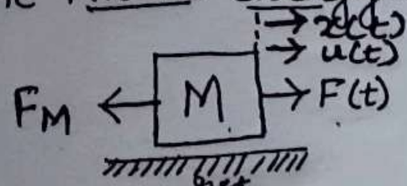
Mathematical eqns → describes the dynamic chr of s/m ⇒ Mathematical Model.

DIFFERENTIAL EQUATIONS

Mechanical s/m
- → Translational s/m ⇒ Motion of the body is along the stright line.
- → Rotational s/m ⇒ Motion of the body is about its own axis. [Circular Path]

I. TRANSLATIONAL S/M

① MASS: Element that stores the Kinetic energy.



$x(t)$ → displacement
$u(t)$ → Velocity
$F(t)$ → applied Force
F_M → opposing Force

$F_M \propto a \Rightarrow F_M = Ma$

$$\boxed{a = \frac{d}{dt}u(t) \;\&\; u(t) = \frac{d}{dt}x(t)}$$

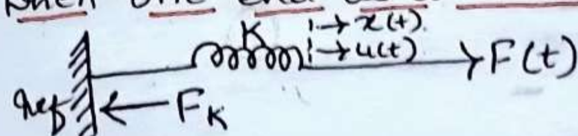
$$\boxed{F_M = M\cdot\frac{d}{dt}u(t) = M\cdot\frac{d^2}{dt^2}x(t)}$$

Neuton's Law $F_M = F(t)$

$$\therefore \boxed{F(t) = M\cdot\frac{d}{dt}u(t) = M\cdot\frac{d^2}{dt^2}x(t)}$$

② SPRING: Element that stores Potential Energy

ⓐ When one end is connected



$F_K \propto x(t) \Rightarrow F_K = K\cdot x(t)$

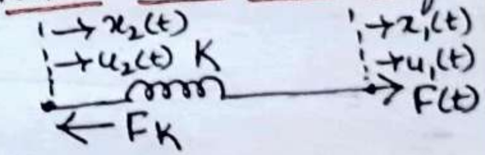
$\because u(t) = \frac{d}{dt}x(t) \Rightarrow x(t) = \int u(t)\,dt$

$\therefore F_K = K\cdot\int u(t)\,dt$

Neuton's law
$F(t) = F_K$

$$\therefore \boxed{F(t) = K\,x(t) = K\int u(t)\,dt}$$

ⓑ when both ends are free

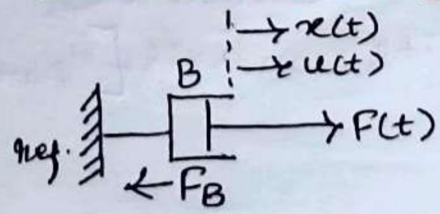


$$\boxed{F(t) = K(x_1(t) - x_2(t))}$$

③ DASH POT : OR FRICTION

Element that resist the motion through Friction.

ⓐ When one end is Connected.



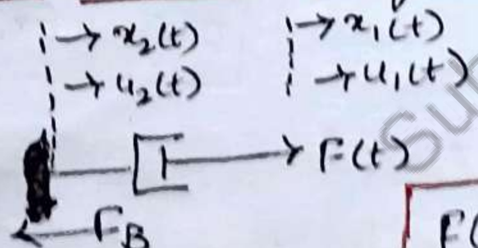
$F_B \propto u(t) \Rightarrow F_B = B\,u(t)$

$F_B = B \cdot \frac{dx(t)}{dt}$

Newton's law $F(t) = F_B$

$$\boxed{F(t) = B \cdot u(t) = B \cdot \frac{d}{dt}x(t)}$$

ⓑ When both ends are free



$$\boxed{\begin{aligned} F(t) &= B \cdot \{u_1(t) - u_2(t)\} \\ F(t) &= B \cdot \left\{\frac{d}{dt}x_1(t) - \frac{d}{dt}x_2(t)\right\} \end{aligned}}$$

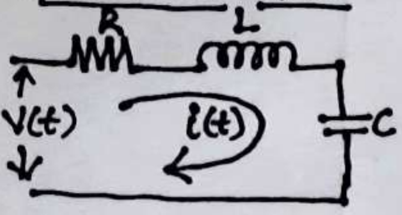
Force-Voltage Analogy [Translational Mechanical system]

→ The s/ms are said to be "Analogus", if the mathematical equations of the two s/ms are same.

Ex:- $f(t) = M\frac{d^2}{dt^2}x(t) + B\frac{d}{dt}x(t) + Kx(t)$ — ①

L.T ⇒ $F(s) = Ms^2X(s) + BsX(s) + KX(s)$

Electrical s/m



KVL,

$V(t) = i(t)R + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt$

L.T ⇒ $V(s) = I(s)R + LsI(s) + \frac{I(s)}{Cs}$

$\because i(t) = \frac{dq}{dt} \Rightarrow I(s) = sQ(s)$

$Q(s) = \frac{I(s)}{s}$

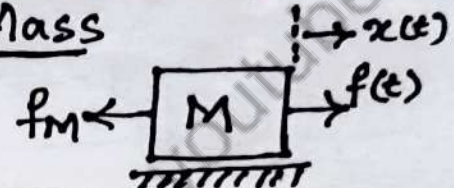
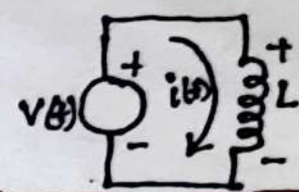
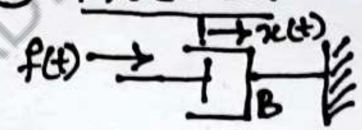
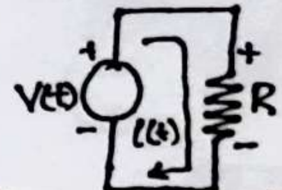
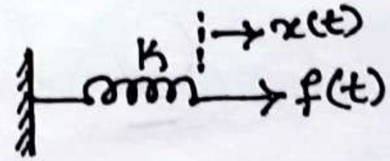
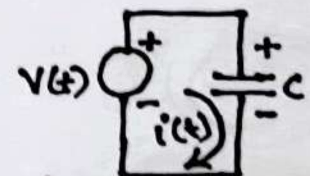
$V(s) = Ls^2Q(s) + RsQ(s) + \frac{1}{C}Q(s)$ → ②

Comparing eqⁿ ① & ②

→ L is analogus to M

→ R is analogus to B

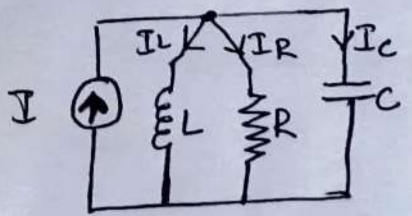
→ 1/C is analogus to K.

Mechanical s/m	Electrical s/m.
① Mass 	
$\boxed{f(t) = M\frac{d^2}{dt^2}x(t)}$	$\boxed{V(t) = L\frac{di(t)}{dt}}$
② Friction 	
$\boxed{f(t) = B\frac{dx(t)}{dt}}$	$\boxed{V(t) = Ri(t)}$
③ 	
$\boxed{f(t) = Kx(t)}$	$\boxed{V(t) = \frac{1}{C}\int i(t)dt}$

Force current Analogy. [Translational Mechanical s/m]

$$f(t) = M\frac{d^2}{dt^2}x(t) + B\frac{d}{dt}x(t) + Kx(t)$$

L.T. ⇒ $\boxed{F(s) = MS^2X(s) + BSX(s) + KX(s)} \rightarrow (1)$



KCL

$$I = I_L + I_R + I_C$$

$$\therefore I = \frac{1}{L}\int V\,dt + \frac{V}{R} + \text{[redacted]} \quad C\frac{dV}{dt}$$

L.T. ⇒ $I(s) = \frac{V(s)}{LS} + \frac{V(s)}{R} + CSV(s)$

$\because V(t) = \frac{d\phi}{dt}$; $\phi \Rightarrow$ Flux

$V(s) = s\,\phi(s) \Rightarrow \phi(s) = \frac{V(s)}{s}$

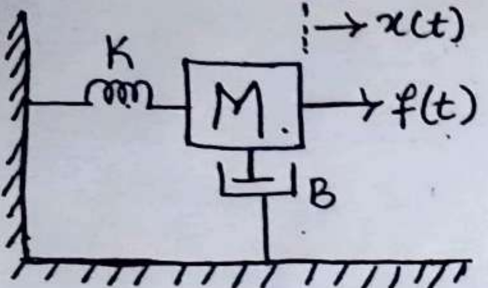
$I(s) = \boxed{Cs^2\phi(s) + \frac{1}{R}s\phi(s) + \frac{1}{L}\phi(s)} \rightarrow (2)$

Comparing eqn (1) & (2)

→ I is analogus F
→ C is analogus M
→ $\frac{1}{R}$ is analogus B
→ $\frac{1}{L}$ is analogus K

Mechanical s/m	Electrical s/m.
(1) Mass: $\boxed{F(t) = M\frac{d^2}{dt^2}x(t)}$	$\boxed{I(t) = C\cdot\frac{d}{dt}V(t)}$
(2) Dash pot: $\boxed{F(t) = B\cdot\frac{d}{dt}x(t)}$	$\boxed{I(t) = \frac{1}{R}V(t)}$
(3) Spring: $\boxed{F(t) = Kx(t)}$	$I(t) = \boxed{\frac{1}{L}\int V(t)\,dt}$

Find the Transfer function of Mechanical S/m



② $f(t) = f_M + f_K + f_B$

$$f(t) = M\frac{d^2}{dt^2}x(t) + Kx(t) + B\frac{d}{dt}x(t)$$

Steps

1. Write the mechanical n/w.
2. Write the differential eqns.
3. Take L.T. of differential eqns
4. Take $\dfrac{\text{L.T. of o/p}}{\text{L.T. of i/p}}$

③ $f(t) \to F(s)$

$x(t) \to X(s)$

$\dfrac{d^2}{dt^2} \to s^2$

$\dfrac{d}{dt} \to s$

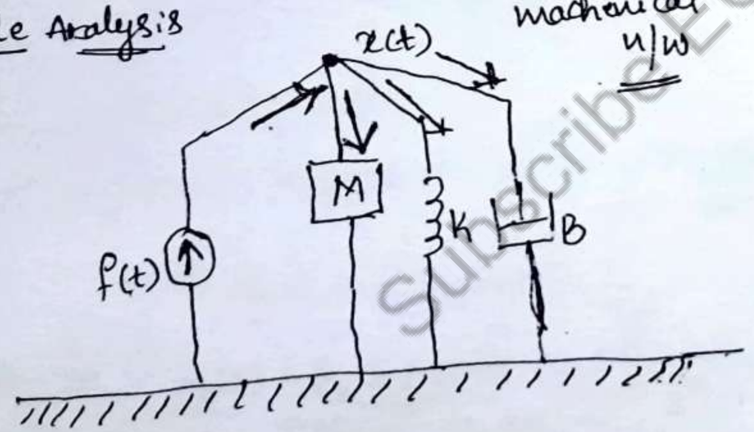
$F(s) = Ms^2X(s) + KX(s) + BsX(s)$

④

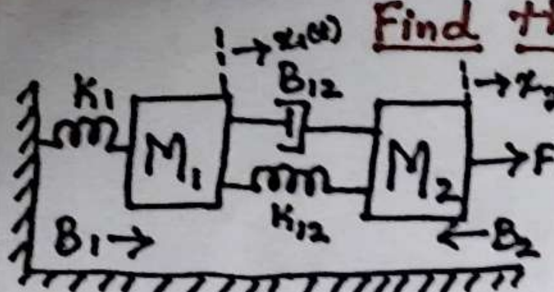
$F(s) = X(s)\left[Ms^2 + K + Bs\right]$

$$\boxed{\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + K + Bs}}$$

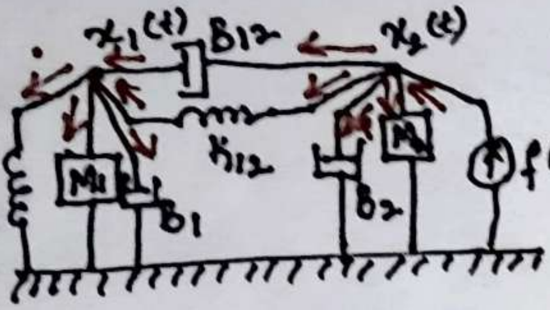
① Node Analysis



Find the Transfer function.



1. Draw the mechanical n/w
2. Write differential eqns
3. Laplace Transform.
4. Find the transfer fn.

① 

Node analysis
→ no of nodes will be equal to no of displacements
$f(t)$ → there two displacements are present in this mech. sys hence we need to take two nodes.

② at $x_2(t)$

$$f(t) = f_{M_2} + f_{B_2} + f_{B_{12}} + f_{K_{12}}$$

$$\Rightarrow f(t) = M_2 \frac{d^2}{dt^2} x_2(t) + B_2 \frac{d}{dt} x_2(t) + B_{12} \frac{d}{dt}(x_2(t) - x_1(t)) + K_{12}(x_2(t) - x_1(t)) \quad ①$$

at $x_1(t)$

$$f_{B_{12}} + f_{K_{12}} = f_{M_1} + f_{B_1} + f_{K_1}$$

$$B_{12} \frac{d}{dt}(x_2(t) - x_1(t)) + K_{12}(x_2(t) - x_1(t)) = M_1 \frac{d^2}{dt^2} x_1(t) + B_1 \frac{d}{dt} x_1(t) + K_1 x_1(t) \quad ②$$

③ L.T.

$$① \Rightarrow F(s) = M_2 s^2 X_2(s) + B_2 s X_2(s) + B_{12} s [X_2(s) - X_1(s)] + K_{12}[X_2(s) - X_1(s)]$$

$$F(s) = X_2(s)[M_2 s^2 + B_2 s + B_{12} s + K_{12}] - X_1(s)[B_{12} s + K_{12}] \quad ③$$

$$② \Rightarrow B_{12} s [X_2(s) - X_1(s)] + K_{12}[X_2(s) - X_1(s)] = M_1 s^2 X_1(s) + B_1 s X_1(s) + K_1 X_1(s)$$

$$X_2(s)[B_{12} s + K_{12}] - X_1(s)[B_{12} s + K_{12}] = X_1(s)[M_1 s^2 + B_1 s + K_1]$$

$$X_2(s)[B_{12} s + K_{12}] = X_1(s)[M_1 s^2 + B_1 s + K_1 + B_{12} s + K_{12}]$$

$$X_2(s) = X_1(s) \frac{[M_1 s^2 + B_1 s + K_1 + B_{12} s + K_{12}]}{[B_{12} s + K_{12}]} \quad ④$$

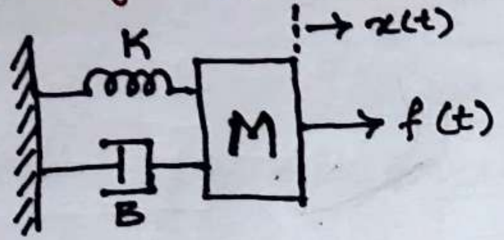
Substitute ④ in ③

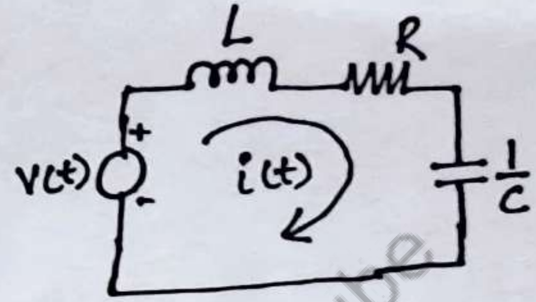
$$F(s) = X_1(s) \frac{[M_1 s^2 + B_1 s + K_1 + B_{12} s + K_{12}][M_2 s^2 + B_2 s + B_{12} s + K_{12}]}{[B_{12} s + K_{12}]} - X_1(s)[B_{12} s + K_{12}]$$

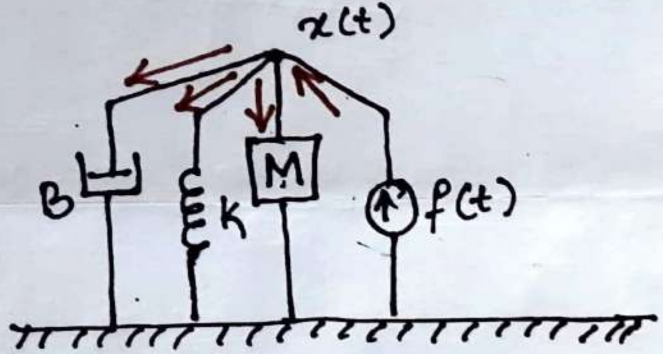
$$F(s) = X_1(s) \left\{ \frac{[M_1 s^2 + B_1 s + K_1 + B_{12} s + K_{12}][M_2 s^2 + B_2 s + B_{12} s + K_{12}] - [B_{12} s + K_{12}]^2}{[B_{12} s + K_{12}]} \right\}$$

$$\boxed{\frac{X_1(s)}{F(s)} = \frac{B_{12} s + K_{12}}{[M_1 s^2 + B_1 s + K_1 + B_{12} s + K_{12}][M_2 s^2 + B_2 s + B_{12} s + K_{12}] - [B_{12} s + K_{12}]^2}}$$

Force - Voltage & Force - Current Analogy







$$f(t) = f_M + f_B + f_K$$

$$f(t) = M\frac{d^2}{dt^2}x(t) + B\frac{d}{dt}x(t) + K\,x(t) \longrightarrow (1)$$

Force-Voltage [#187]

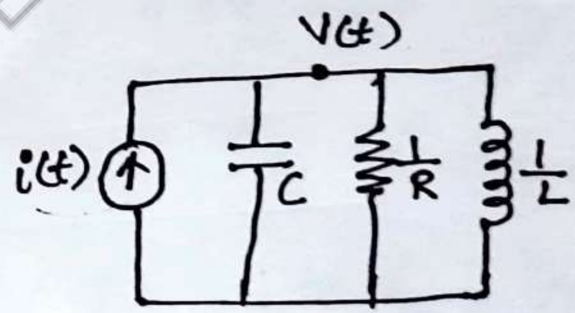
$f(t) \to v(t)$, $M \to L$, $B \to R$, $K \to \frac{1}{C}\int$, $x(t) \to i(t)$

$$\boxed{v(t) = L\frac{d}{dt}i(t) + R\,i(t) + \frac{1}{C}\int i(t)\,dt}$$

Force - Current [#188]

$f(t) \to i(t)$, $M \to C$, $B \to \frac{1}{R}$, $K \to \frac{1}{L}\int$, $x(t) \to v(t)$

$$\boxed{i(t) = C\frac{d}{dt}v(t) + \frac{1}{R}v(t) + \frac{1}{L}\int v(t)\,dt}$$



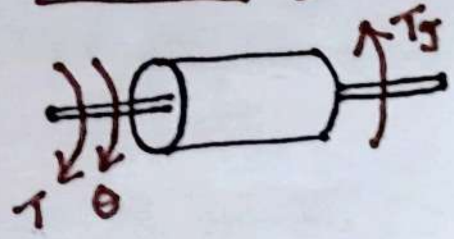
Rotationa
Mechanic.

Rotational Mechanical s/m

[Differential equations]

→ Motion of the body is in it's own axis.

① Moment of inertia (J) OR Rotational Mass



T → Torque

θ → Angular displacement

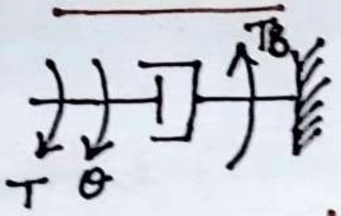
T_J → opposing Torque.

$$T_J \propto \frac{d^2}{dt^2}\theta \Rightarrow T_J = J\frac{d^2\theta}{dt^2}$$

Newton's law $T = T_J$

$$\boxed{T = J\frac{d^2}{dt^2}\theta}$$

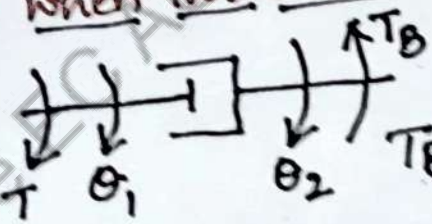
② Rotational Dashpot (B)



(i) when one end is fixed

$$T_B \propto \frac{d\theta}{dt} \Rightarrow T_B = B\cdot\frac{d\theta}{dt}$$

(ii) when two ends are Free



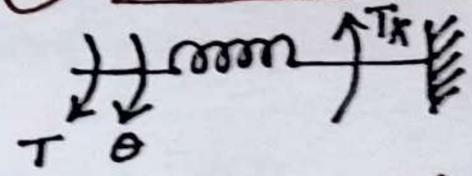
$$T_B = B\cdot\frac{d}{dt}(\theta_1-\theta_2)$$

Newton's law $T = T_B$

$$\boxed{T = B\cdot\frac{d}{dt}\theta}$$

$$\boxed{T = B\cdot\frac{d}{dt}(\theta_1-\theta_2)}$$

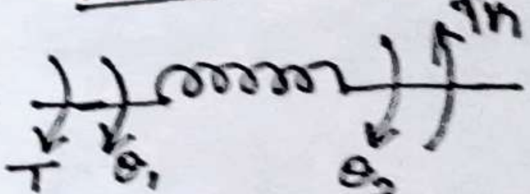
③ Rotational Spring (K)



(i) when one end is fixed

$$T_K \propto \theta \Rightarrow T_K = K\cdot\theta$$

(ii) when two ends are Free



$$T_K \propto K\cdot(\theta_1-\theta_2)$$

Newton's law $T = T_K$

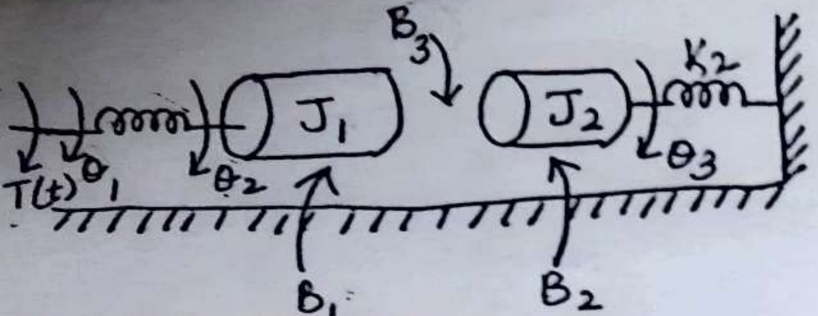
$$\boxed{T = K\cdot\theta}$$

$$\boxed{T = K(\theta_1-\theta_2)}$$

13

Rotational s/m to Torque-Voltage & Torque Current Analogous s/m

Mechanical	Electrical Analogous	
Rotational Mechanical s/m	Torque-Voltage Analogy	Torque-Current Analogy
Torque [T(t)]	Voltage [V(t)]	Current [i(t)]
Angular displacement [$\theta(t)$]	Current [i(t)]	Voltage [V(t)]
Moment of Inertia [J] $J \frac{d^2}{dt^2} \theta(t)$	Inductance [L] $L \frac{d}{dt} i(t)$	Capacitance [C] $C \frac{d}{dt} v(t)$
Rotational Spring [K] $K\theta$	Reciprocal of capacitance [1/C] $\frac{1}{C}\int i(t)\,dt$	Reciprocal of Inductance [1/L] $\frac{1}{L}\int V(t)\,dt.$
Rotational Dashpot [B] $B\frac{d}{dt}\theta$	Resistance [R] $R\,i(t)$	Conductance [$\frac{1}{R}$] $\frac{1}{R} V(t)$

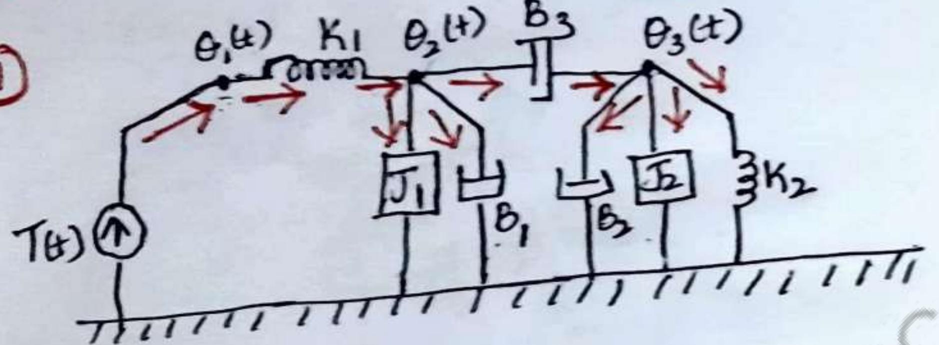


① Write mechanical n/w

② Write the Differential eqns

③ Write T-V & T-I Analogy.

①



②
$$T(t) = K_1(\theta_1(t) - \theta_2(t)) \rightarrow ①$$

$$K_1(\theta_1(t) - \theta_2(t)) = J_1 \frac{d^2}{dt^2}\theta_2(t) + B_1 \frac{d}{dt}\theta_2(t) + B_3 \frac{d}{dt}(\theta_2(t) - \theta_3(t)) \rightarrow ②$$

$$B_3 \frac{d}{dt}(\theta_2(t) - \theta_3(t)) = J_2 \frac{d^2}{dt^2}\theta_3(t) + B_2 \frac{d}{dt}\theta_3(t) + K_2\theta_3(t) \rightarrow ③$$

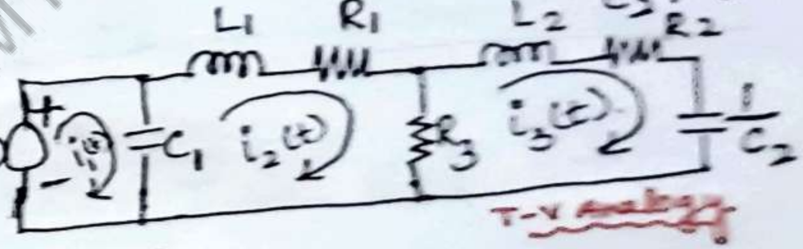
③ T-V Analogy

$T(t) \rightarrow V(t)$, $J \rightarrow L$, $B \rightarrow R$, $K \rightarrow \frac{1}{C}$, $\int \theta(t) \rightarrow i(t)$

$$① \Rightarrow V(t) = \frac{1}{C_1}\int (i_1(t) - i_2(t))\,dt \rightarrow ④$$

$$② \Rightarrow \frac{1}{C_1}\int (i_1(t) - i_2(t))\,dt = L_1 \frac{d}{dt} i_2(t) + R_1 i_2(t) + R_3(i_2(t) - i_3(t)) \rightarrow ⑤$$

$$③ \Rightarrow R_3(i_2(t) - i_3(t)) = L_2 \frac{d}{dt} i_3(t) + R_2 i_3(t) + \frac{1}{C_2}\int i_3(t)\,dt \rightarrow ⑥$$



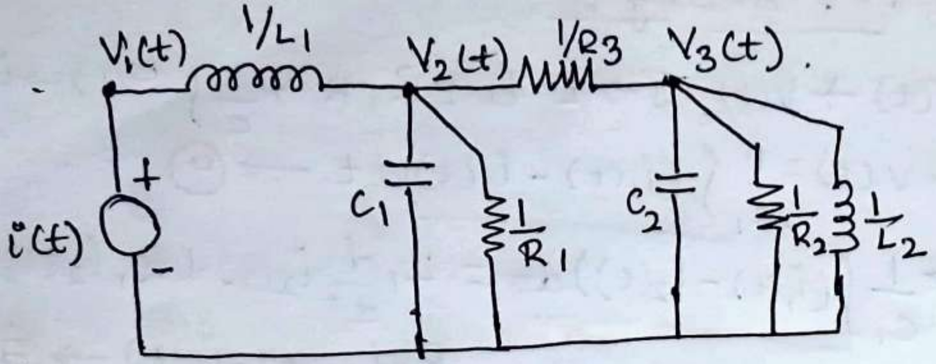
T-I Analogy

$T(t) \rightarrow i(t)$, $J \rightarrow C$, $B \rightarrow \frac{1}{R}$, $K \rightarrow \frac{1}{L}$, $\int \theta(t) \rightarrow V(t)$

$$① \Rightarrow i(t) = \frac{1}{L_1}\int [V_1(t) - V_2(t)]\,dt \rightarrow ⑦$$

$$② \Rightarrow \frac{1}{L_1}\int (V_1(t) - V_2(t))\,dt = C_1 \frac{d}{dt} V_2(t) + \frac{1}{R_1} V_2(t) + \frac{1}{R_3}(V_2(t) - V_3(t)) \rightarrow ⑧$$

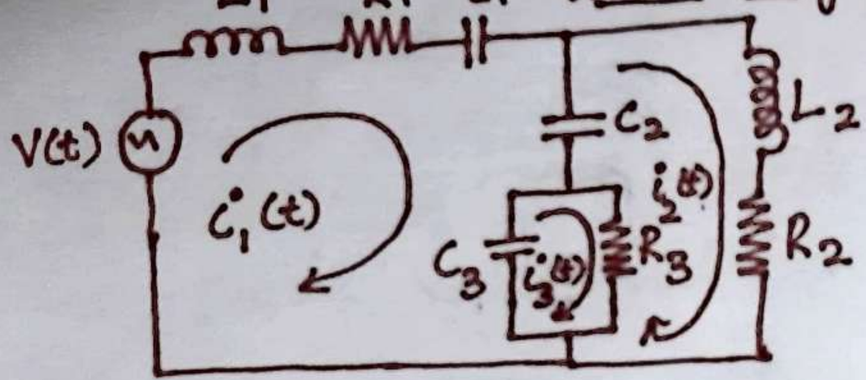
$$③ \Rightarrow \frac{1}{R_3}(V_2(t) - V_3(t)) = C_2 \frac{d}{dt} V_3(t) + \frac{1}{R_2} V_3(t) + \frac{1}{L_2}\int V_3(t)\,dt \rightarrow ⑨$$



T-I Analogy.

Obtain the Analogous Mechanical s/m

: Force Voltage Anology



Apply KVL,

$$V(t) = L_1 \frac{d}{dt} i_1(t) + R_1 i_1(t) + \frac{1}{C_1}\int i_1(t)\,dt + \frac{1}{C_2}\int [i_1(t) - i_2(t)]\,dt + \frac{1}{C_3}\int [i_1(t) - i_3(t)]\,dt \quad (1)$$

$$0 = R_3[i_3(t) - i_2(t)] + \frac{1}{C_3}\int [i_3(t) - i_1(t)]\,dt$$

$$\frac{1}{C_3}\int [i_1(t) - i_3(t)]\,dt = R_3[i_3(t) - i_2(t)] \quad (2)$$

$$0 = L_2 \frac{d}{dt} i_2(t) + R_2 i_2(t) + R_3[i_2(t) - i_3(t)] + \frac{1}{C_2}\int [i_2(t) - i_1(t)]\,dt$$

$$R_3[i_3(t) - i_2(t)] = L_2 \frac{d}{dt} i_2(t) + R_2 i_2(t) + \frac{1}{C_2}\int [i_2(t) - i_1(t)]\,dt \quad (3)$$

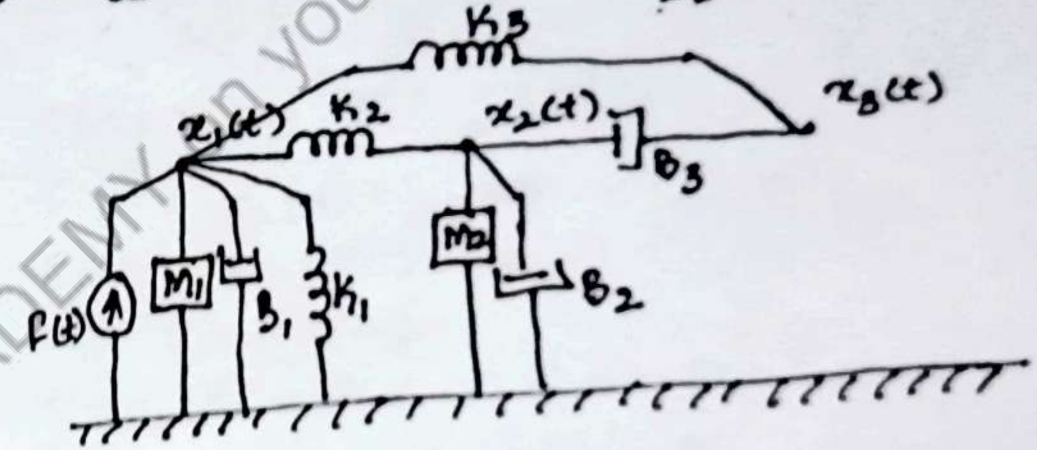
F-V Analogy:

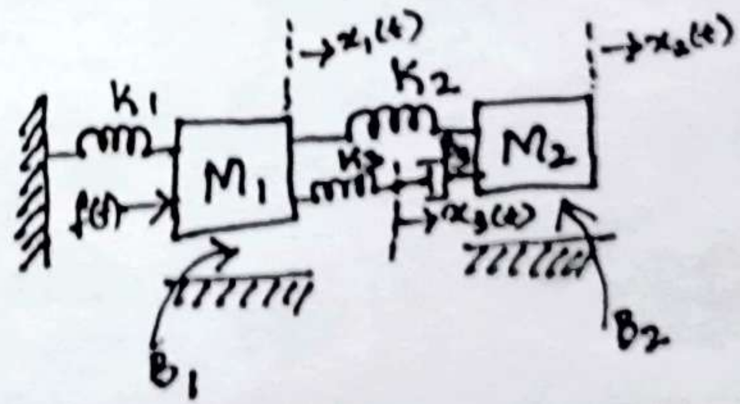
$$\boxed{F(t) \Rightarrow V(t),\ M \Rightarrow L,\ B \Rightarrow R,\ K \Rightarrow \frac{1}{C}\int,\ x(t) \Rightarrow i(t)}$$

$$(1) \Rightarrow F(t) = M_1 \frac{d^2}{dt^2} x_1(t) + B_1 x_1(t) + K_1 x_1(t) + K_2[x_1(t) - x_2(t)] + K_3[x_1(t) - x_3(t)] \quad (4)$$

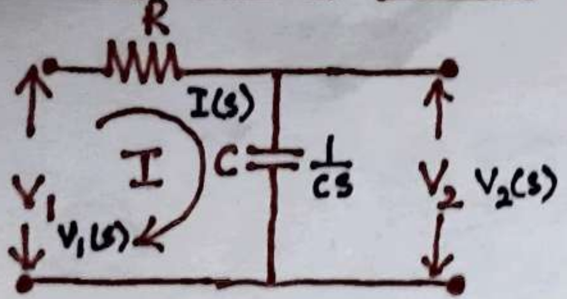
$$(2) \Rightarrow K_3[x_1(t) - x_3(t)] = B_3 \frac{d}{dt}[x_3(t) - x_2(t)] \quad (5)$$

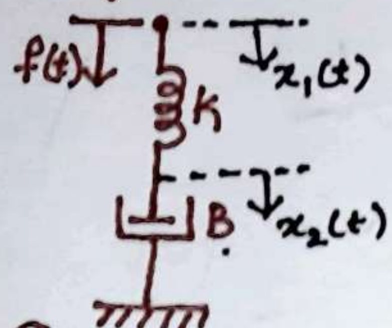
$$(3) \Rightarrow B_3 \frac{d}{dt}[x_3(t) - x_2(t)] = M_2 \frac{d^2}{dt^2} x_2(t) + B_2 \frac{d}{dt} x_2(t) + K_2[x_2(t) - x_1(t)] \quad (6)$$





S.T. two s/ms are Analogus





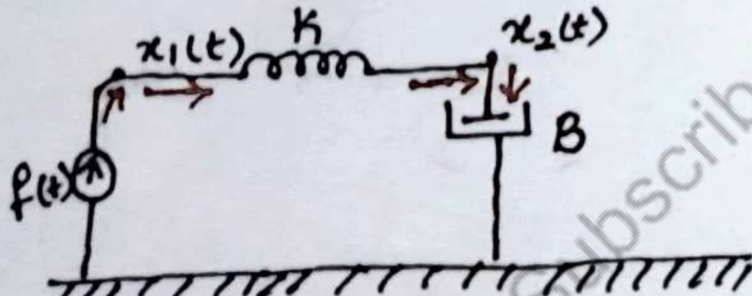
$$V_1(s) = I(s)R + \frac{1}{Cs} I(s) \rightarrow (1)$$

$$V_2(s) = \frac{1}{Cs} I(s) \Rightarrow I(s) = Cs \cdot V_2(s) \rightarrow (2)$$

Put (2) in (1)

$$V_1(s) = Cs \cdot V_2(s)\left[R + \frac{1}{Cs}\right] = \cancel{Cs} \cdot V_2(s)\left[\frac{RCs+1}{\cancel{Cs}}\right]$$

$$\frac{V_1(s)}{V_2(s)} = RCs+1 \Rightarrow \boxed{\frac{V_2(s)}{V_1(s)} = \frac{1}{(RCs)+1}} \rightarrow (A)$$



$$f(t) = K(x_1(t) - x_2(t))$$

$$f(t) = Kx_1(t) - Kx_2(t)$$

$$L.T \Rightarrow F(s) = KX_1(s) - KX_2(s) \rightarrow (3)$$

$$0 = B\frac{d}{dt}x_2(t) + K(x_2(t) - x_1(t))$$

$$0 = B\frac{d}{dt}x_2(t) + Kx_2(t) - Kx_1(t)$$

$$L.T \Rightarrow 0 = BsX_2(s) + KX_2(s) - KX_1(s)$$

$$KX_1(s) = X_2(s)[Bs+K]$$

$$X_1(s) = X_2(s)\left[\frac{Bs+K}{K}\right] \rightarrow (4)$$

Put (4) in (3)

$$F(s) = \cancel{K} \cdot X_2(s)\left[\frac{Bs+K}{\cancel{K}}\right] - KX_2(s)$$

$$F(s) = X_2(s)[Bs + \cancel{K} - \cancel{K}]$$

$$F(s) = BsX_2(s) \rightarrow (5)$$

Put (5) in (3)

$$BsX_2(s) = KX_1(s) - KX_2(s)$$

$$BsX_2(s) + KX_2(s) = KX_1(s)$$

$$X_2(s)[Bs+K] = KX_1(s)$$

$$\frac{X_2(s)}{X_1(s)} = \frac{K}{Bs+K}$$

$$\boxed{\frac{X_2(s)}{X_1(s)} = \frac{1}{\left(\frac{B}{K}\right)s+1}} \rightarrow (B)$$

Comparing (A) & (B) → The transfer fun is similar

Hence the "given s/ms are Analogus"

4.8 Gear Trains

A gear train is a mechanical device that transmits energy from one part of a system to another in such a way that force, torque, speed and displacement may be altered. The inertia and friction of the gears are neglected in the ideal case. Consider a gear system as shown in the Fig. 4.8.1.

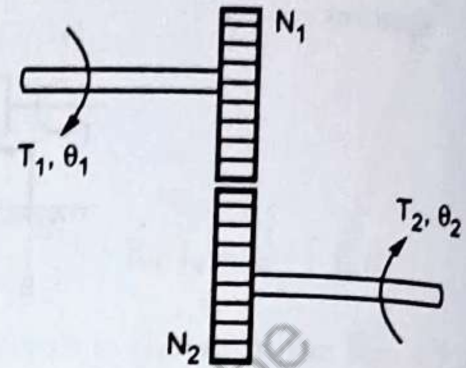


Fig. 4.8.1 Gear system

The number of teeth on the surface of the gears is proportional to the radii r_1 and r_2 of the gears.

i.e. $$\boxed{r_1 N_2 = r_2 N_1}$$

The distance travelled along the surface of each gear is same.

i.e. $$\boxed{\theta_1 r_1 = \theta_2 r_2}$$

The work done by one gear is same as the other.

i.e. $T_1 \theta_1 = T_2 \theta_2$

$\therefore$ we can say $$\boxed{\frac{T_1}{T_2} = \frac{\theta_2}{\theta_1} = \frac{r_1}{r_2} = \frac{N_1}{N_2}}$$

Remarks :

1) The numbers of teeth N are proportional to the radius r of a gear.
2) The distance travelled on each gear is same.
3) Work done = Tθ by each gear is same.

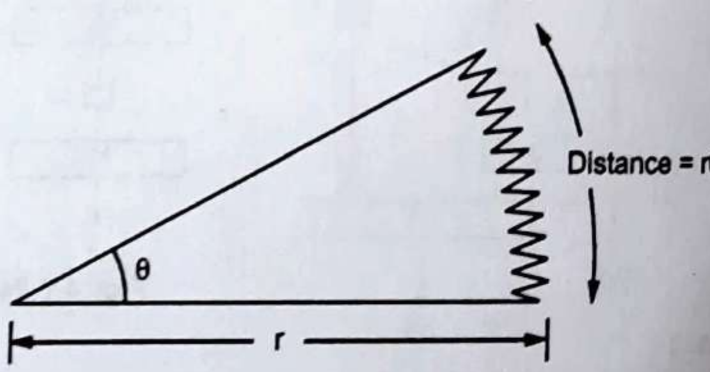


Fig. 4.8.2

4.8.1 Gear Train with Inertia and Friction

In practice, gears do have inertia and friction which cannot be neglected. Consider such practical gear arrangement connected to the load, shown below.

T = Applied torque θ_1, θ_2 = Angular displacements

T_1, T_2 = Torque transmitted to gears J_1, J_2 = Inertia of gears

N_1, N_2 = Number of teeth B_1, B_2 = Friction coefficients.

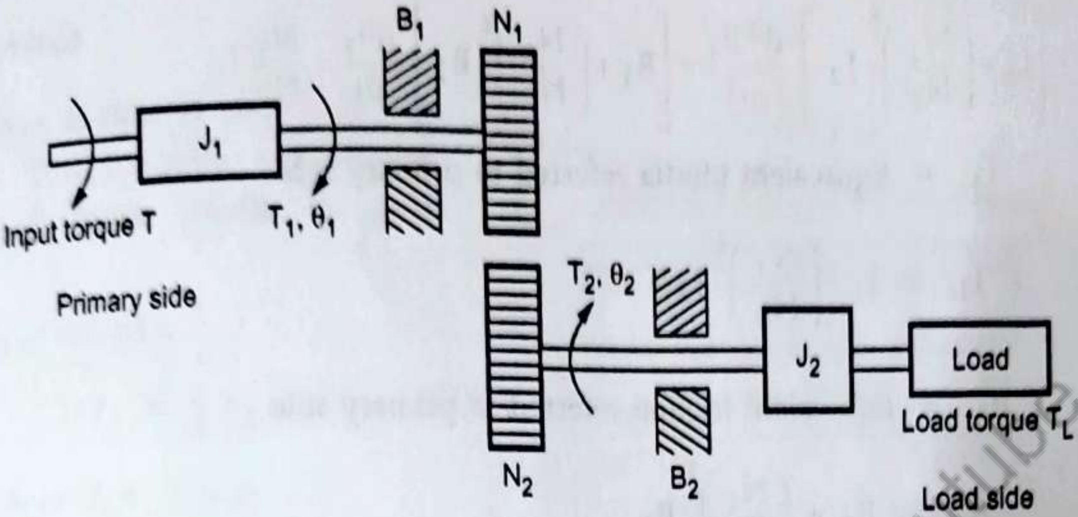


Fig. 4.8.3

Torque equation of side 1 is,

$$T = J_1 \frac{d^2 \theta_1(t)}{dt^2} + B_1 \frac{d\theta_1(t)}{dt} + T_1(t) \quad \text{... (4.8.1)}$$

Torque equation of side 2 is,

$$T_2 = J_2 \frac{d^2 \theta_2(t)}{dt^2} + B_2 \frac{d\theta_2(t)}{dt} + T_L(t) \quad \text{... (4.8.2)}$$

Now $\quad \frac{T_1}{T_2} = \frac{N_1}{N_2} = \frac{\theta_2}{\theta_1}$ i.e. $T_2 = \frac{N_2}{N_1} T_1$

Substituting in equation (4.8.2)

$$\therefore \quad \frac{N_2}{N_1} T_1 = J_2 \frac{d^2 \theta_2}{dt^2} + B_2 \frac{d\theta_2}{dt} + T_L$$

$$\therefore \quad T_1 = \frac{N_1}{N_2} J_2 \frac{d^2\theta_2}{dt^2} + \frac{N_1}{N_2} B_2 \frac{d\theta_2}{dt} + \frac{N_1}{N_2} T_L \quad \text{... (4.8.3)}$$

Substituting value of T_1 in equation (4.8.1)

$$T = J_1 \frac{d^2 \theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + \frac{N_1}{N_2} J_2 \frac{d^2 \theta_2}{dt_2} + \frac{N_1}{N_2} B_2 \frac{d\theta_2}{dt} + \frac{N_1}{N_2} T_L$$

Substituting $\theta_2 = \frac{N_1}{N_2} \theta_1$

$$\therefore T = J_1 \frac{d^2 \theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + \frac{N_1}{N_2} J_2 \frac{N_1}{N_2} \frac{d^2 \theta_1}{dt_2} + \frac{N_1}{N_2} \cdot B_2 \frac{N_1}{N_2} \frac{d\theta_1}{dt} + \frac{N_1}{N_2} T_L$$

$$\therefore\ T = \left[J_1 + \left(\frac{N_1}{N_2}\right)^2 J_2\right]\frac{d^2\theta_1}{dt^2} + \left[B_1 + \left(\frac{N_1}{N_2}\right)^2 B_2\right]\frac{d\theta_1}{dt} + \frac{N_1}{N_2}T_L$$

$\therefore$ J_{1e} = Equivalent inertia referred to primary side

$$\therefore\ \boxed{J_{1e} = J_1 + \left(\frac{N_1}{N_2}\right)^2 J_2}$$

and B_{1e} = Equivalent friction referred to primary side

$$\therefore\ \boxed{B_{1e} = B_1 + \left(\frac{N_1}{N_2}\right)^2 B_2}$$

$$\therefore\ \boxed{T = J_{1e}\frac{d^2\theta_1}{dt_2} + B_{1e}\frac{d\theta_1}{dt} + \left(\frac{N_1}{N_2}\right)T_L}$$

Similarly the equation can be written referred to load side also, where applied torque gets transferred to load as $\left(\frac{N_2}{N_1}T\right)$

$$\boxed{\left(\frac{N_2}{N_1}\right)T = J_{2e}\frac{d^2\theta_2}{dt^2} + B_{2e}\frac{d\theta_2}{dt} + T_L}$$

where $J_{2e} = J_2 + \left(\frac{N_2}{N_1}\right)^2 J_1$ and $B_{2e} = B_2 + \left(\frac{N_2}{N_1}\right)^2 B_1$

4.8.2 Belt or Chain Drives

Belt and chain drives perform same function as that of gear train. Assuming that there is no slippage between belt and pulleys we can write,

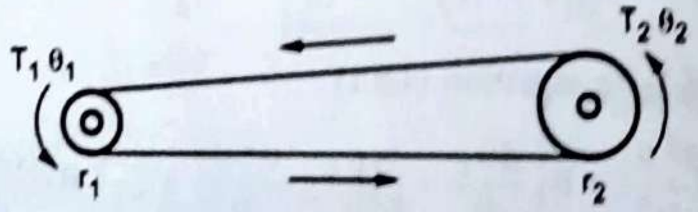


Fig. 4.8.4

$$\boxed{\frac{T_1}{T_2} = \frac{r_1}{r_2} = \frac{\theta_2}{\theta_1}}\ \text{for such drive}$$

4.8.3 Levers

The lever system is shown in the Fig. 4.8.5 This transmits translational motion and forces, similar to gear trains.

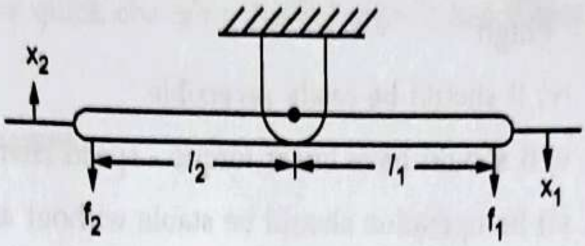


Fig. 4.8.5

By law of moment,

$$\boxed{f_1 \ell_1 = f_2 \ell_2}$$

By work done $f_1 x_1 = f_2 x_2$

Hence $\boxed{\dfrac{f_1}{f_2} = \dfrac{\ell_2}{\ell_1} = \dfrac{x_2}{x_1}}$

Review Questions

1. *Write a note on gear train used in mechanical systems.*
2. *Derive the expressions for the equivalent inertia and friction referred to primary side for a gear train with inertia and friction.*

4.9 Servomotors

The servosystem is one in which the output is some mechanical variable like position, velocity or acceleration. Such systems are generally automatic control systems which work on the error signals. The error signals are amplified to drive the motors used in such systems. These motors used in servosystems are called servomotors. These motors are usually coupled to the output shaft i.e. load through gear train for power matching.

These motors are used to convert electrical signal applied, into the angular velocity or movement of shaft.

4.9.1 Requirements of Good Servomotor

The servomotors which are designed for use in feedback control systems must have following requirements :

i) Linear relationship between electrical control signal and the rotor speed over a wide range.

ii) Inertia of rotor should be as low as possible. A servomotor must stop running without any time delay, if control signal to it is removed. For low inertia, it is designed with large length to diameter ratio, for rotors. Compared to its frame size, the rotor of a servomotor has very small diameter.

iii) Its response should be as fast as possible. For quickly changing error signals, it must react with good response. This is achieved by keeping torque to weight ratio high.

iv) It should be easily reversible.

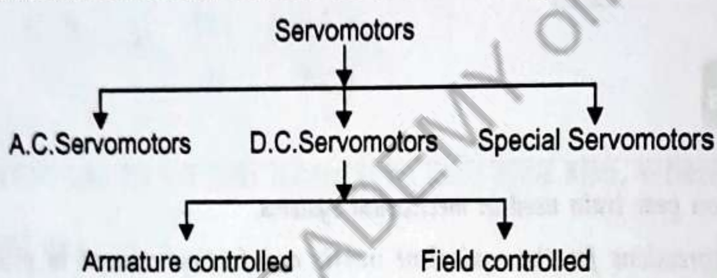
v) It should have linear torque - speed characteristics.

vi) Its operation should be stable without any oscillations or overshoots.

4.9.2 Types of Servomotors

The servomotors are basically classified depending upon the nature of the electric supply to be used for its operation.

The types of servomotors are as shown in the following chart :



4.10 D.C. Servomotor

Basically d.c. servomotor is more or less same as normal d.c.motor. There are some minor differences between the two. All d.c. servomotors are essentially separately excited type. This ensures linear torque-speed characteristics.

The control of d.c. servomotor can be from field side or from armature side. Depending upon this, these are classified as field controlled d.c. servomotor and armature controlled d.c. servomotor.

4.10.1 Field Controlled D.C. Servomotor

In this motor, the controlled signal obtained from the servoamplifier is applied to the field winding. With the help of constant current source, the armature current is maintained constant. The arrangement is shown in the Fig. 4.10.1.

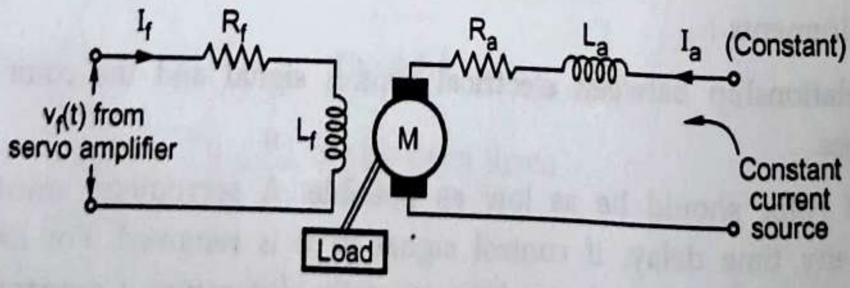


Fig. 4.10.1 Field controlled d.c. servomotor

This type of motor has large L_f /R_f ratio where L_f is reactance and R_f is resistance of field winding. Due to this the time constant of the motor is high. This means it can not give rapid response to the quick changing control signals hence this is uncommon in practice.

4.10.1.1 Features of Field Controlled D.C. Servomotor

It has following features :

i) Preferred for small rated motors.

ii) It has large time constant.

iii) It is open loop system. This means any change in output has no effect on the input.

iv) Control circuit is simple to design.

4.10.2 Armature Controlled D.C. Servomotor

In this type of motor, the input voltage `V_a' is applied to the armature with a resistance of R_a and inductance L_a. The field winding is supplied with constant current I_f. Thus armature input voltage controls the motor shaft output. The arrangement is shown in the Fig. 4.10.2.

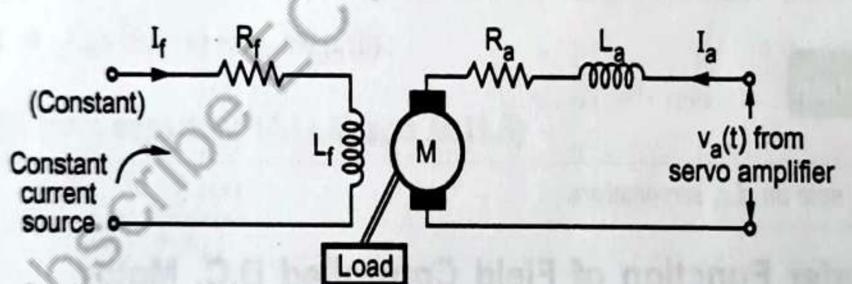


Fig. 4.10.2 Armature controlled d.c. servomotor

The constant field can be supplied with the help of parmanent magnets. In such case no field coils are necessary.

4.10.2.1 Features of Armature Controlled D.C. Servomotor

It has following features :

i) Suitable for large rated motors.

ii) It has small time constant hence its response is fast to the control signal.

iii) It is closed loop system.

iv) The back e.m.f. provides internal damping which makes motor operation more stable.

v) The efficiency and overall performance is better than field controlled motor.

As the armature controlled d.c. servomotor is closed loop system, in comparison with open loop field controlled system, generally armature controlled motors are used.

4.10.3 Characteristics of D.C. Servomotors

The characteristics of d.c. servomotors are mainly similar to the torque-speed characteristics of a.c. servomotor. The characteristics are shown in the Fig. 4.10.3.

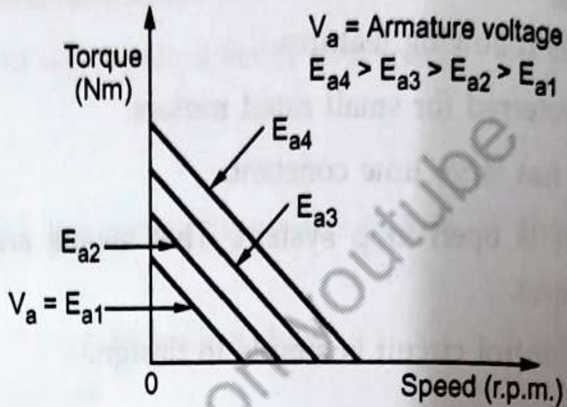


Fig. 4.10.3 Torque-speed characteristics for an armature controlled d.c. servomotor

4.10.4 Applications of D.C. Servomotor

These are widely used in air craft control systems, electromechanical actuators, process controllers, robotics, machine tools etc.

Review Question

1. *Write a note on d.c. servomotors.*

4.11 Transfer Function of Field Controlled D.C. Motor

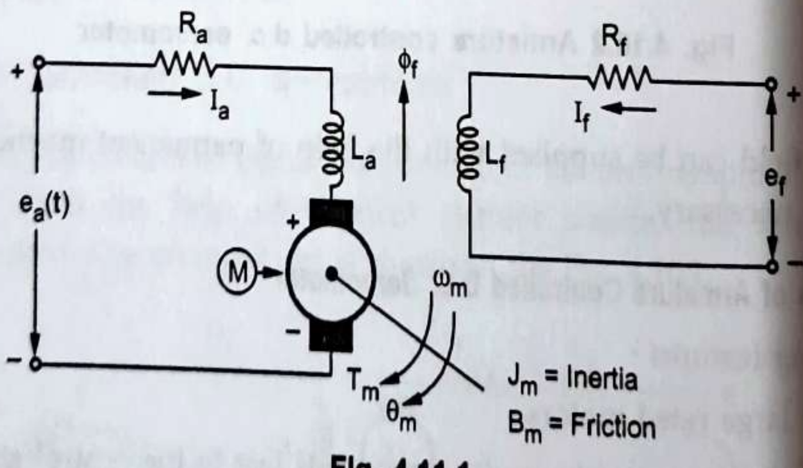


Fig. 4.11.1

Assumptions :

(1) Constant armature current is fed into the motor.

(2) $\phi_f \propto I_f$. Flux produced is proportional to field current.

$$\boxed{\phi_f = K_f I_f}$$

$\therefore$

(3) Torque is proportional to product of flux and armature current.

$$\boxed{T_m \propto \phi I_a}$$

$$\therefore T_m = K' \phi I_a = K' K_f I_f I_a = \boxed{K_m K_f I_f} \quad \text{... (4.11.1)}$$

Where $K_m = K' I_a$ = Constant

Apply Kirchhoff's law to field circuit, $L_f \dfrac{di_f}{dt} + R_f I_f = e_f$ ... (4.11.2)

Now shaft torque T_m is used for driving load against the inertia and frictional torque.

$$T_m = J_m \frac{d^2\theta_m}{dt^2} + B_m \frac{d\theta_m}{dt} \quad \text{... (4.11.3)}$$

Finding Laplace Transforms of equations (4.11.1), (4.11.2) and (4.11.3) we get,

$$T_m(s) = K_m K_f I_f(s) \quad \text{... (4.11.4)}$$

$$E_f(s) = (sL_f + R_f) I_f(s) \quad \text{... (4.11.5)}$$

$$T_m(s) = J_m s^2 \theta_m(s) + B_m s \theta_m(s) \quad \text{... (4.11.6)}$$

Eliminate $I_f(s)$ from equations (4.11.4) and (4.11.5)

$$T_m(s) = \frac{K_m K_f E_f(s)}{(sL_f + R_f)} \quad \text{... (4.11.7)}$$

Eliminate $T_m(s)$ from equations (4.11.6) and (4.11.7),

$$(s^2 J_m + sB_m)\theta_m(s) = \frac{K_m K_f E_f(s)}{(sL_f + R_f)}$$

Input = $E_f(s)$ and Output = Rotational displacement $\theta_m(s)$

$\therefore$ Transfer function = $\dfrac{\theta_m(s)}{E_f(s)}$

$$\frac{\theta_m(s)}{E_f(s)} = \frac{K_m K_f}{(J_m s^2 + sB_m)(R_f + sL_f)} = \frac{K_m K_f}{sR_f B_m [1 + s\tau_m][1 + s\tau_f]}$$

Where, $\tau_m = \dfrac{J_m}{B_m}$ = Motor time constant, $\tau_f = \dfrac{L_f}{R_f}$ = Field time constant

$$\boxed{\text{T.F.} = \frac{\theta_m(s)}{E_f(s)} = \frac{K_f}{R_f[1 + s\tau_f]} \cdot \frac{K_m}{B_m(1 + s\tau_m)} \cdot \frac{1}{s}}$$

Block diagram for field controlled d.c. motor is as shown in Fig. 4.11.2.

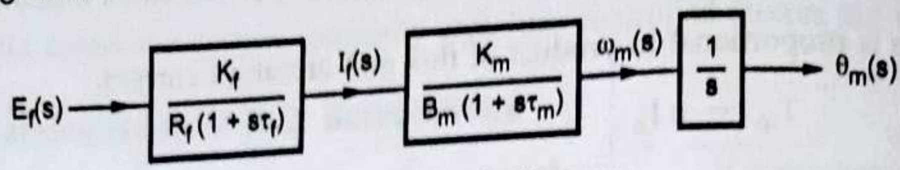


Fig. 4.11.2 Block diagram

Review Question

1. *Derive the transfer function of field controlled d.c. servomotor.*

4.12 Transfer Function of Armature Controlled D.C. Motor

VTU : July-04, 09, 12, Jan.-06, 11

Assumptions :

(i) Flux is directly proportional to current through field winding,

$$\phi_m = K_f I_f = \text{Constant}$$

(ii) Torque produced is proportional to product of flux and armature current.

$$T = K'_m \phi I_a$$

$$T = K'_m K_f I_f I_a$$

(iii) Back e.m.f. is directly proportional to shaft velocity ω_m, as flux ϕ is constant.

as $\omega_m = \frac{d\theta(t)}{dt}$

$$E_b = K_b \omega_m(s) = K_b s\theta_m(s)$$

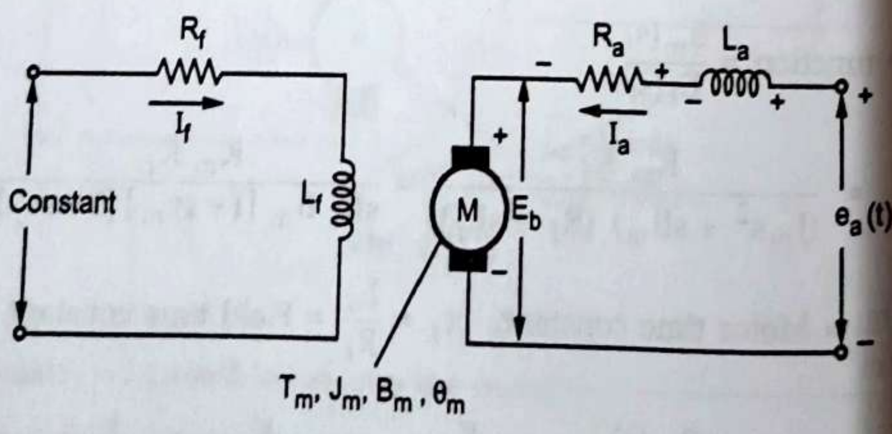


Fig. 4.12.1

Apply Kirchhoff's law to armature circuit :

$$e_a = E_b + I_a (R_a) + L_a \frac{di_a}{dt}$$

Take Laplace transform,

$$\therefore \quad E_a(s) = E_b(s) + I_a(s)[R_a + s L_a] \quad \text{i.e.} \quad I_a(s) = \frac{E_a(s) - E_b(s)}{R_a + s L_a}$$

$$I_a(s) = \frac{E_a(s) - K_b\, s\,\theta_m(s)}{R_a + s L_a}$$

Now $\quad T_m = K'_m K_f I_f I_a$

$$T_m = K'_m K_f I_f \left\{ \frac{E_a - K_b\, s\,\theta_m(s)}{R_a + s L_a} \right\}$$

Also $\quad T_m = \{J_m s^2 + s B_m\}\theta_m(s) \quad$ ... from equation (3)

Equating equations of T_m,

$$\frac{K'_m K_f I_f E_a(s)}{(R_a + s L_a)} = \frac{K'_m K_f I_f K_b\, s\,\theta_m(s)}{(R_a + s L_a)} + (J_m s^2 + s B_m)\theta_m(s)$$

$$\therefore \quad \frac{K'_m K_f I_f}{(R_a + s L_a)} E_a(s) = \left[\frac{K'_m K_f I_f K_b\, s}{(R_a + s L_a)} + J_m s^2 + s B_m \right] \theta_m(s)$$

$$\therefore \quad \boxed{\frac{\theta_m(s)}{E_a(s)} = \frac{\dfrac{K_m}{s R_a B_m (1 + s\tau_m)(1 + s\tau_a)}}{1 + \dfrac{K_m \cdot sK_b}{s R_a B_m (1 + s\tau_m)(1 + s\tau_a)}} = \frac{G(s)}{1 + G(s)H(s)}}$$

where $\tau_m = J_m/B_m$ and $\quad \tau_a = \dfrac{L_a}{R_a}$ while $K_m = K'_m K_f$

$$G(s) = \frac{K_m}{s R_a B_m (1 + s\tau_m)(1 + s\tau_a)} \quad \text{and} \quad H(s) = sK_b$$

Therefore it can be represented in its block diagram form as in Fig. 4.12.2.

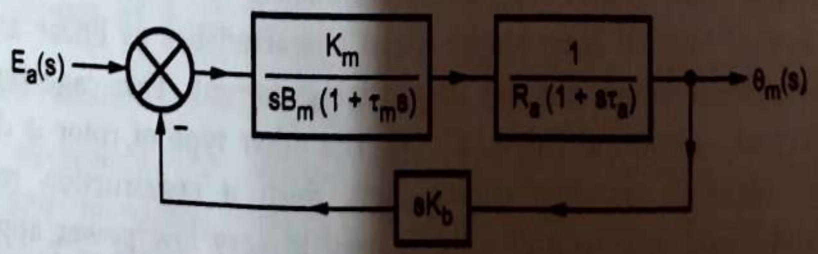


Fig. 4.12.2 Block diagram

Key Point *Field controlled d.c. motor is open loop while armature controlled is closed loop system. Hence armature controlled d.c. motors are preferred over field controlled type.*

Review Question

1. *Obtain the block diagram representation of the transfer function of armature controlled d.c. servomotor.* **VTU : Jan.-06, 11, July-04, 09, 12, Marks 6**

4.13 A.C. Servomotor

VTU : Jan.-03, Dec.-05

Most of the servomotors used in low power servomechanisms are a.c. servomotors. The a.c. servomotor is basically two phase induction motor. The output power of a.c. servomotor varies from fraction of watt to few hundred watts. The operating frequency is 50 Hz to 400 Hz.

4.13.1 Construction

It is mainly divided into two parts namely stator and rotor.

The stator carries two windings, uniformly distributed and displaced by 90°, in space. One winding is called **main winding or fixed winding or reference winding**. This is excited by a constant voltage a.c. supply. The other winding is called **control winding**. It is excited by variable control voltage, which is obtained from a servoamplifier. This voltage is 90° out of phase with respect to the voltage applied to the reference winding. This is necessary to obtain rotating magnetic field. The schematic stator is shown in the Fig 4.13.1.

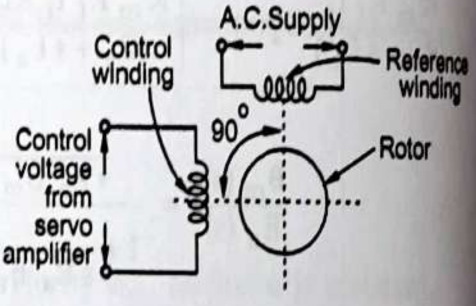
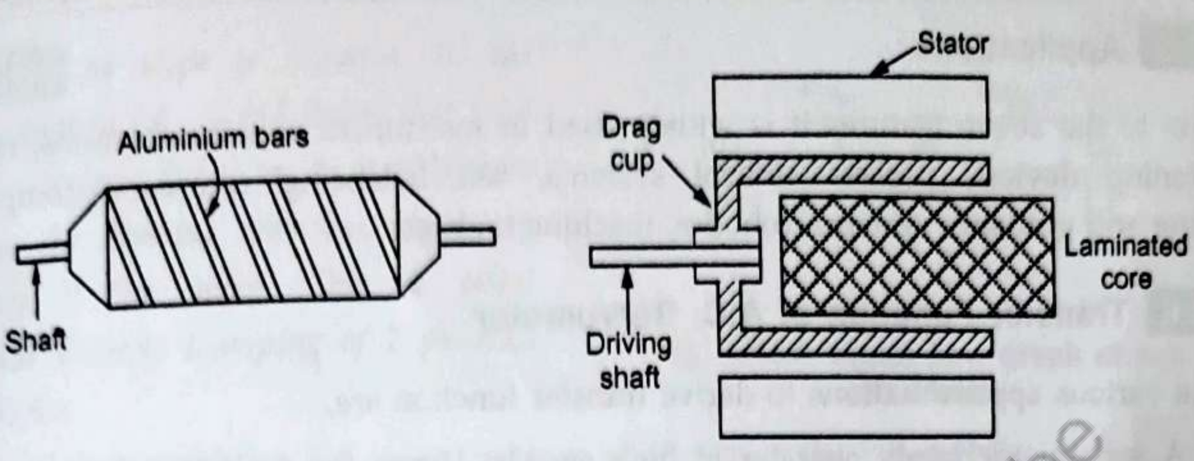


Fig. 4.13.1 Stator of A.C. servomotor

4.13.2 Rotor

The rotor is generally of two types. The one is usual squirrel cage rotor. This has small diameter and large length. Aluminium conductors are used to keep weight small. Its resistance is very high to keep torque-speed characteristics as linear as possible. Air gap is kept very small which reduces magnetizing current. This cage type of rotor is shown with skewed bars in the Fig. 4.13.2 (a). The other type of rotor is drag cup type. There are two air gaps in such construction. Such a construction reduces inertia considerably and hence such type of rotor is used in very low power applications. The aluminium is used for the cup construction. The construction is shown in the Fig. 4.13.2 (b).



(a) Squirrel cage rotor (b) Drag cup type rotor

Fig. 4.13.2

4.13.3 Torque-speed Characteristics

The torque-speed characteristics of a two phase induction motor, mainly depends on the ratio of reactance to resistance. For small X to R ratio i.e. high resistance low reactance motor, the characteristics is much more linear while it is nonlinear for large X to R ratio as shown in the Fig. 4.13.3.

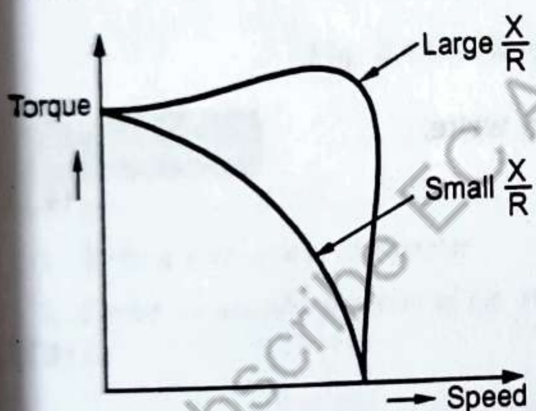


Fig. 4.13.3

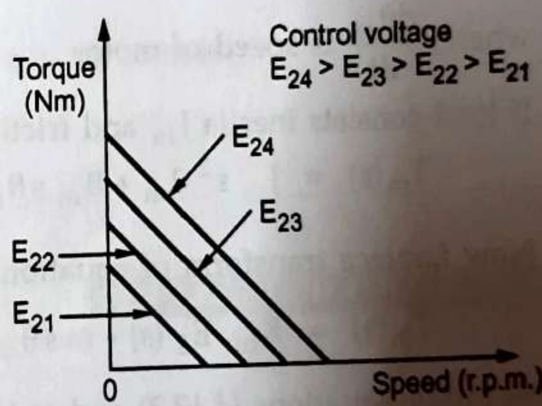


Fig. 4.13.4

In practice, design of the motor is so as to get almost linear torque-speed characteristics. The Fig. 4.13.4 shows the torque-speed characteristics for various control voltages. The torque varies almost linearly with speed. All the characteristics are equally spaced for equal increments of control voltage. It is generally operated with low speeds.

4.13.4 Features of A. C. Servomotor

The a.c. servomotor has following features :

i) Light in weight. ii) Robust construction. iii) Reliable and stable operation. iv) Smooth and noise free operation. v) Large torque to weight ratio. vi) Large R to X ratio i.e. small X to R ratio. vii) No brushes or slip rings hence maintenance free. viii) Simple driving circuits.

4.13.5 Applications

Due to the above features it is widely used in instrument servomechanisms, remote positioning devices, process control systems, self balancing recorders, computers, tracking and guidance systems, robotics, machine tools etc.

4.13.6 Transfer Function of A.C. Servomotor

The various approximations to derive transfer function are,

(i) A servomotor rarely operates at high speeds. Hence for a given value of control voltage, $T \propto N$ characteristics are perfectly linear.

(ii) In order that $T \propto N$ characteristics are directly proportional to voltage applied to its control phase, we assume $T \propto N$ characteristics are straight lines and equally spaced.

Torque at any speed 'N' is,

$$T_m = K_{tm} E_{2t} + m \frac{d\theta_m}{dt} \qquad \text{... (4.13.1)}$$

where, $\frac{d\theta_m}{dt}$ is speed of motor.

If load consists inertia J_m and friction B_m we can write,

$$T_m(s) = J_m s^2 \theta_m + B_m s \theta_m \qquad \text{.... (4.13.2)}$$

Now Laplace transform of equation (4.13.1) is

$$T_m(s) = K_{tm} E_2(s) + m s \theta_m(s) \qquad \text{... (4.13.3)}$$

Equating equations (4.13.2) and (4.13.3)

$$\therefore \; K_{tm} E_2(s) + m s \theta_m(s) = J_m s^2 \theta_m(s) + B_m s \theta_m(s)$$

$$\therefore \quad \frac{\theta_m(s)}{E_2(s)} = \frac{K_{tm}}{s(sJ_m - m + B_m)} = \frac{K_{tm}}{s(B_m - m)\left[1 + \frac{sJ_m}{(B_m - m)}\right]}$$

$$\therefore \quad \boxed{\frac{\theta_m(s)}{E_2(s)} = \frac{K_m}{s(1 + \tau_m s)}}$$

where $\quad K_m = \frac{K_{tm}}{B_m - m}$

and $\quad \tau_m = \frac{J_m}{B_m - m}$

Key Point *As slope is negative, in the above equation [$B_m - m$] shows that total friction increases due to m. As it adds more friction, the damping improves, improving stability of the motor. This is called* ***Internal Electric Damping of 2 ph A.C.*** *servomotor.*

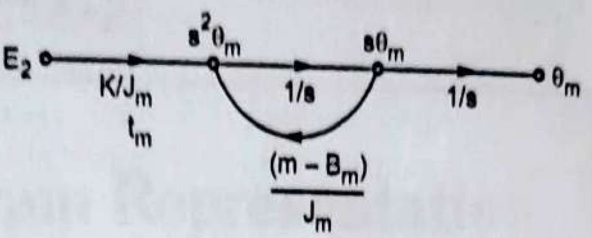


Fig. 4.13.5 Signal flow graph of a.c. servomotor

Signal flow graph for A.C. servomotor is as shown in the Fig. 4.13.5.

Hence block diagram of A.C. servomotor is

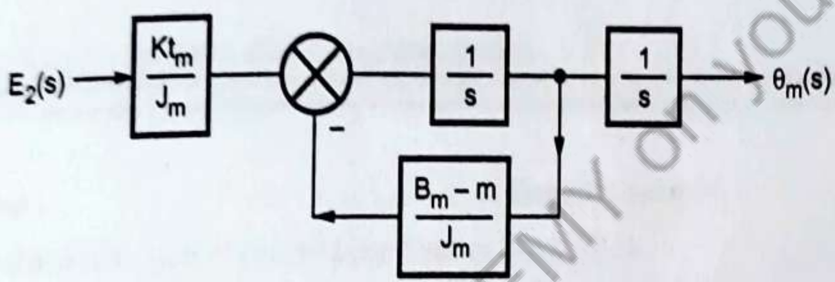


Fig. 4.13.6 Block diagram of a.c. servomotor

Review Questions

1. *Write a note on a.c. servomotor.*
2. *Derive the transfer function of a.c. servomotor.* **VTU : Jan.-03, Dec.-05, Marks 6**

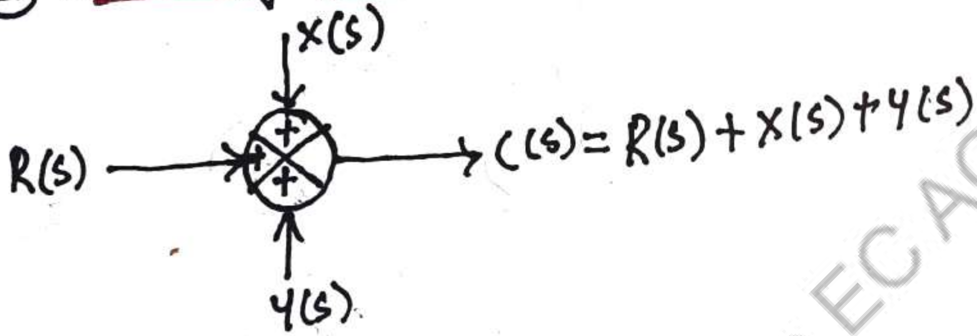
Introduction to Block Diagram Representation

→ control s/m → no elements → "Block"

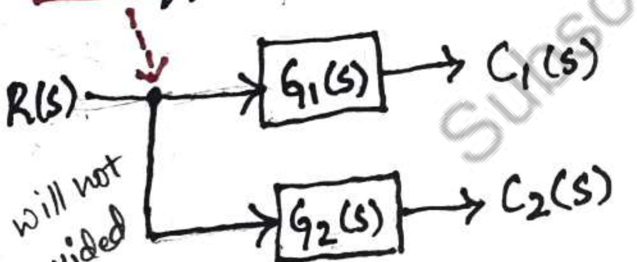
Elements :

① Block

② Summing Point



③ Take off Point (or) Branch Point



i/p will not be devided

Advantages

→ Simple to construct Block diagram for Complex s/ms

→ Easy to find fun of of individual elements

→ overall fun of the s/m can be easily studies

→ Easily Calculate T.F. of the closed loop.

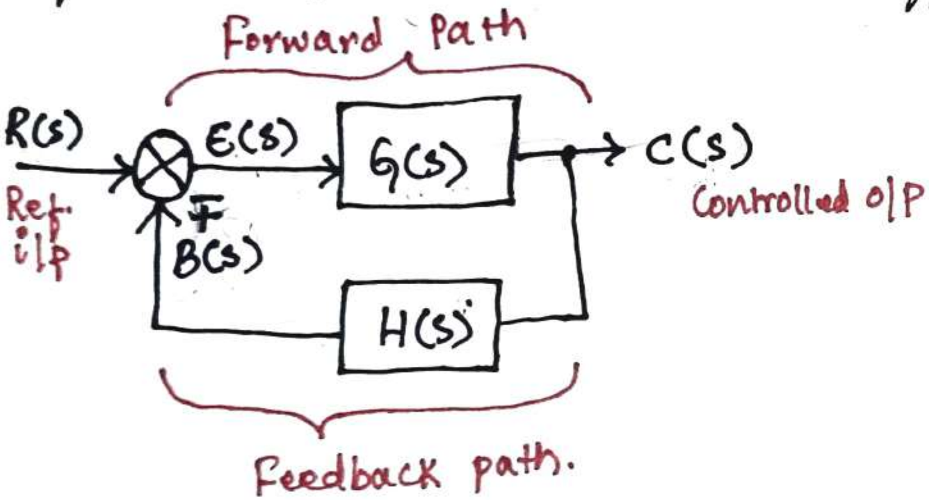
Disadvantages.

→ Does not include any information about physical condition of the s/m

→ Source of energy is not shown

Simple or Canonical form of closed loop System:

A block diagram which consist of one block in forward path, one block in feedback path and one take off point & one summing point.



$B(s) \rightarrow$ Feed back Signal
$E(s) \rightarrow$ Error Signal.

Transfer fun:

$E(s) = R(s) \mp B(s) \longrightarrow (1)$

$B(s) = C(s) \cdot H(s) \longrightarrow (2)$

$C(s) = E(s) \cdot G(s) \longrightarrow (3)$

Put (2) in (1)

$$E(s) = R(s) \mp C(s) \cdot H(s)$$

$$(3) \Rightarrow E(s) = \frac{C(s)}{G(s)}$$

$$\frac{C(s)}{G(s)} = R(s) \mp C(s) \cdot H(s)$$

$$C(s) = R(s)\,G(s) \mp C(s)\,G(s)\,H(s)$$

$$C(s) \pm C(s)\,G(s)\,H(s) = R(s) \cdot G(s)$$

$$C(s)\left[1 \pm G(s)\,H(s)\right] = R(s) \cdot G(s)$$

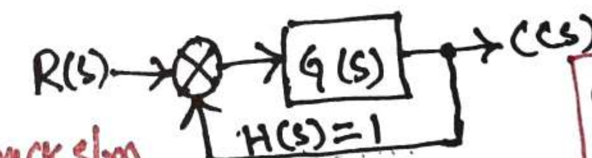
$$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)\,H(s)}} = \frac{G}{1 \pm GH}$$

For −ve F.B. s/m ↳ + sign

For +ve FB s/m ↳ − sign.

If $H(s) = 1$

Unity Feedback s/m



$$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)}}$$

For negative feedback control s/m, Starting from fundamentals, show that closed loop transfer fun $M(s)$ is given by,

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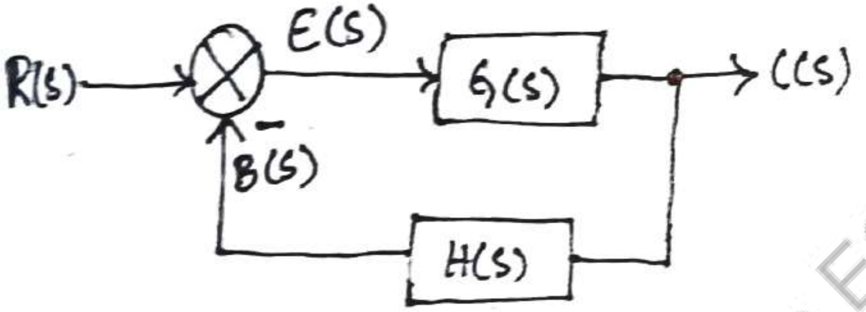
$$M(s) = \frac{N_g D_h}{(D_g D_h + N_g N_h)} \quad \text{where} \quad G(s) = \frac{N_g}{D_g}, \quad H(s) = \frac{N_h}{D_h}.$$



$$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}}$$

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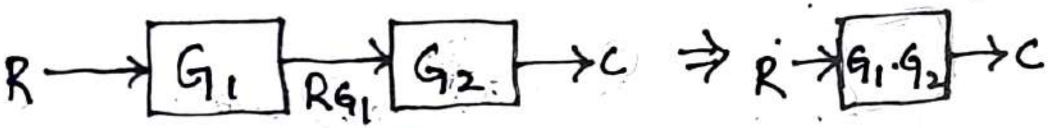
$$M(s) = \frac{\frac{N_g}{D_g}}{1+\frac{N_g}{D_g}\cdot\frac{N_h}{D_h}} = \frac{\frac{N_g}{\cancel{D_g}}}{\frac{D_g D_h + N_g N_h}{\cancel{D_g} D_h}}$$

$$\boxed{M(s) = \frac{N_g D_h}{D_g D_h + N_g N_h}}$$

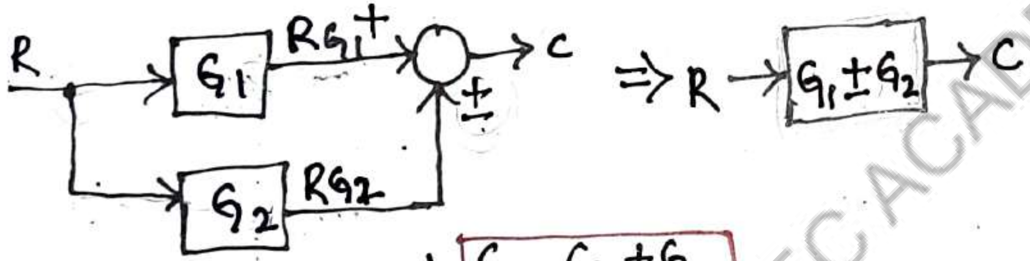
Rules for Block diagram Reduction:

① Blocks in Cascade



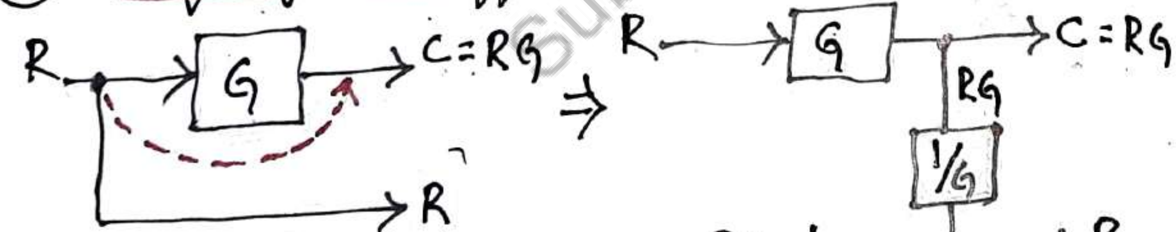
$$C = RG_1 \cdot G_2 \Rightarrow \boxed{\frac{C}{R} = G_1 \cdot G_2}$$

② Blocks in parallel



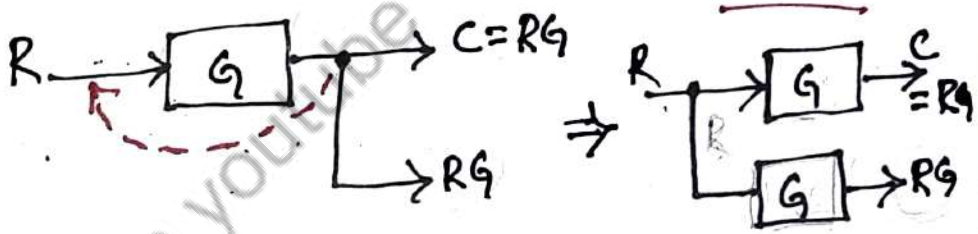
$$C = RG_1 \pm RG_2 \Rightarrow \boxed{\frac{C}{R} = G_1 \pm G_2}$$

③ Shifting Take off point after a block

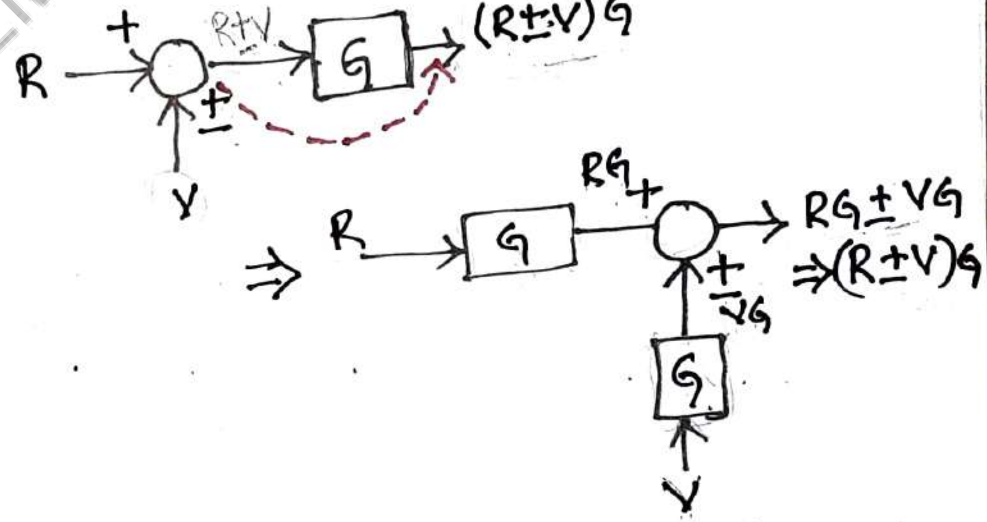


$$RG \cdot \frac{1}{G} = R$$

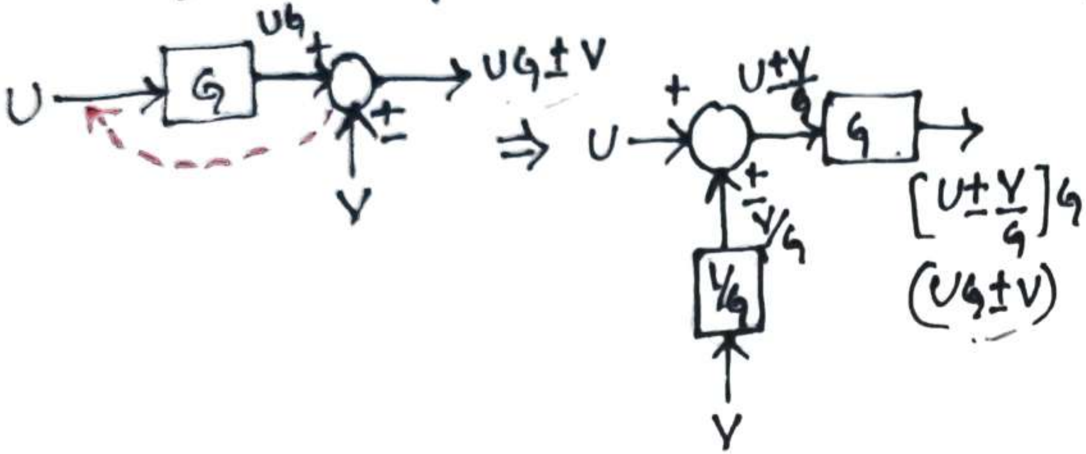
④ Shifting take off point before a block



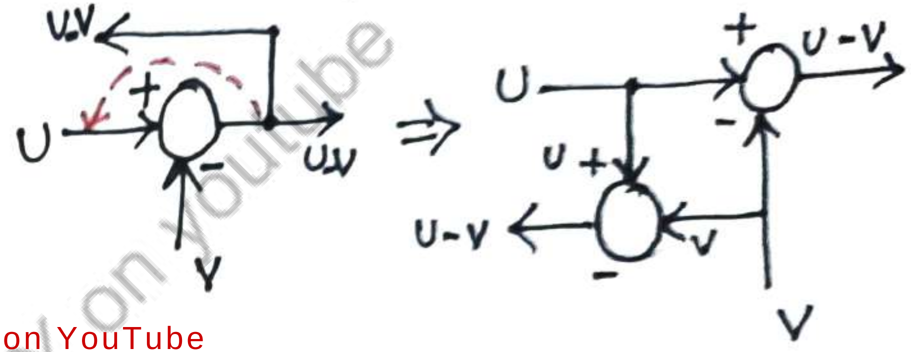
⑤ Moving Summing point after block



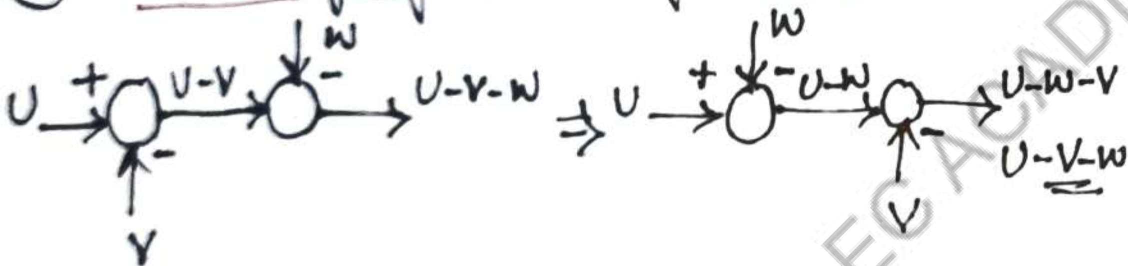
(7) Moving Summing point before block



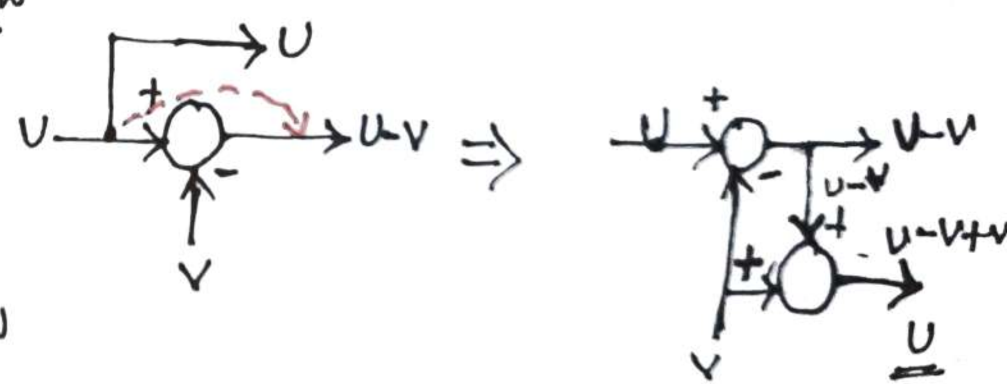
(10) Shifting take off point before Summing point



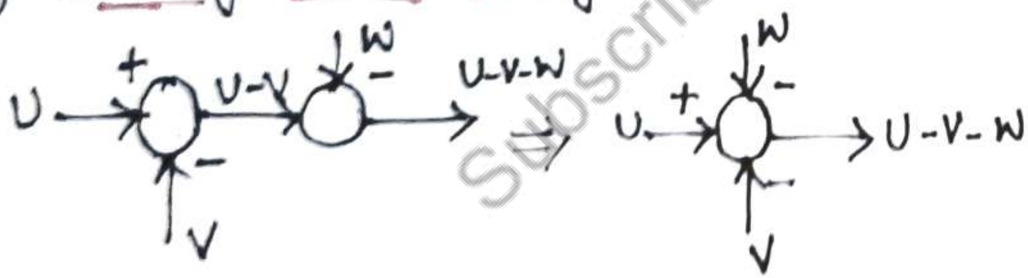
(8) Rearranging Summing point



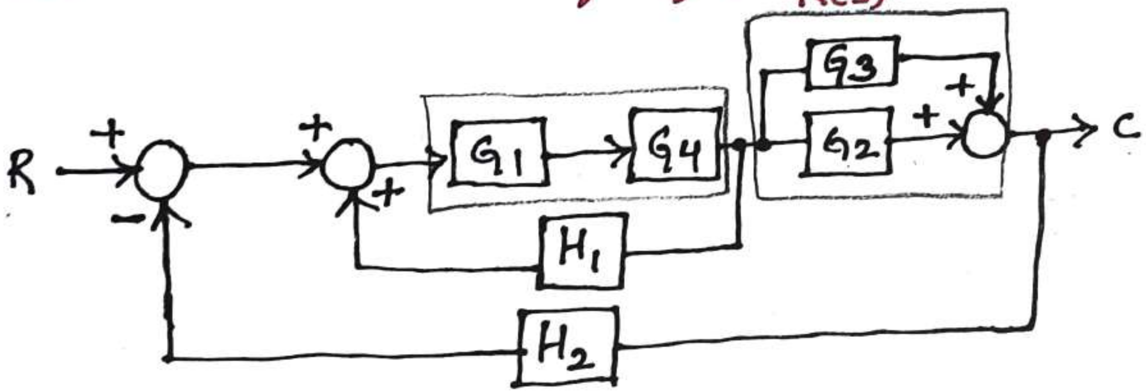
(11) Shifting take off point after Summing point

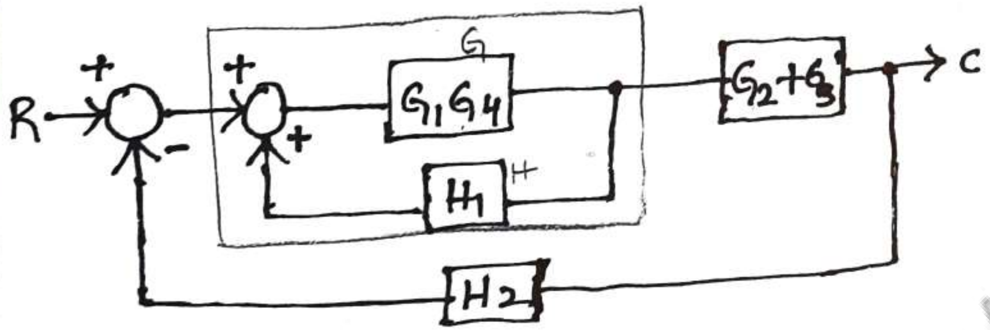


(9) Adding the Summing Point



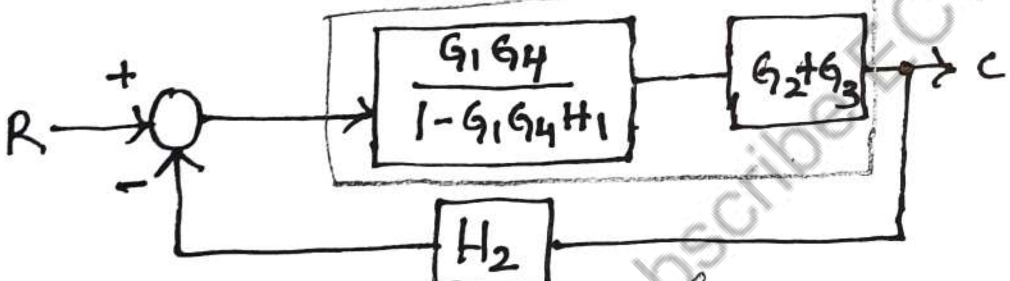
Determine the Transfer fun $\frac{C(s)}{R(s)}$

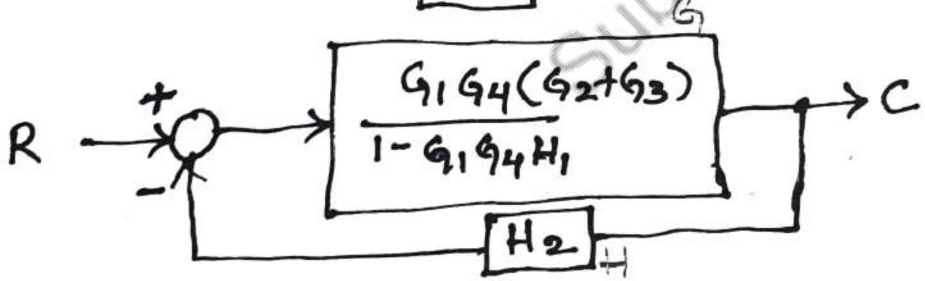




+Ve FB

$\frac{G}{1-GH}$





-VeFB

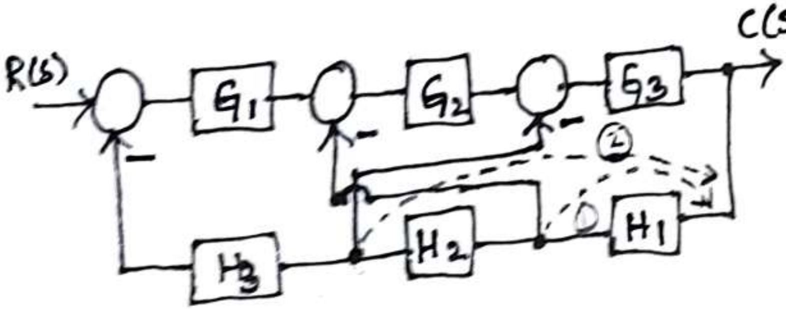
$\frac{G}{1+GH}$

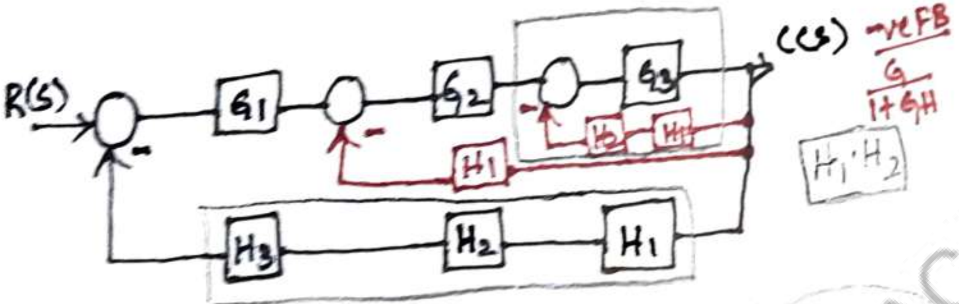
$$\frac{C(s)}{R(s)} = \frac{\dfrac{G_1G_4(G_2+G_3)}{1-G_1G_4H_1}}{1+\dfrac{G_1G_4(G_2+G_3)}{1-G_1G_4H_1}\cdot H_2}$$

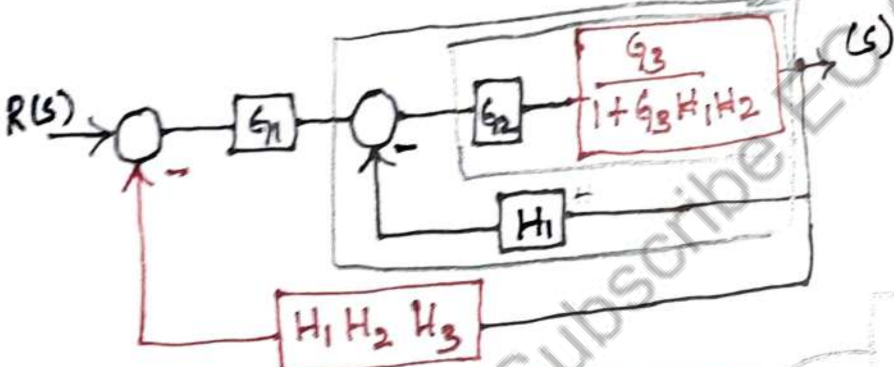
$$\frac{C(s)}{R(s)} = \frac{\dfrac{G_1G_4(G_2+G_3)}{\cancel{1-G_1G_4H_1}}}{\dfrac{1-G_1G_4H_1+G_1G_4(G_2+G_3)H_2}{\cancel{1-G_1G_4H_1}}}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{G_1G_4(G_2+G_3)}{1-G_1G_4H_1+G_1G_4(G_2+G_3)H_2}}$$

Obtain $\frac{C(s)}{R(s)}$ using Block diagram reduction rules.

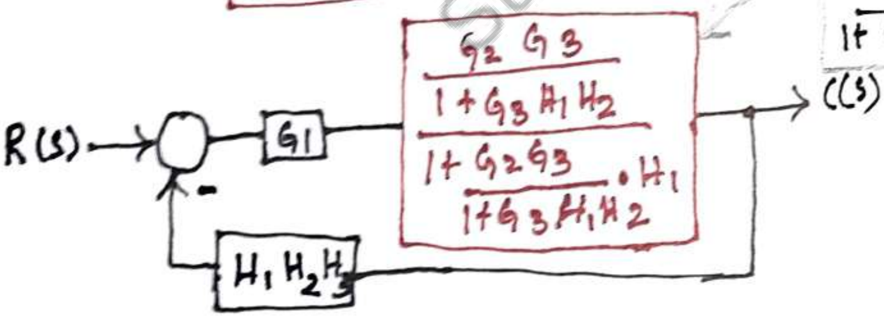


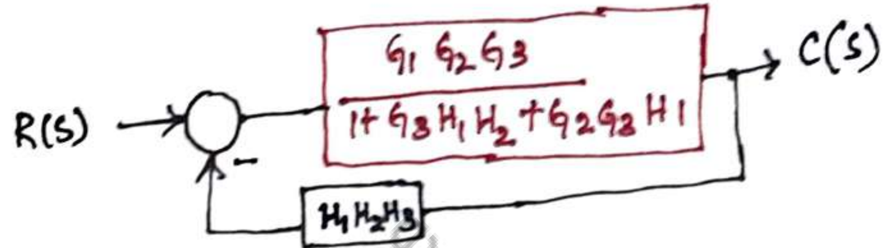




$$\frac{G_2G_3}{1+G_3H_1H_2}$$

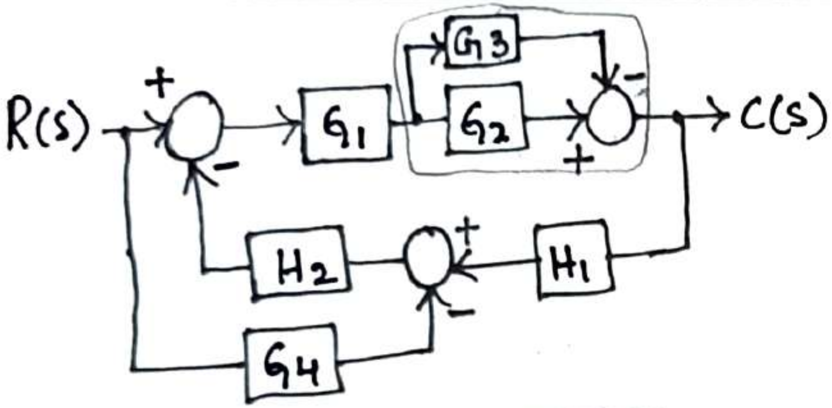
$$\frac{G_2G_3}{1+G_3H_1H_2+G_2G_3H_1}$$

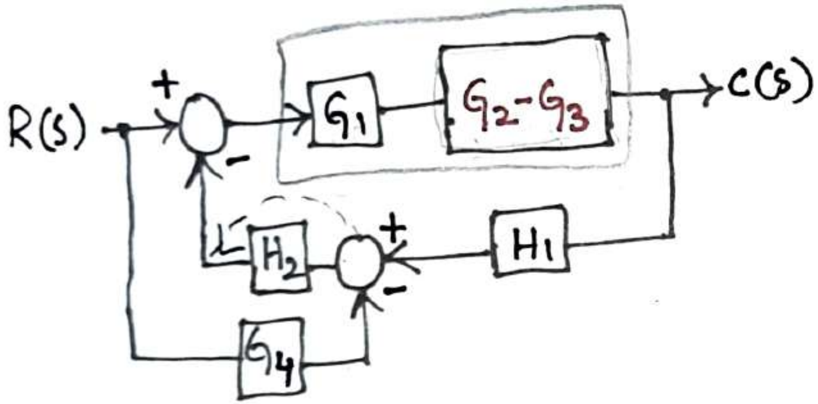


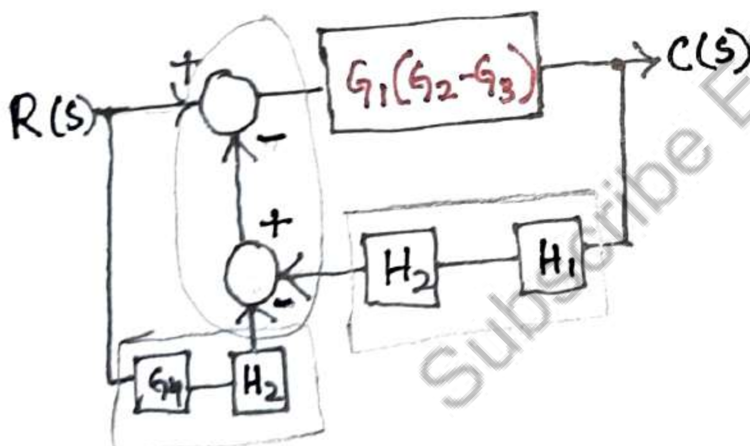


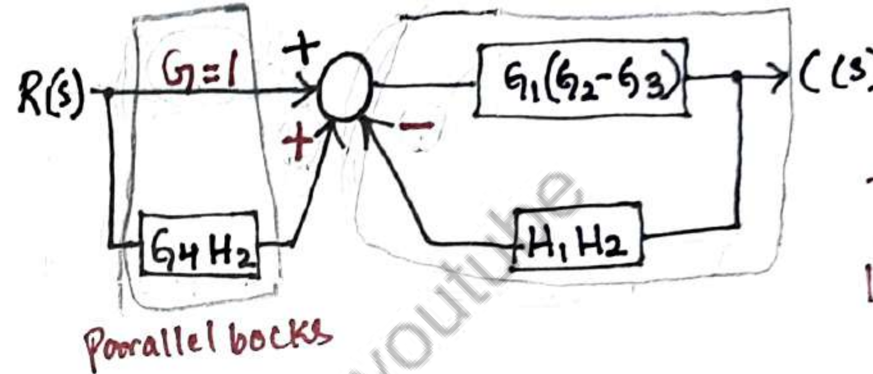
$$\frac{C(s)}{R(s)} = \frac{\frac{G_1G_2G_3}{1+G_3H_1H_2+G_2G_3H_1}}{1+\frac{G_1G_2G_3\cdot H_1H_2H_3}{1+G_3H_1H_2+G_2G_3H_1}}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{G_1G_2G_3}{1+G_3H_1H_2+G_2G_3H_1+G_1G_2G_3\cdot H_1H_2H_3}}$$



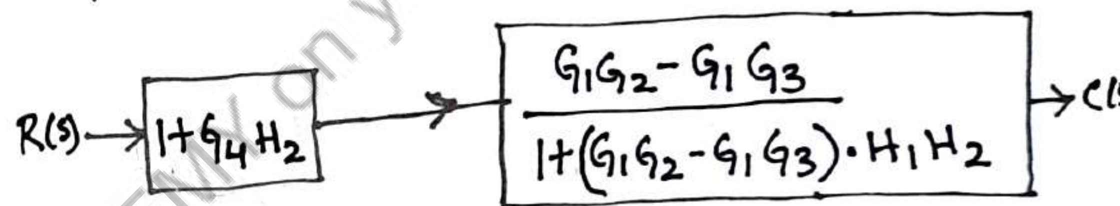






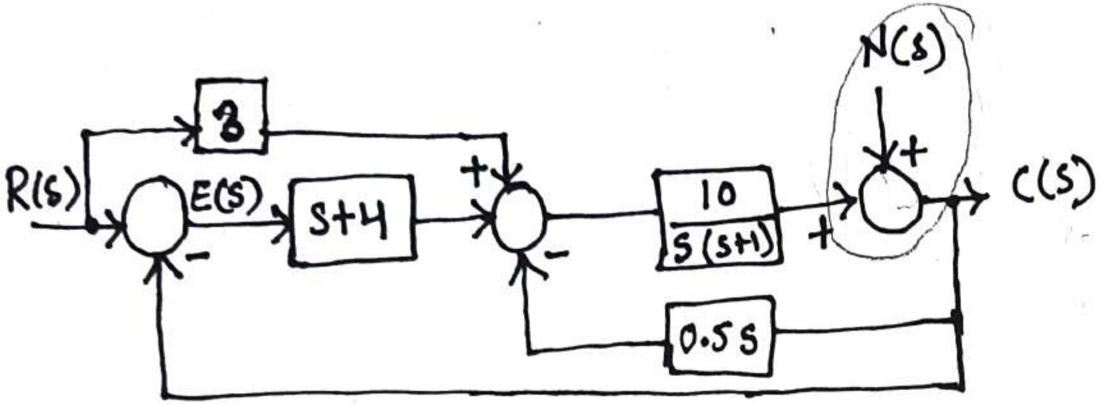
-ve

$$\frac{G}{1+GH}$$

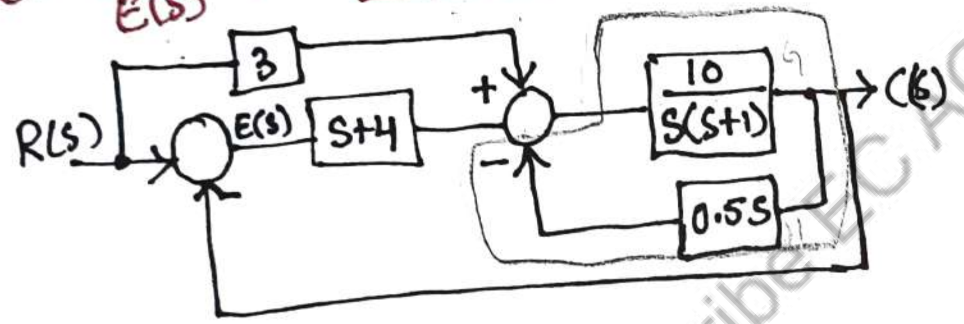


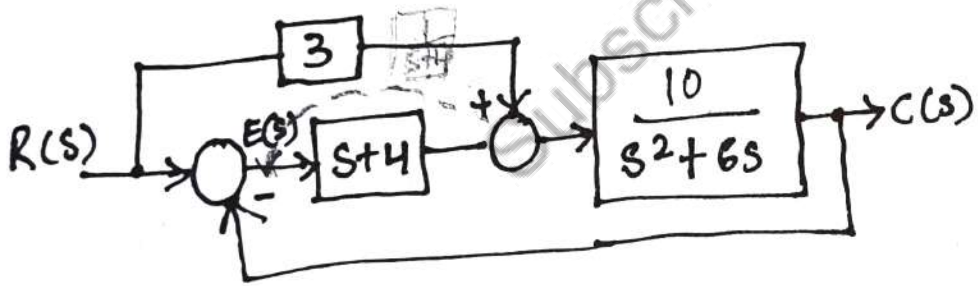
$$\boxed{\frac{C(s)}{R(s)} = \frac{(1+G_4H_2)(G_1G_2-G_1G_3)}{1+(G_1G_2-G_1G_3)\cdot H_1H_2}}$$

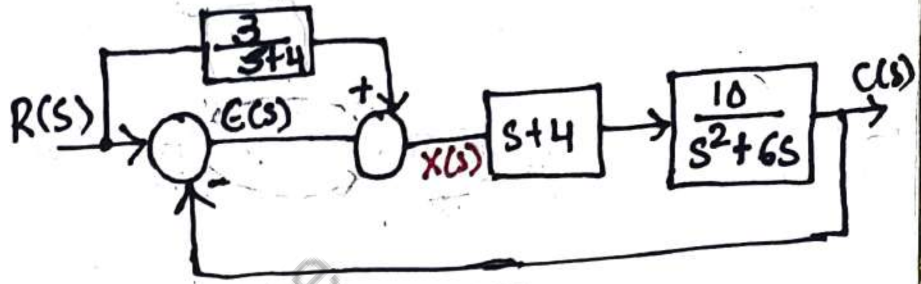
Find (i) $\frac{C(s)}{E(s)}$ if $N(s)=0$ (ii) $\frac{C(s)}{R(s)}$ if $N(s)=0$



(i) $\frac{C(s)}{E(s)}$ with $\boxed{N(s)=0}$







$$\therefore E(s) = R(s) - C(s) \rightarrow (1)$$

$$C(s) = X(s)\cdot\left[\frac{(s+4)10}{s^2+6s}\right] \rightarrow (2)$$

$$X(s) = E(s) + \left[\frac{3}{s+4}\right]R(s) \rightarrow (3)$$

$$\frac{s^2+6s}{10(s+4)}C(s) = E(s) + \left[\frac{3}{s+4}\right]\left[E(s)+C(s)\right]$$

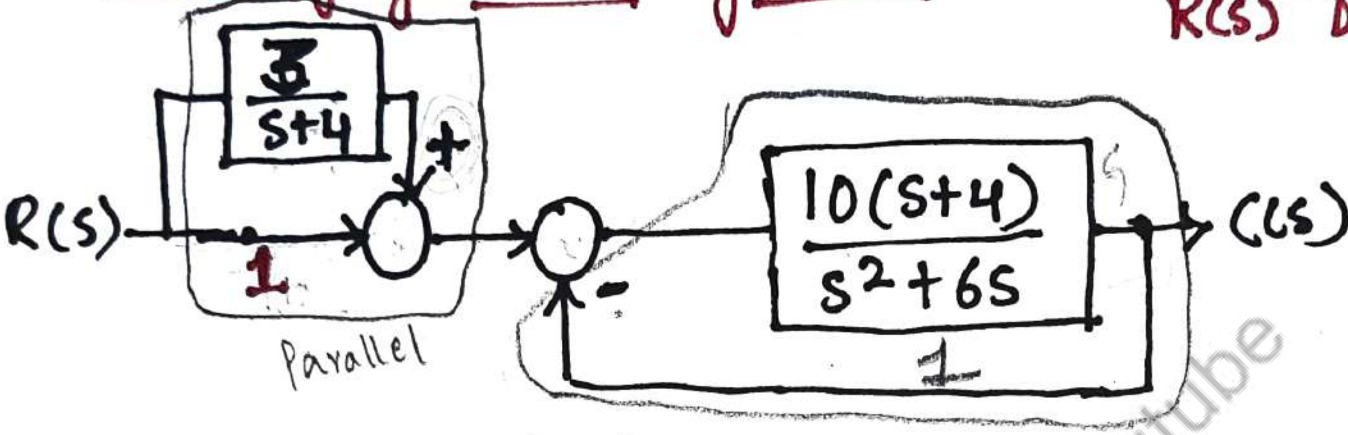
$$\frac{s^2+6s}{10(s+4)}C(s) = E(s) + \frac{3}{s+4}E(s) + \frac{3}{s+4}C(s)$$

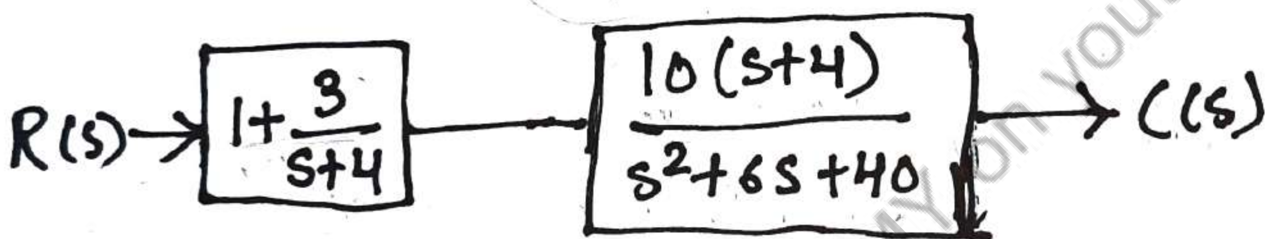
$$\left[\frac{s^2+6s}{10(s+4)} - \frac{3}{s+4}\right]C(s) = \left[1+\frac{3}{s+4}\right]E(s)$$

$$\left[\frac{s^2+6s-30}{10(s+4)}\right]C(s) = \left[\frac{s+7}{\cancel{(s+4)}}\right]E(s)$$

$$\therefore \boxed{\frac{C(s)}{E(s)} = \frac{10(s+7)}{s^2+6s-30}}$$

Exchanging summing points (ii) $\frac{C(s)}{R(s)}$ if $\boxed{N=0}$





$$\frac{C(s)}{R(s)} = \left(\frac{s+7}{s+4}\right) \times \left[\frac{10(s+4)}{s^2+6s+40}\right]$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{10\cdot(s+7)}{s^2+16s+40}}$$

Assigment

(iii) $\frac{C(s)}{N(s)}$ if $R(s)=0$

$$\boxed{\frac{C(s)}{N(s)} = \frac{3(s+1)}{s^2+16s+40}}$$

Signal Flow Graph

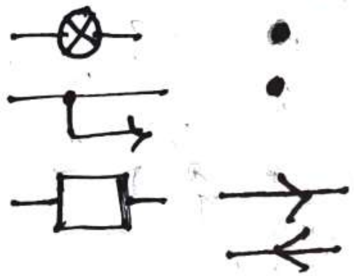
"Graphical representation of the Algebraic eqns that represents a System"

SFG

Summing point

Takeoff point

Signal (or) block



Ex:-

$R(s) \rightarrow \boxed{G(s)} \rightarrow C(s)$

Basic Elements [TERMINOLOGIES]

⇒ **BRANCH:** Line that joins two nodes

It has both gain & Direction.

$x_1 \xrightarrow{4} x_2$

$\boxed{x_2 = 4x_1}$

⇒ **NODE:** Point that represents either a variable (or) signal.

* **Input Node [source node]**

A node which has only outgoing branches

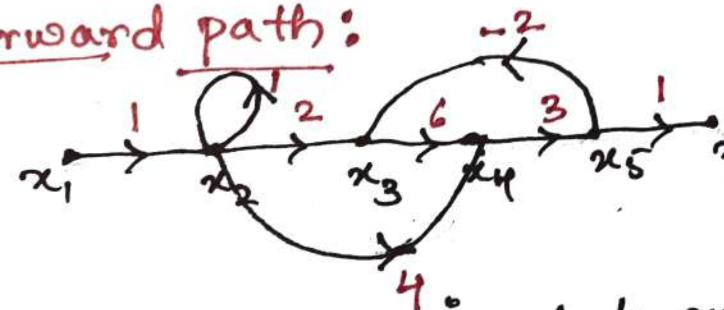
* **Output Node [sink node]**

A node which has only incoming branches.

* **Mixed Node [chain node]**

A node which has both incoming and outgoing branches

⇒ **Forward path:**



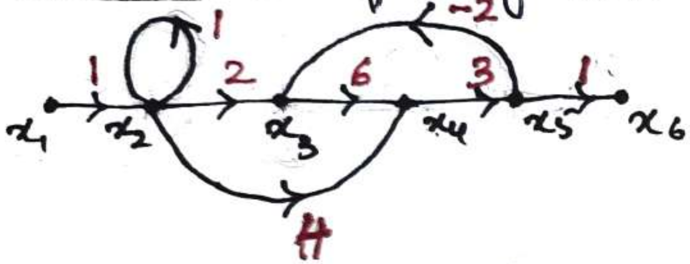
A path from input to output node

$x_1 - x_2 - x_3 - x_4 - x_5 - x_6$

$x_1 - x_2 - x_4 - x_5 - x_6$

⇒ Feedback loop:-

A path originates at a node and terminates at same node, travelling through at least one other node, with out tracing any node twice



$x_3 - x_4 - x_5 - x_3$

⇒ Self loop:

A feedback loop consisting of only one node. x_2

⇒ Non-touching loop:

If there is no node common between two or more loops.

$\underline{\underline{x_2}}$ $\quad$ $x_3 - \underline{x_4 - x_5} - x_3$

⇒ Loop gain:

It is the product of all gain of branch that form a loop

loop: $x_2 \Rightarrow \text{gain} = \boxed{1}$

$x_3 - x_4 - x_5 - x_3 \Rightarrow \text{gain} = 6 \times 3 \times -2$
$= \boxed{-36}$

⇒ Path gain:

Product of branch gain while going through a Forward path

$x_1 - x_2 - x_3 - x_4 - x_5 - x_6 = 1 \times 2 \times 6 \times 3 \times 1$
$= \boxed{36}$

$x_1 - x_2 - x_4 - x_5 - x_6 = 1 \times 4 \times 3 \times 1$
$= \boxed{12}$

⇒ DUMMY NODE:

A branch having gain one (1) can be added at i/p & o/p.

Properties of Signal Flow Graph:

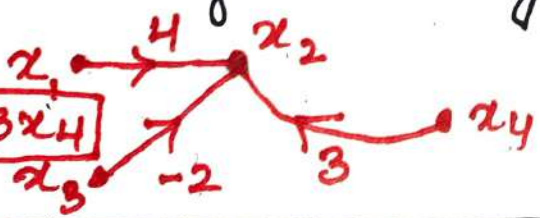
1. Signal flow graph is applicable only to Linear time invariant systems [LTI]

2. The signal will flow along the branches and along the arrow associated with the branches.

3. The signal gets multiplied when travelling along the branches, by the gain.

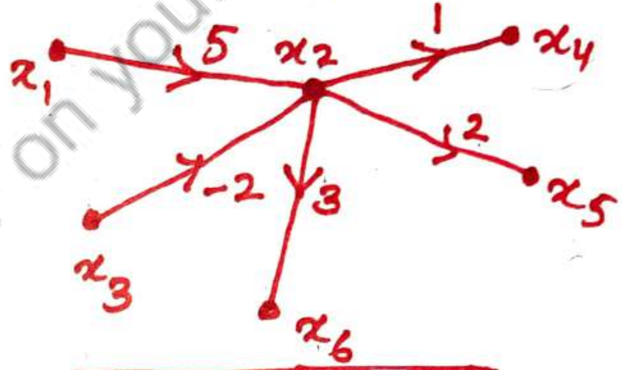
$$\boxed{x_2 = 4x_1}$$

4. The value of node is an algebraic sum of all the signal entering the node



$$\therefore \boxed{x_2 = 4x_1 - 2x_3 + 3x_4}$$

5. The value of any node is avilable to all branches leaving that node.



$$\therefore \boxed{x_2 = 5x_1 - 2x_3}$$

$x_4 = x_2$

$x_5 = 2x_2$

$x_6 = 3x_2$

6. For a given system the signal flow graph is not unique.

Procedure to Obtain Signal Flow Graph

① From Linear equations

$x_2 = 4x_1 + 3x_3 + 2x_4 \rightarrow (1)$

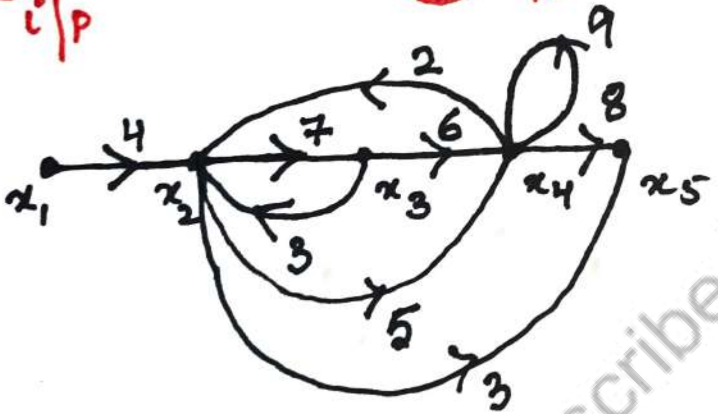
$x_3 = 7x_2 \rightarrow (2)$

$x_4 = 5x_2 + 6x_3 + 9x_4 \rightarrow (3)$

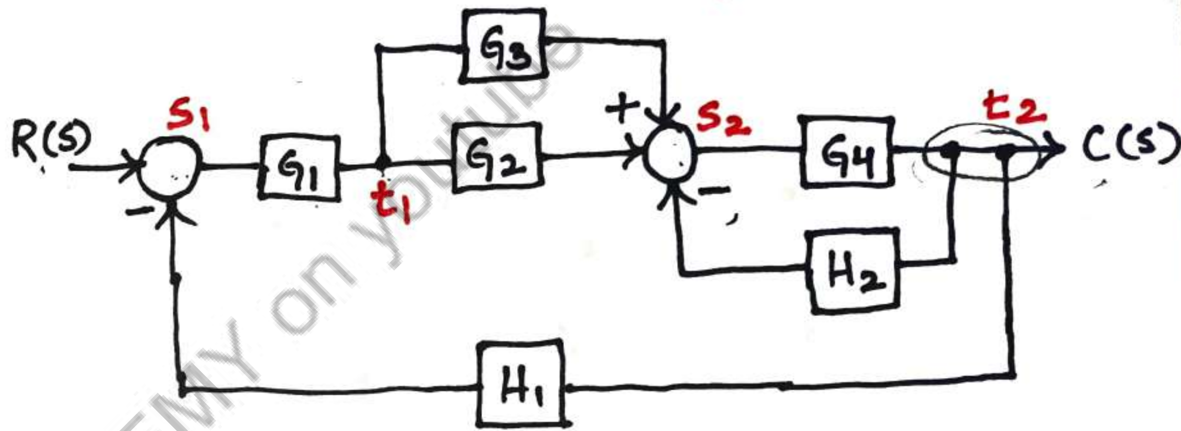
$x_5 = 3x_2 + 8x_4 \rightarrow (4)$

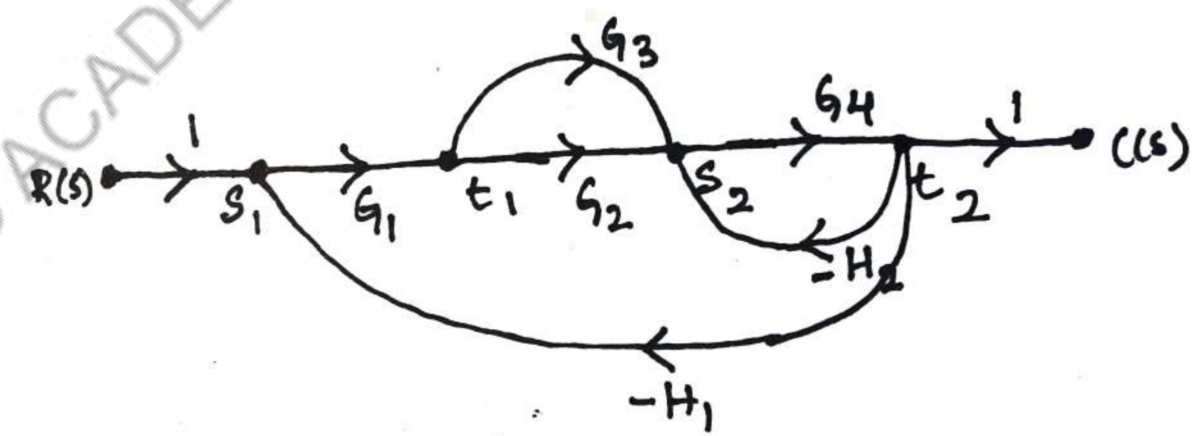
x_1 x_2 x_3 x_4 x_5 o/p

i/p



② From Block diagram.





Mason's Gain Formula:

→ Block diagram ⇒ Reduction Rules ⇒ T.F.

→ SFG ⇒ Formula ⇒ T.F.
↳ MGF.

$$\boxed{\text{Overall TF} = \frac{\sum T_K \Delta_K}{\Delta}}$$

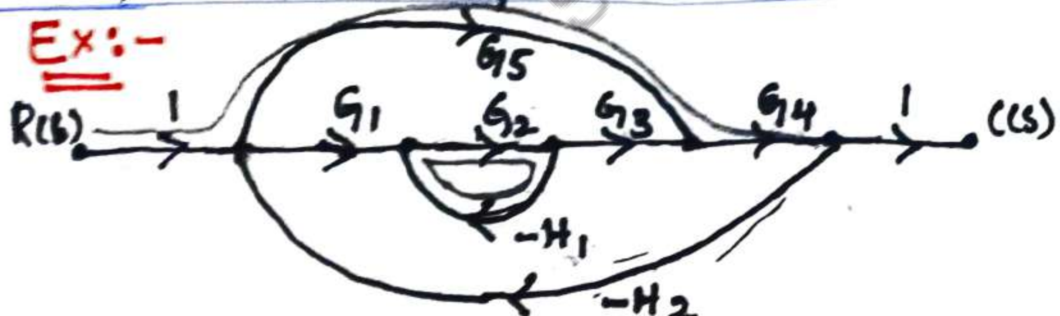
K → No. of Forward path

T_K → Gain of K^{th} Forward path

Δ = 1 − (Sum of individual loop gain) + (Sum of Product of non touching loop gains taken two at a time) − (Sum of Product of non touching loop gains taken three at a time) + ...

Δ_K = 1 − (loop gain which does not touch forward path).

Ex:-



① **No. of Forward path:** $K = 2$

$$TF = \frac{T_1\Delta_1 + T_2\Delta_2}{\Delta}$$

$T_1 = G_1G_2G_3G_4$

$T_2 = G_5G_4$

② **Individual Feedback loop gain:**

$L_1 = -G_2H_1$ $\quad L_2 = -G_1G_2G_3G_4H_2$

$L_3 = -G_5G_4H_2$

③ **Non-touching loop pairs:**

two non touching loops

$\boxed{L_1 \times L_3} = -G_2H_1 \times -G_5G_4H_2 = \boxed{G_2G_4G_5H_1H_2}$

three non touching loops ×

④ **Find Δ:**

$\Delta = 1 - (-G_2H_1 - G_1G_2G_3G_4H_2 - G_5G_4H_2) + (G_2G_4G_5H_1H_2)$

$$\boxed{\Delta = 1 + G_2H_1 + G_1G_2G_3G_4H_2 + G_5G_4H_2 + G_2G_4G_5H_1H_2}$$

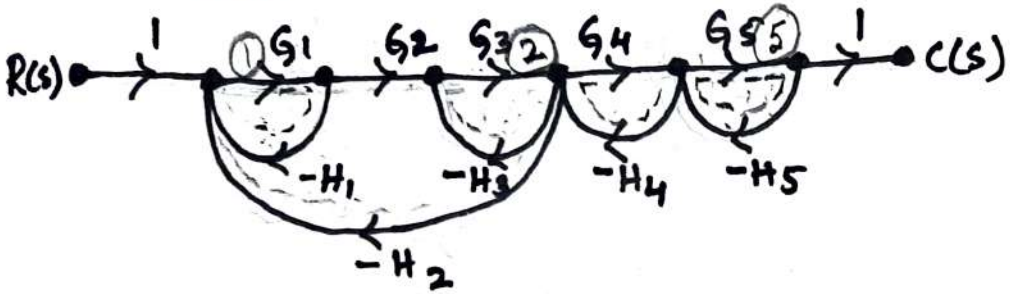
⑤ **Find Δ_1 & Δ_2:**

$\Delta_1 = 1 - 0 \Rightarrow \boxed{\Delta_1 = 1}$ $\quad \Delta_2 = 1 - [-G_2H_1]$

$\boxed{\Delta_2 = 1 + G_2H_1}$

$$\boxed{\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4 + G_5G_4[1 + G_2H_1]}{1 + G_2H_1 + G_1G_2G_3G_4H_2 + G_5G_4H_2 + G_2G_4G_5H_1H_2}}$$

Find $\frac{C(s)}{R(s)}$ by Mason's Gain Formula.



$$MGF \Rightarrow TF = \frac{\sum T_K \Delta_K}{\Delta} = \frac{T_1 \Delta_1}{\Delta}$$

① no of Forward path : K = 1

$T_1 = G_1G_2G_3G_4G_5$

② Feedback loop gain :

$L_1 = -G_1H_1 \quad L_2 = -G_3H_3 \quad L_3 = -G_1G_2G_3H_2$

$L_4 = -G_4H_4 \quad L_5 = -G_5H_5.$

③ Non touching loop pairs :

two nontouching loops

$L_1 \& L_2, \; L_1 \& L_4, \; L_1 \& L_5, \; L_2 \& L_5, \; L_3 \& L_5$

three nontouching loops

$L_1, L_2 \& L_5$

④ Find Δ :

$$\Delta = 1 - [L_1 + L_2 + L_3 + L_4 + L_5] + [L_1L_2 + L_1L_4 + L_1L_5 + L_2L_5 + L_3L_5] - [L_1L_2L_5]$$

$$\boxed{\begin{aligned}\Delta = 1 - [-G_1H_1 - G_3H_3 - G_1G_2G_3H_2 - G_4H_4 - G_5H_5] \\ + [G_1G_3H_1H_3 + G_1G_4H_1H_4 + G_1G_5H_1H_5 + \\ G_3G_5H_3H_5 + G_1G_2G_3G_5H_2H_5] - [G_1G_3G_5H_1H_3H_5]\end{aligned}}$$

⑤ Find Δ_1 :

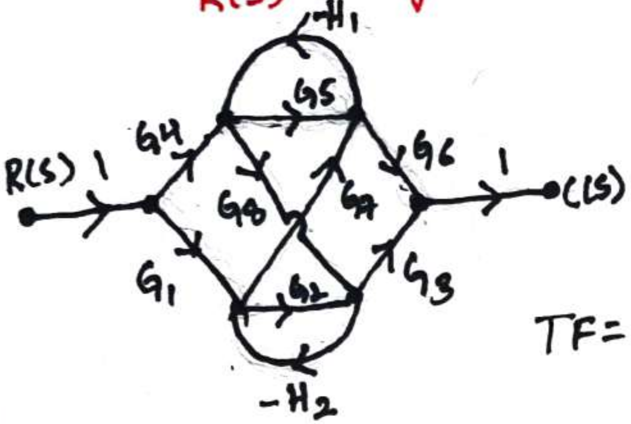
$\Delta_1 = 1 - [\;\;]$

$\Delta_1 \rightarrow T_1$ all loops are touching

$$\boxed{\Delta_1 = 1}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4G_5}{\begin{aligned}1 + G_1H_1 + G_3H_3 + G_1G_2G_3H_2 + G_4H_4 + G_5H_5 \\ + G_1G_3H_1H_3 + G_1G_4H_1H_4 + G_1G_5H_1H_5 + \\ G_3G_5H_3H_5 + G_1G_2G_3G_5H_2H_5 + G_1G_3G_5H_1H_3H_5\end{aligned}}}$$

Find $\frac{C(S)}{R(S)}$ using Mason's gain Formula.



MGF

$$TF = \frac{\sum T_K \Delta_K}{\Delta}$$

$$TF = \frac{T_1\Delta_1 + T_2\Delta_2 + T_3\Delta_3 + T_4\Delta_4 + T_5\Delta_5 + T_6\Delta_6}{\Delta}$$

① No. of Forward path : K = 6

$T_1 = G_1 G_2 G_3$ $\quad$ $T_4 = G_4 G_8 G_3$

$T_2 = G_4 G_5 G_6$ $\quad$ $T_5 = G_4 G_8 (-H_2) G_7 G_6$

$T_3 = G_1 G_7 G_6$ $\quad$ $T_6 = G_1 G_7 (-H_1) G_8 G_3$

② Feedback loop gain :

$L_1 = -G_5 H_1$ $\quad$ $L_2 = -G_2 H_2$ $\quad$ $L_3 = G_7 G_5 H_1 H_2$

③ Non touching loop pairs :

two non touching loop pairs

$$\boxed{L_1 \cdot L_2 = G_2 G_5 H_1 H_2}$$

④ Find Δ :-

$$\Delta = 1 - [-G_5 H_1 - G_2 H_2 + G_7 G_5 H_1 H_2] + [G_2 G_5 H_1 H_2]$$

$$\boxed{\Delta = 1 + G_5 H_1 + G_2 H_2 - G_7 G_5 H_1 H_2 + G_2 G_5 H_1 H_2}$$

⑤ Find Δ_1 to Δ_6 :-

Δ_1 :- $T_1 \rightarrow L_1$ is nontouching

$\Delta_1 = 1 - [-G_5 H_1] \Rightarrow \boxed{\Delta_1 = 1 + G_5 H_1}$

Δ_2 :- $T_2 \rightarrow L_2$ is nontouching

$\Delta_2 = 1 - [-G_2 H_2] = \boxed{\Delta_2 = 1 + G_2 H_2}$

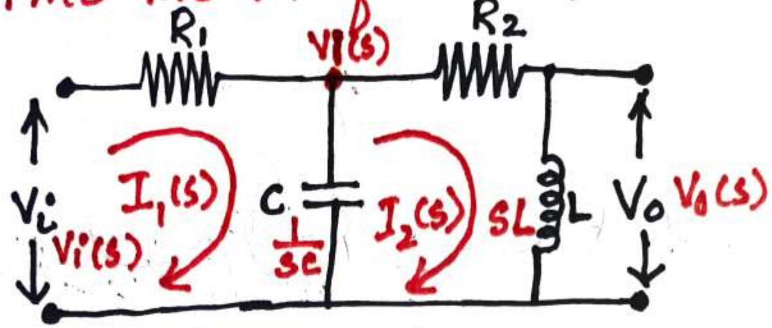
T_3, T_4, T_5 & T_6 all loops are touching.

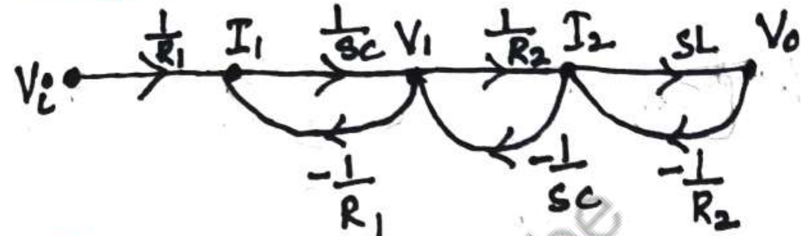
$$\boxed{\Delta_3 = \Delta_4 = \Delta_5 = \Delta_6 = 1}$$

$$\boxed{\frac{C(S)}{R(S)} = \frac{G_1 G_2 G_3 [1 + G_5 H_1] + G_4 G_5 G_6 [1 + G_2 H_2] + G_1 G_7 G_6 + G_4 G_8 G_3 + G_4 G_8 (-H_2) G_7 G_6 + G_1 G_7 (-H_1) G_8 G_3}{1 + G_5 H_1 + G_2 H_2 - G_7 G_5 H_1 H_2 + G_2 G_5 H_1 H_2}}$$

Mason's Gain Formula for Electrical n/w:

Find the T.F. for the n/w.





Procedure:

1. Redraw the n/w in s-domain.
2. Write eqns for Branch current and node voltages.
3. Write Signal Flow graph using the eqns
4. Use MGF & Find the T.F. of the n/w.

$$I_1(s) = \frac{V_i(s) - V_1(s)}{R_1} = \left[\frac{1}{R_1} V_i(s)\right] + \left[-\frac{1}{R_1} V_1(s)\right] \rightarrow (1)$$

$$V_1(s) = [I_1(s) - I_2(s)]\frac{1}{sC} = \left[\frac{1}{sC} I_1(s)\right] + \left[-\frac{1}{sC} I_2(s)\right] \rightarrow (2)$$

$$I_2(s) = \frac{V_1(s) - V_0(s)}{R_2} = \left[\frac{1}{R_2} V_1(s)\right] + \left[-\frac{1}{R_2} V_0(s)\right] \rightarrow (3)$$

$$V_0(s) = I_2(s)\cdot SL = SL \cdot I_2(s) \rightarrow (4)$$

(1) $K = 1$

$$\frac{V_o}{V_i} = \frac{T_1 \Delta_1}{\Delta}$$

$$T_1 = \frac{1}{R_1}\cdot\frac{1}{sC}\cdot\frac{1}{R_2}\cdot SL$$

$$\boxed{T_1 = \frac{L}{R_1 R_2 C}}$$

(2) $L_1 = -\frac{1}{sR_1C}$ $\quad L_2 = -\frac{1}{sR_2C}$

$$L_3 = -\frac{SL}{R_2}$$

(3) L_1 & L_3

(4) $\Delta = 1 - [L_1 + L_2 + L_3] + [L_1 \cdot L_3]$

$$\boxed{\Delta = 1 + \frac{1}{sR_1C} + \frac{1}{sR_2C} + \frac{SL}{R_2} + \frac{L}{R_1R_2C}}$$

(5) $\boxed{\Delta_1 = 1}$

(6)
$$\frac{V_o}{V_i} = \frac{\dfrac{L}{R_1 R_2 C}}{1 + \dfrac{1}{sR_1C} + \dfrac{1}{sR_2C} + \dfrac{SL}{R_2} + \dfrac{L}{R_1R_2C}}$$

$$\boxed{\frac{V_o}{V_i} = \frac{SL}{SR_1R_2C + R_2 + R_1 + s^2LR_1C + SL}}$$

Comparison of Block diagram and Signal Flow graph Methods.

Block diagram

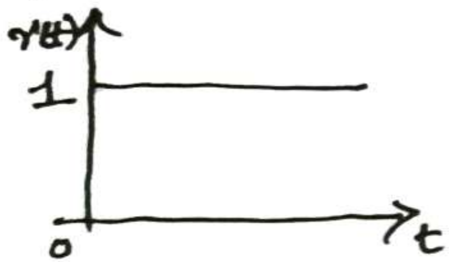
1. Applicable only to LTI s/m
2. Each element is represented by block
3. Summing point and takeoff point are Separate.
4. Self loop do not exist.
5. Time Consuming
6. Feedback path is present
7. Block diagram reduction rules are used

Signal Flow graph

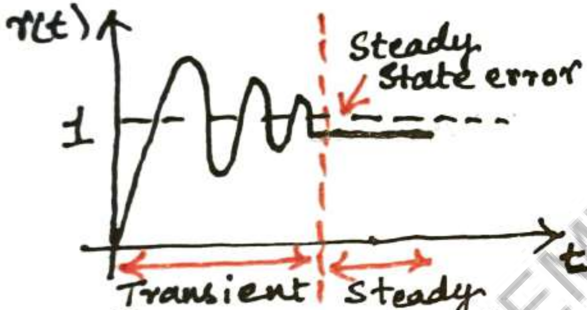
1. Applicable only to LTI s/m
2. Each Variable is represented by node.
3. Summing point & takeoff point are not present. They are represented by nodes.
4. Self loop can exist.
5. Requires less time.
6. Feedback loop is present.
7. Mason's gain formula is used.

Time response of Feedback Control s/m:

Time response: response (O/P) of the s/m when subjected to input, which is the fun of time.



(a) i/p signal.



(b) o/p signal (response)

(i) Transient response

variation in the o/p for the time it takes to achive its Final value

$$\boxed{\lim_{t \to \infty} C_t(t) = 0}$$

(ii) Steady state response

It is the part of time response which remains after Transient response vanish. $[C_{ss}(t)]$

$\therefore$ total time response

$$\boxed{c(t) = C_t(t) + C_{ss}(t)}$$

Steady state error (e_{ss}):

The difference b/w the desired o/p and the actual o/p.

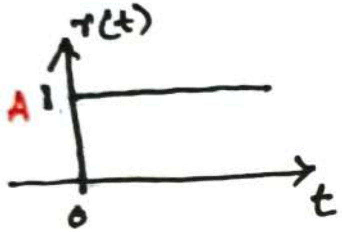
Input test signals:

→ Step i/p
→ Ramp i/p
→ Parabolic i/p
→ Impulse i/p
→ Sinusoidal i/p.

Standard test input signals:

1> Step input:

$$r(t) = \begin{cases} 1 \; ; \; t \geq 0 \\ 0 \; ; \; t < 0 \end{cases}$$



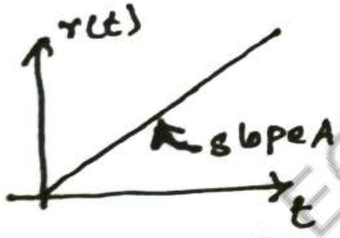
$$\boxed{L.T.\{r(t)\} = \frac{1}{s}}$$

$r(t) = A$

$$\boxed{L.T.\{r(t)\} = \frac{A}{s}}$$

2> Ramp input:

$$r(t) = \begin{cases} At \; ; \; t \geq 0 \\ 0 \; ; \; t < 0 \end{cases}$$



if $A = 1$ then $r(t) = t$

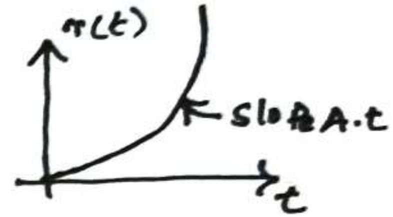
$$\therefore \boxed{L.T.\{r(t)\} = \frac{1}{s^2}}$$

if $r(t) = At$

$$\boxed{L.T.\{r(t) = \frac{A}{s^2}}$$

3> Parabolic input:

$$r(t) = \begin{cases} \frac{A}{2} t^2 \; ; \; t \geq 0 \\ 0 \; ; \; t < 0 \end{cases}$$



if $A = 1$ then $r(t) = \frac{t^2}{2}$

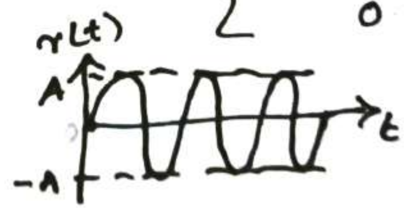
$$\therefore \boxed{L.T.\{r(t)\} = \frac{1}{s^3}}$$

if $r(t) = \frac{A}{2} t^2$

$$\boxed{L.T.\{r(t)\} = \frac{A}{s^3}}$$

4> Sinusoidal input:

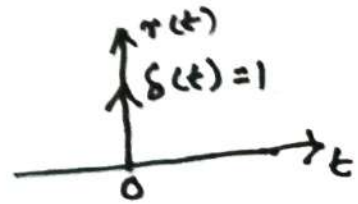
$$r(t) = \begin{cases} A \sin \omega t \; ; \; t \geq 0 \\ 0 \; ; \; t < 0 \end{cases}$$



$$\therefore \boxed{L.T.\{r(t)\} = \frac{A\omega}{s^2 + \omega^2}}$$

5> Impulse input:

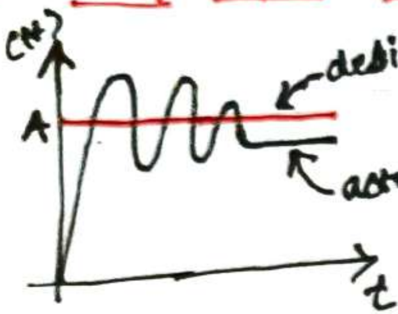
$$r(t) = \begin{cases} \delta(t) \; ; \; t = 0 \\ 0 \; ; \; t \neq 0 \end{cases}$$



$$\boxed{L.T.\{r(t)\} = 1}$$

Static Error Coefficients:

① Step input with Magnitude A:



$$R(s) = \frac{A}{s}$$

$$e_{ss} = \lim_{s \to 0} \frac{s \cdot R(s)}{1+G(s)H(s)} = \lim_{s \to 0} \frac{\cancel{s} \cdot A/\cancel{s}}{1+G(s)H(s)}$$

$$e_{ss} = \lim_{s \to 0} \frac{A}{1+G(s)H(s)} \Rightarrow e_{ss} = \frac{A}{1+\boxed{\lim_{s \to 0} G(s)H(s)}}$$

K_p "Positional Error Co-effient"

$$\boxed{e_{ss} = \frac{A}{1+K_p}}$$

② Ramp input with Magnitude A:

$$R(s) = \frac{A}{s^2}$$

$$e_{ss} = \lim_{s \to 0} \frac{s \cdot R(s)}{1+G(s)H(s)}$$

$$e_{ss} = \lim_{s \to 0} \frac{\cancel{s} \cdot \frac{A}{s^{\cancel{2}}}}{1+G(s)H(s)} = \lim_{s \to 0} \frac{A}{s[1+G(s)H(s)]}$$

$$e_{ss} = \lim_{s \to 0} \frac{A}{s+sG(s)H(s)} = \frac{A}{\boxed{\lim_{s \to 0} sG(s)H(s)}}$$

K_v 'Velocity error Co-effient'

$$\boxed{e_{ss} = \frac{A}{K_v}}$$

③ parabolic input with magnitude A:

$$R(s) = \frac{A}{s^3}$$

$$e_{ss} = \lim_{s \to 0} \frac{s \cdot R(s)}{1+G(s)H(s)}$$

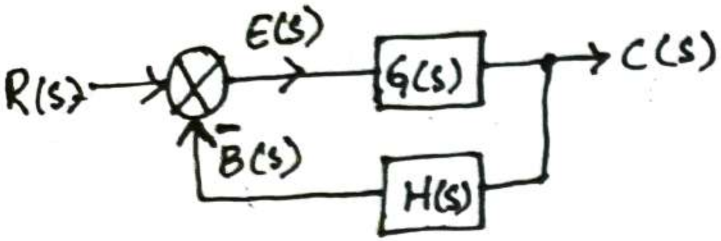
$$e_{ss} = \lim_{s \to 0} \frac{\cancel{s} \cdot \frac{A}{s^{\cancel{3}} s^2}}{1+G(s)H(s)} = \lim_{s \to 0} \frac{A}{s^2+s^2G(s)H(s)}$$

$$e_{ss} = \frac{A}{\boxed{\lim_{s \to 0} s^2G(s)H(s)}}$$

K_a 'acceleration error Co-effient'

$$\boxed{e_{ss} = \frac{A}{K_a}}$$

Steady State Error of closed loop System:



$E(s) = R(s) - B(s) \quad \because B(s) = C(s) \cdot H(s)$

$E(s) = R(s) - C(s) \cdot H(s) \quad \because C(s) = E(s) \cdot G(s)$

$E(s) = R(s) - E(s)\, G(s) \cdot H(s)$

$E(s) + E(s)\, G(s)\, H(s) = R(s)$

$E(s)\left[1 + G(s)\, H(s)\right] = R(s)$

$$\boxed{E(s) = \frac{R(s)}{1 + G(s)\, H(s)}}$$

if $H(s) = 1$ $\quad \boxed{E(s) = \frac{R(s)}{1 + G(s)}}$

Time domain: error signal $e(t)$

$\therefore$ $\boxed{\text{Steady state error: } e_{ss} = \lim_{t \to \infty} e(t)}$

Final Value theorem

$$\boxed{\lim_{t \to \infty} f(t) = \lim_{s \to 0} s F(s)}$$

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} s E(s)$$

$$\boxed{e_{ss} = \lim_{s \to 0} \frac{s R(s)}{1 + G(s) H(s)}}$$

Steady state Error depends on

→ $R(s)$ → Reference i/p

→ $G(s) H(s)$ → open loop Transfer fun

→ Nonlinearities present if any.

Type and order of the System:

$\frac{G(s)}{1+G(s)H(s)} \rightarrow$ Closed loop T.F.

$G(s)\cdot H(s) \rightarrow$ open loop T.F.

"Poles" and "Zeros"

$\frac{s+2}{s(s+1)(s+4)}$

$s+2 \rightarrow$ roots in numerator ⇒ ZEROES

$s(s+1)(s+4) \rightarrow$ roots in denominator ⇒ POLES

Nr ⇒ $s=-2$ ⇒ 1 zeroes.

Dr ⇒ $\boxed{s=0}$, $s=-1$, $s=-4$ ⇒ 3 POLES

POLES in the origin.

TYPE: It is the no of poles in the origin in the openloop Transfer fun of a unity feedback

$$G(s)H(s) = \frac{K(1+T_1 s)(1+T_2 s)}{s^j(1+T_a s)(1+T_b s)}$$

$K \rightarrow$ Resultant s/m gain

$\boxed{j \rightarrow \text{Type of s/m}}$

$$G(s)H(s) = \frac{K(1+T_1 s)(1+T_2 s)}{(1+T_a s)(1+T_b s)}$$

$\boxed{j=0 \quad \therefore \text{Type}=0}$

$$G(s)H(s) = \frac{K(1+T_1 s)(1+T_2 s)}{s(1+T_a s)(1+T_b s)}$$

$\boxed{j=1 \quad \therefore \text{Type}=1}$

Ex:- $\frac{20(1+s)}{s^2(2+s)(4+s)}$ $\boxed{j=2 \quad \therefore \text{Type}=2}$

ORDER: Highest power of 's' present in the chr. eqn of a s/m.

$G(s) = \frac{K}{s(1+Ts)}$ & $H(s)=1$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)\cdot H(s)} = \frac{\frac{K}{s(1+Ts)}}{1+\frac{K}{s(1+Ts)}\cdot 1} = \frac{\frac{K}{\cancel{s(1+Ts)}}}{\frac{s(1+Ts)+K}{\cancel{s(1+Ts)}}}$$

$\frac{C(s)}{R(s)} = \frac{K}{Ts^2+s+K}$ Chr eqn ⇒ $Ts^2+s+K=0$

$\boxed{\text{Order} = 2}$

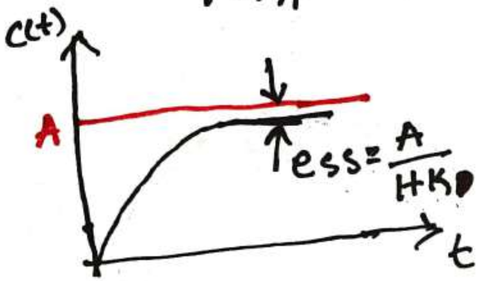
Analysis of TYPE 0, 1 and 2 Systems for step i/p of Magnitude 'A':

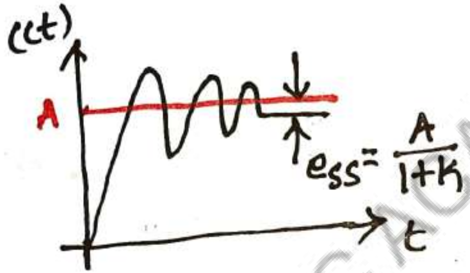
TYPE 0:

$$G(s)H(s) = \frac{K(1+T_1 s)(1+T_2 s)\ldots}{(1+T_a s)(1+T_b s)\ldots}$$

Step i/p $\quad K_p = \lim_{s\to 0} G(s)H(s) = K$

$$e_{ss} = \frac{A}{1+K_p} = \frac{A}{1+K}$$





Increasing the value 'K'

TYPE 1:

$$G(s)H(s) = \frac{K(1+T_1 s)(1+T_2 s)\ldots}{s(1+T_a s)(1+T_b s)\ldots}$$

step i/p $\quad K_p = \lim_{s\to 0} G(s)H(s) = \infty$

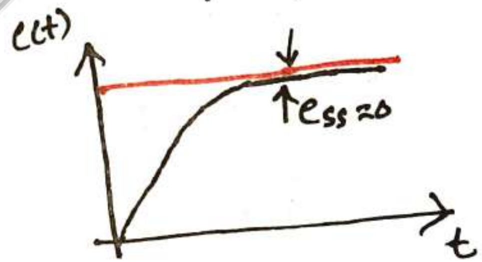
$$e_{ss} = \frac{A}{1+K_p} = \frac{A}{1+\infty} = 0$$

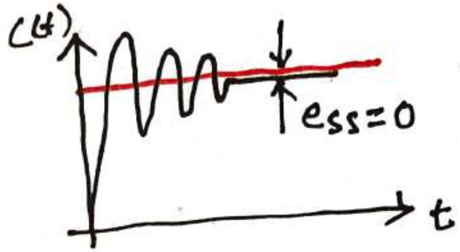
TYPE 2:

$$G(s)H(s) = \frac{K(1+T_1 s)(1+T_2 s)\ldots}{s^2(1+T_a s)(1+T_b s)\ldots}$$

step i/p $\quad K_p = \lim_{s\to 0} G(s)H(s) = \infty$

$$e_{ss} = \frac{A}{1+K_p} = \frac{A}{1+\infty} = 0$$





Type 1 and above s/ms which follows Step i/p of any magnitude will have negligeble small error

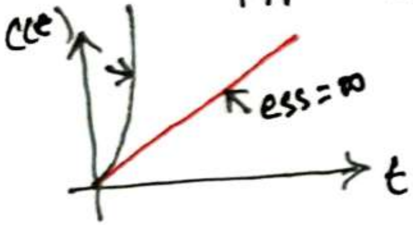
Analysis of TYPE 0, 1 and 2 Systems for RAMP input:

TYPE 0:

$$G(s)H(s) = \frac{K(1+T_1 s)(1+T_2 s)\ldots}{(1+T_a s)(1+T_b s)\ldots}$$

Ramp i/p $K_v = \lim_{s \to 0} s \cdot G(s) \cdot H(s) = 0$

$$e_{ss} = \frac{A}{K_v} = \frac{A}{0} = \infty$$



Type 0 will not follow ramp i/p of any magnitude and will give large error.

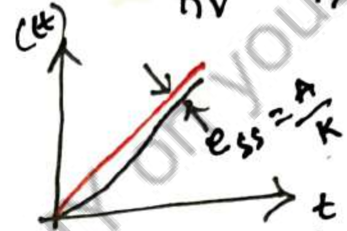
Hence ramp i/p should not applied to Type 0 s/m.

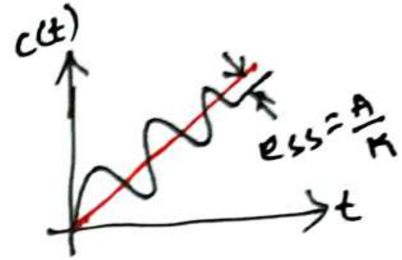
TYPE 1:

$$G(s)H(s) = \frac{K(1+T_1 s)(1+T_2 s)\ldots}{s(1+T_a s)(1+T_b s)\ldots}$$

Ramp i/p $K_v = \lim_{s \to 0} s \cdot G(s)H(s) = K$

$$e_{ss} = \frac{A}{K_v} = \frac{A}{K}$$





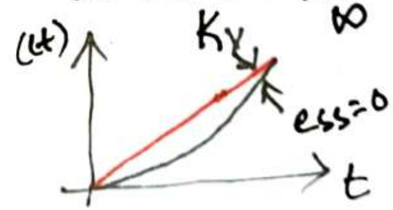
Increasing 'K' ⇒ reduce the error

TYPE 2:

$$G(s)H(s) = \frac{K(1+T_1 s)(1+T_2 s)\ldots}{s^2(1+T_a s)(1+T_b s)\ldots}$$

Ramp i/p $K_v = \lim_{s \to 0} s \cdot G(s)H(s) = \infty$

$$e_{ss} = \frac{A}{K_v} = \frac{A}{\infty} = 0$$



For any type of s/m more than one, the error is negligeble for ramp i/p

Analysis of TYPE 0, 1 and 2 Systems for Parabolic i/p with magnitude 'A'

TYPE 0 :

$$G(s)H(s) = \frac{K(1+T_1 s)(1+T_2 s)\dots}{(1+T_a s)(1+T_b s)\dots}$$

$$K_a = \lim_{s \to 0} s^2 G(s)H(s) = 0$$

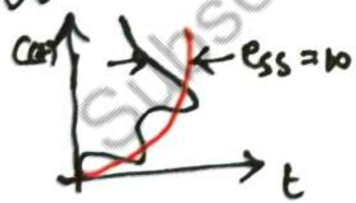
$$e_{ss} = \frac{A}{K_a} = \frac{A}{0} = \infty$$

TYPE 1 :

$$G(s)H(s) = \frac{K(1+T_1 s)(1+T_2 s)\dots}{s(1+T_a s)(1+T_b s)\dots}$$

$$K_a = \lim_{s \to 0} s^2 G(s)H(s) = 0$$

$$e_{ss} = \frac{A}{K_a} = \frac{A}{0} = \infty$$



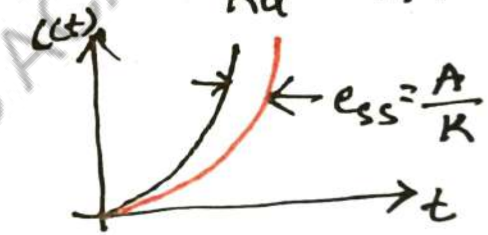
Type 0, 1 s/m → have very large steady state error → "Uncontrollable" if i/p is parabolic

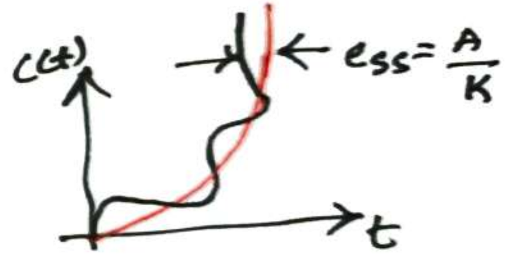
TYPE 2 :

$$G(s)H(s) = \frac{K(1+T_1 s)(1+T_2 s)\dots}{s^2(1+T_a s)(1+T_b s)\dots}$$

$$K_a = \lim_{s \to 0} s^2 G(s)H(s) = K$$

$$e_{ss} = \frac{A}{K_a} = \frac{A}{K}.$$





Parabolic i/p to Type 2 s/ms and above the error is negligible (or) very small.

1) A Unity FB s/m has $\boxed{G(s) = \frac{20(1+s)}{s^2(2+s)(4+s)}}$, Find it SSE co-efficients and error when the applied input $\boxed{r(t) = 40+2t+5t^2}$

$H(s)=1$

$$G(s)H(s) = \frac{20(1+s)}{s^2(2+s)(4+s)} = \frac{20(1+s)}{s^2\left[2(1+\frac{s}{2})4(1+\frac{s}{4})\right]}$$

$$= \frac{\overset{2.5}{\cancel{20}}(1+s)}{s^2[8(1+0.5s)(1+0.25s)]}$$

$$= \frac{2.5(1+s)}{s^2[(1+0.5s)(1+0.25s)]} \qquad \frac{K(1+T_1 s)}{s^2(1+T_a s)(1+T_b s)}$$

Type 2 s/m:

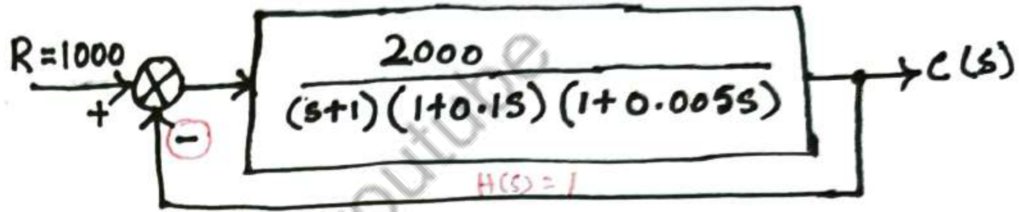
$$K_p = \lim_{s\to 0} G(s)H(s) = \lim_{s\to 0} \frac{2.5(1+s)}{s^2[(1+0.5s)(1+0.25s)]} = \infty$$

$$K_v = \lim_{s\to 0} sG(s)H(s) = \lim_{s\to 0} s \cdot \frac{2.5(1+s)}{s^2[(1+0.5s)(1+0.25s)]} = \infty$$

$$K_a = \lim_{s\to 0} s^2 G(s)H(s) = \lim_{s\to 0} s^2 \cdot \frac{2.5(1+s)}{s^2[(1+0.5s)(1+0.25s)]} = \frac{2.5}{1}$$

$\boxed{K_a = 2.5}$

2) For the block diagram [represents a heat treating oven]. The set point [desired temp] is 1000°c. What is s.s temp?



$H(s)=1$ & $G(s) = \frac{2000}{(s+1)(1+0.1s)(1+0.005s)}$

$R = 1000$

step → (A)

$$G(s)H(s) = \frac{2000}{(s+1)(1+0.1s)(1+0.005s)}$$

$j=0$ $\boxed{\text{Type } 0}$

Step i/P

$$K_P = \lim_{s\to 0} G(s)H(s) = \lim_{s\to 0} \frac{2000}{(s+1)(1+0.1s)(1+0.005s)}$$

$$K_P = \frac{2000}{1} \Rightarrow \boxed{K_p = 2000}$$

$$e_{ss} = \frac{A}{1+K_p} \Rightarrow \frac{1000}{1+2000} \Rightarrow \boxed{e_{ss} = 0.0499}$$

$t_{ss} \Rightarrow$ desired temp $- e_{ss} \Rightarrow t_{ss} = 1000 - 0.0499$

$\boxed{t_{ss} = 999.95°c}$

$K_p = \infty$, $K_v = \infty$ & $K_a = 2.5$

Step $\rightarrow A$

ramp $\rightarrow At$

Par. $\rightarrow \frac{A}{2} t^2$

i/p $\Rightarrow r(t) = 40 + 2t + 5t^2$

$r(t) = A_1 + A_2 t + A_3/_2\, t^2$

$r(t) = A_1 + A_2 t + \frac{A_3}{2} t^2$

40 steps, 2 ramp & 10 parabolic

$\frac{A_3}{2} = 5$

$\boxed{A_3 = 10}$

$e_{ss} = e_{ss1} + e_{ss2} + e_{ss3}$

$$e_{ss} = \frac{A_1}{1+K_p} + \frac{A_2}{K_v} + \frac{A_3}{K_a}$$

$$e_{ss} = \frac{40}{1+\infty} + \frac{2}{\infty} + \frac{10}{2.5}$$

$e_{ss} = 0 + 0 + 4$

$\boxed{e_{ss} = 4}$

Generalised Error Co-efficient:

Dynamic Error Co-efficient:

Error transfer fun

$$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)H(s)} \longrightarrow (1)$$

Taylor's series to eqn (1)

$$\frac{1}{1+G(s)H(s)} = K_0 + K_1 s + K_2 s^2 + K_3 s^3 + \dots \longrightarrow (2)$$

$K_0, K_1, K_2 \dots \rightarrow$ Dynamic Error Co-efficients

Take limits on both side as $s \to 0$

$$\boxed{\lim_{s \to 0} \frac{1}{1+G(s)H(s)} = K_0}$$

differentiate eqn (2) w.r.t 's'

$$\frac{d}{ds}\left[\frac{1}{1+G(s)H(s)}\right] = K_1 + 2K_2 s + 3K_3 s^2 + \dots \longrightarrow (3)$$

Take limit $s \to 0$ on both side

$$\boxed{\lim_{s \to 0} \frac{d}{ds}\left[\frac{1}{1+G(s)H(s)}\right] = K_1}$$

differentiate eqn (3) w.r.t s

$$\frac{d^2}{ds^2}\left[\frac{1}{1+G(s)H(s)}\right] = 2 \cdot 1 K_2 + 3 \cdot 2 K_3 s + \dots$$

Take limit $s \to 0$ on both side

$$\lim_{s \to 0} \frac{d^2}{ds^2}\left[\frac{1}{1+G(s)H(s)}\right] = 2K_2$$

$$\boxed{K_2 = \frac{1}{2!} \lim_{s \to 0} \frac{d^2}{ds^2}\left[\frac{1}{1+G(s)H(s)}\right]}$$

In general,

$$\boxed{K_n = \frac{1}{n!} \lim_{s \to 0} \frac{d^n}{ds^n}\left[\frac{1}{1+G(s)H(s)}\right]}$$

Put eqn (2) in (1)

$$\frac{E(s)}{R(s)} = K_0 + K_1 s + K_2 s^2 + \cdots$$

$$E(s) = K_0 R(s) + K_1 s R(s) + K_2 s^2 R(s) + \cdots$$

Take inverse laplace transform.

generalised error series,

$$e(t) = K_0 r(t) + K_1 \frac{d}{dt} r(t) + K_2 \frac{d^2}{dt^2} r(t) + \cdots$$

$$\boxed{e(t) = K_0 r(t) + K_1 r'(t) + K_2 r''(t) + \cdots}$$

SSE, $\boxed{e_{ss} = \lim_{t \to \infty} e(t)}$

The openloop T.F. of control s/m with unity feedback is $G(s) = \dfrac{10}{s(1+0.1s)}$. Evaluate error series for the s/m and also find S.S.E. with i/p $r(t) = 1+2t+t^2$

The error T.F. $\dfrac{E(s)}{R(s)} = \dfrac{1}{1+G(s)H(s)} = \dfrac{1}{1+\dfrac{10}{s(1+0.1s)}\cdot 1}$

$$\frac{E(s)}{R(s)} = \frac{1}{\dfrac{s(1+0.1s)+10}{s(1+0.1s)}} = \frac{s(1+0.1s)}{s(1+0.1s)+10} = \frac{s+0.1s^2}{s+0.1s^2+10}$$

$$= \frac{\cancel{0.1}(s^2+10s)}{\cancel{0.1}(s^2+10s+100)} \Rightarrow \boxed{\frac{E(s)}{R(s)} = \frac{s^2+10s}{s^2+10s+100}}$$

$$K_0 = \lim_{s\to 0} \frac{1}{1+G(s)H(s)} \Rightarrow K_0 = \lim_{s\to 0} \frac{s^2+10s}{s^2+10s+100}$$

$$\boxed{K_0 = 0}$$

$$K_1 = \lim_{s\to 0} \frac{d}{ds}\left[\frac{1}{1+G(s)H(s)}\right] = \lim_{s\to 0} \frac{d}{ds}\left[\frac{s^2+10s}{s^2+10s+100}\right]$$

$$K_1 = \lim_{s\to 0} \frac{(s^2+10s+100)(2s+10) - (s^2+10s)(2s+10)}{(s^2+10s+100)^2}$$

$$K_1 = \lim_{s\to 0} \frac{2s+10\,(\cancel{s^2}+\cancel{10s}+100-\cancel{s^2}-\cancel{10s})}{(s^2+10s+100)^2} = \frac{10\times 100}{100^2}$$

$$\boxed{K_1 = 0.1}$$

$$K_2 = \frac{1}{2!}\lim_{s\to 0} \frac{d^2}{ds^2}\left[\frac{1}{1+G(s)H(s)}\right]$$

$$K_2 = \frac{1}{2}\lim_{s\to 0} \frac{d}{ds}\left\{\frac{d}{ds}\frac{1}{1+G(s)H(s)}\right\}$$

$$K_2 = \frac{1}{2}\lim_{s\to 0} \frac{d}{ds}\left\{\frac{100(2s+10)}{(s^2+10s+100)^2}\right\}$$

$$K_2 = \frac{1}{2}\lim_{s\to 0} 100\left\{\frac{(s^2+10s+100)^2(2) - (2(s)+10)\times 2\,\cancel{(s^2+10s+100)}\,(2s+10)}{(s^2+10s+100)^{\cancel{4}\,3}}\right\}$$

$$K_2 = \frac{1}{2}\lim_{s\to 0} 200\left\{\frac{s^2+10s+100-(2s+10)^2}{(s^2+10s+100)^3}\right\}$$

$$K_2 = \frac{1}{2}\cdot 200\left\{\frac{100-100}{100^3}\right\} \qquad \boxed{K_2 = 0}$$

error series

$$e(t) = K_0 r(t) + K_1 r'(t) + K_2 r''(t) + \cdots$$

$$e(t) = 0 + 0.1[2+2t] + 0$$

$$\boxed{e(t) = 0.2 + 0.2t}$$

$r(t) = 1+2t+t^2$

$r'(t) = 2+2t$

$r''(t) = 2.$

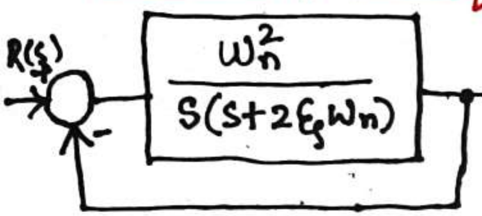
$r'''(t) = 0$

SSE ⇒

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{t\to\infty} 0.2+0.2t$$

$$\boxed{e_{ss} = \infty}$$

Time Response of 2^{nd} order control s/m:



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)\cdot H(s)}$$

$$= \frac{\frac{\omega_n^2}{s(s+2\xi\omega_n)}}{1+\frac{\omega_n^2}{s(s+2\xi\omega_n)}\cdot 1}$$

$$\therefore \boxed{\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}}$$

$\omega_n \rightarrow$ Natural frq [rad/s]

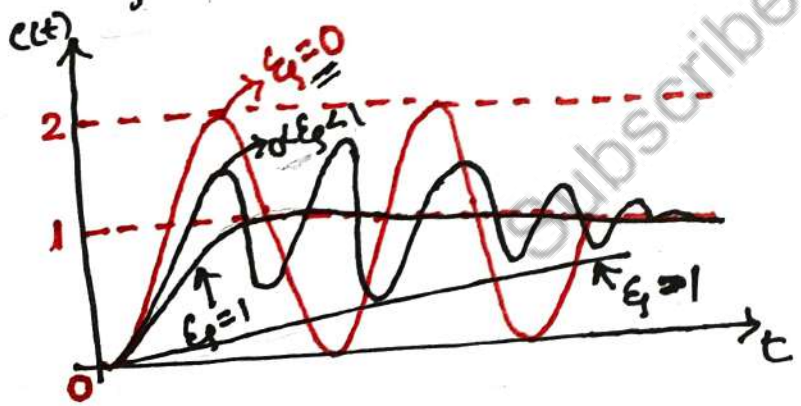
$\xi \rightarrow$ Damping ratio.

if $\xi = 0 \rightarrow$ Undamped s/m

$0<\xi<1 \rightarrow$ under damped s/m

$\xi>1 \rightarrow$ overdamped s/m

$\xi=1 \rightarrow$ Critically damped s/m.



$\rightarrow$ roots of Denominator of $\frac{C(s)}{R(s)} \rightarrow$ "Poles"

roots of Numerator of $\frac{C(s)}{R(s)} \rightarrow$ "Zeroes"

$\rightarrow$ Time response of any s/m is characterised by Poles of the T.F.

$1+G(s)H(s) \rightarrow$ Characteristic eqn

$1+G(s)H(s)=0$ $\quad as^2+bs+c=0$

$$\boxed{x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}}$$

$s^2+2\xi\omega_n s+\omega_n^2=0$

$a=1 \quad b=2\xi\omega_n \quad c=\omega_n^2$

$$s = \frac{-2\xi\omega_n \pm\sqrt{4\xi^2\omega_n^2-4\omega_n^2}}{2} = \frac{2\left[-\xi\omega_n\pm\sqrt{\xi^2\omega_n^2-\omega_n^2}\right]}{2}$$

$$\boxed{s = -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}}$$

1) $\xi=0$ $\quad s = \pm\omega_n\sqrt{-1}$ $\quad j=\sqrt{-1}$ $\quad s=\pm j\omega_n$

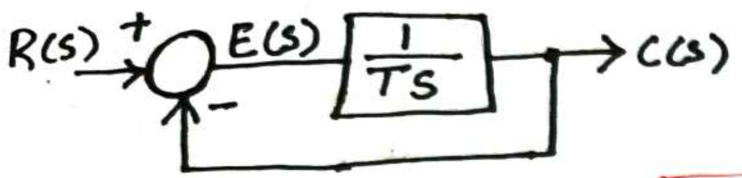
roots are imajnary

2) $\xi=1$ $\quad s=-\omega_n$ real negative & equal poles

3) $0<\xi<1$ Poles $\rightarrow$ complex conjugate

4) $\xi>1$ Poles $\rightarrow$ real negative and unequal.

Time response of first order control s/m:



$$\frac{C(s)}{R(s)} = \frac{1/Ts}{1+\frac{1}{Ts}\cdot 1} = \frac{1}{Ts+1} \quad \therefore \boxed{\frac{C(s)}{R(s)} = \frac{1}{sT+1}}$$

$T \rightarrow$ Time constant of the s/m

$$C(s) = R(s)\cdot\frac{1}{sT+1}$$

$\because R(t) = 1 \quad \therefore L\{R(t)\}$

step i/p $= R(s) = \frac{1}{s}$

$$C(s) = \frac{1}{s}\cdot\frac{1}{sT+1}$$

use partial fraction.

$$C(s) = \frac{1}{s}\cdot\frac{1}{sT+1} = \frac{A}{s} + \frac{B}{sT+1} \rightarrow (1)$$

$$A = \lim_{s\to 0}[s\cdot C(s)] = \lim_{s\to 0}\left[s\cdot\frac{1}{s}\cdot\frac{1}{sT+1}\right] \Rightarrow A = 1$$

$$B = \lim_{s\to -\frac{1}{T}}[(sT+1)C(s)] = \lim_{s\to -\frac{1}{T}}\left[(sT+1)\frac{1}{s}\cdot\frac{1}{sT+1}\right]$$

$$B = -T$$

(1) $\Rightarrow$ $C(s) = \frac{1}{s} - \frac{T}{sT+1}$

$$C(s) = \frac{1}{s} - \frac{1}{s+\frac{1}{T}}$$

inverse L.T.

$$\boxed{C(t) = 1 - e^{-t/T}} \rightarrow (2)$$

$t \geq 0$

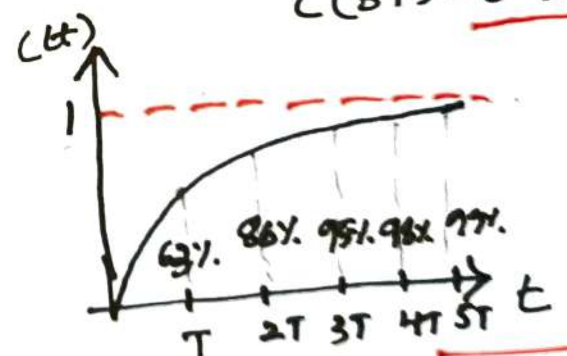
At $t = T$ $\quad C(T) = 1 - e^{-T/T} = 1 - e^{-1}$

$C(T) = 0.63$

At $t = 2T$ $\quad C(2T) = 0.86$

At $t = 3T, 4T$ & $5T$ then.

$C(3T) = 0.95$ to $C(5T) = 0.99$



if $t = \infty$ then $\boxed{C(t) = 1}$

$\therefore$ steady state is reached only after infinite time.

Response of an underdamped s/m $[0<\xi<1]$:

2nd order s/m → unit step i/p

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$C(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}\cdot R(s) \longrightarrow ①$$

$R(s) \to$ Unit step response

$$R(s) = \frac{1}{s}$$

$$C(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}\cdot\frac{1}{s}$$

$$C(s) = \frac{\omega_n^2}{s(s^2+2\xi\omega_n s+\omega_n^2)}$$

Partial Fractions to RHS

$$C(s) = \frac{\omega_n^2}{s(s^2+2\xi\omega_n s+\omega_n^2)} = \frac{K_1}{s} + \frac{K_2 s+K_3}{s^2+2\xi\omega_n+\omega_n^2} \longrightarrow ②$$

$$\omega_n^2 = K_1\cdot(s^2+2\xi\omega_n s+\omega_n^2)+(K_2 s+K_3)s.$$

Put $s=0$

$$\omega_n^2 = K_1\omega_n^2$$

$$K_1 = \frac{\cancel{\omega_n^2}}{\cancel{\omega_n^2}} \Rightarrow \boxed{K_1=1}$$

Comparing the co-efficients of s^2

$$s^2;\ 0 = K_1+K_2 \Rightarrow K_2=-K_1$$

$$\boxed{K_2=-1}$$

Comparing the coefficients of s

$$s;\ 0 = 2\xi\omega_n K_1+K_3$$

$$K_3 = -2\xi\omega_n K_1$$

$$\boxed{K_3=-2\xi\omega_n} \quad \because K_1=1$$

$$C(s) = \frac{1}{s} + \frac{-s-2\xi\omega_n}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$C(s) = \frac{1}{s} - \frac{s+2\xi\omega_n}{\underbrace{s^2+2\xi\omega_n s+(\xi\omega_n)^2}_{(a+b)^2}-(\xi\omega_n)^2+\omega_n^2}$$

$$C(s) = \frac{1}{s} - \frac{(s+\xi\omega_n)+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_n^2-\xi^2\omega_n^2}$$

$$C(s) = \frac{1}{s} - \frac{(s+\xi\omega_n)+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_n^2(1+\xi^2)}$$

$$C(s) = \frac{1}{s} - \frac{(s+\xi\omega_n)+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2}$$

$\because \omega_d = \omega_n\sqrt{1-\xi^2} \Rightarrow \omega_d^2 = \omega_n^2(1+\xi^2)$

$$C(s) = \frac{1}{s} - \left[\frac{(s+\xi\omega_n)}{(s+\xi\omega_n)^2+\omega_d^2} + \frac{\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2}\right]$$

Multiply ω_d for both Nr. & Dr.

$$C(s) = \frac{1}{s} - \frac{(s+\xi\omega_n)}{(s+\xi\omega_n)^2+\omega_d^2} - \frac{\xi\omega_n\cdot\omega_d}{\omega_d(s+\xi\omega_n)^2+\omega_d^2}$$

Take inverse laplace transform.

$$c(t) = 1 - e^{-\xi\omega_n t}\cos\omega_d t - \frac{\xi\omega_n}{\omega_n\sqrt{1-\xi^2}}\, e^{-\xi\omega_n t}\sin\omega_d t$$

$$c(t) = 1 - e^{-\xi\omega_n t}\cos\omega_d t - \frac{\xi}{\sqrt{1-\xi^2}}\, e^{-\xi\omega_n t}\cdot\sin\omega_d t.$$

let $\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}$ as common.

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\cos\omega_d t\sqrt{1-\xi^2} + \xi\sin\omega_d t\right]$$

Put $\sin\theta = \sqrt{1-\xi^2}$ $\therefore \cos\theta = \xi$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sin\theta\cos\omega_d t + \cos\theta\sin\omega_d t\right]$$

$$\boxed{c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sin(\omega_d t+\theta)\right]} \rightarrow (3)$$

if the step i/p amplitude 'A'

$$\boxed{c(t) = A\left[1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sin(\omega_d t+\theta)\right]\right]}$$

$\because \omega_d = \omega_n\sqrt{1-\xi^2}$ & $\theta = \tan^{-1}\left[\frac{\sqrt{1-\xi^2}}{\xi}\right]$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin\left[\omega_n\sqrt{1-\xi^2}\,t + \tan^{-1}\left[\frac{\sqrt{1-\xi^2}}{\xi}\right]\right]$$

The error signal.

$$e(t) = r(t) - c(t) \qquad \underline{\underline{r=1}}$$

$$e(t) = 1 - \left[1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \sin\left[\omega_n\sqrt{1-\xi^2}\,t + \tan^{-1}\left[\frac{\sqrt{1-\xi^3}}{\xi}\right]\right]\right]$$

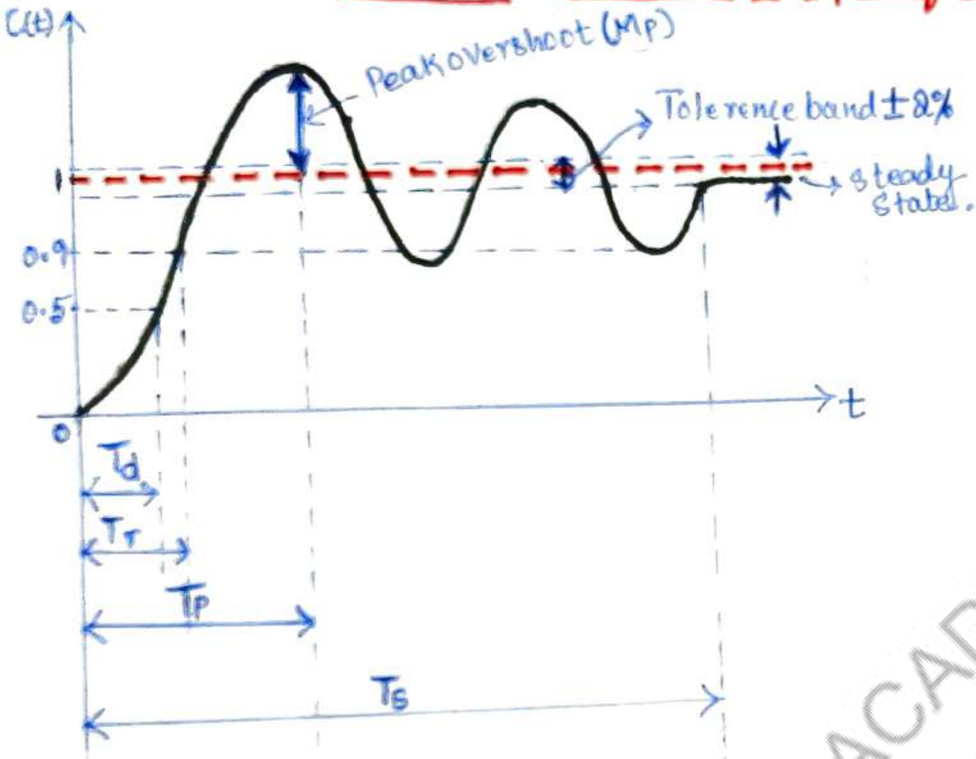
(or)

$$e(t) = \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \cdot \sin\left[\omega_n\sqrt{1-\xi^2}\,t + \tan^{-1}\left[\frac{\sqrt{1-\xi^2}}{\xi}\right]\right]$$

∴ The steady state error.

$$\boxed{e_{ss} = \lim_{t\to\infty} \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \left[\sin\left\{\omega_n\sqrt{1-\xi^2}\,t + \tan^{-1}\left[\frac{\sqrt{1-\xi^2}}{\xi}\right]\right]\right]}$$

Time Response [Transient Response] Specification:



The response specifications are:

a) Raise Time: [Tr]

It is the time required by the response to raise from 0% to 90% → Over damped s/m
0% to 100% → Underdamped s/m.
at very first time.

b) Delay Time: [Td]

time required by the response to reach 50% of final value at very first time.

c) Peak Time: [Tp]

Time required for the response of the s/m to reach first Peak of overshoot.

d) Peak overshoot: [MP]

It is the max value of the response curve, measured from unity.

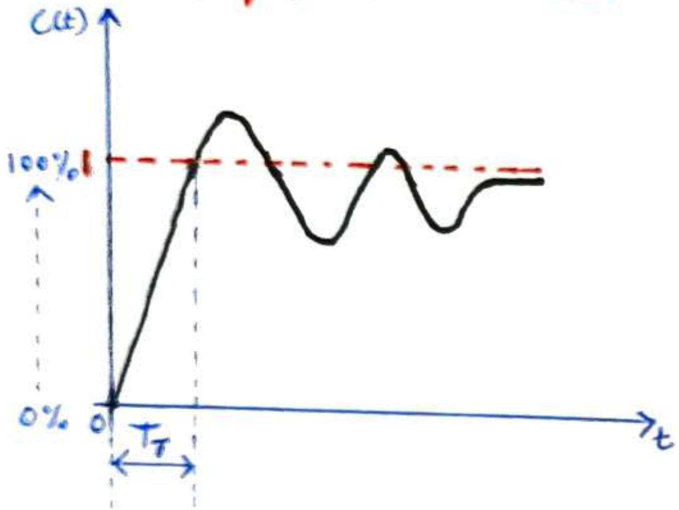
$$\boxed{M_p = C(t_p) - C(\infty)}$$

$$\boxed{\%M_p = \frac{C(t_p) - C(\infty)}{C(\infty)} \times 100}$$

e) Settling time: [Ts]

Time required by the response curve to reach the steady state and stay within specified range of it's final value.
(within tolerence band, ±2%)

Expression for Rise Time $[T_r]$:



Response of 2^{nd} order s/m with unit step i/p as,

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta) \longrightarrow ①$$

$$c(t) = c(t_r) = 1$$

$$① \Rightarrow \quad 1 = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$$

$$0 = \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$$

$$\sin(\omega_d t + \theta) = 0$$

$$\omega_d t + \theta = n\pi \quad ; n = 1, 2, 3 \ldots\ldots$$

$$\omega_d t = n\pi - \theta$$

$$\boxed{n = 1}$$

$$\omega_d t = \pi - \theta$$

$$t = \frac{\pi - \theta}{\omega_d}$$

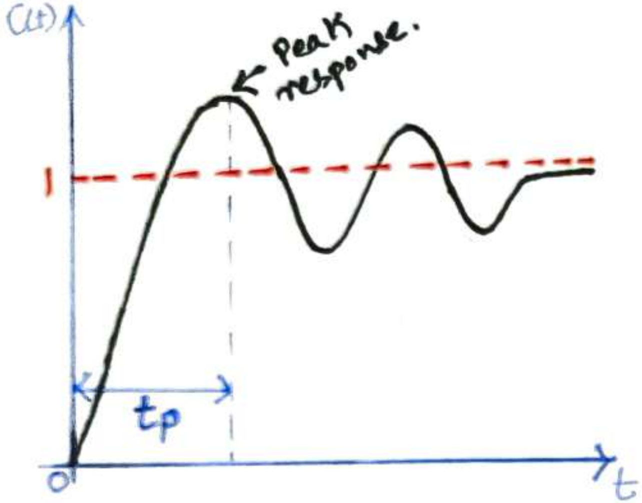
$$\boxed{t_r = \frac{\pi - \theta}{\omega_d}} \qquad \because t = t_r$$

W.K.T $\theta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$\boxed{t_r = \frac{\pi - \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}}{\omega_n\sqrt{1-\xi^2}}}$$

Expression for Peak Time: $[T_P]$



2^{nd} order c.s. with unit step i/p

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$$

$$\left.\frac{dc(t)}{dt}\right|_{t=t_p} = 0 - \left[\frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}}(-\xi\omega_n)\sin(\omega_d t_p+\theta) + \frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}}\cos(\omega_d t_p+\theta)\cdot\omega_d\right] = 0$$

When $t = t_p$, the slope of $c(t) = 0$

$$\frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}}\left[(\xi\omega_n)\sin(\omega_d t_p+\theta) + \omega_d\cos(\omega_d t_p+\theta)\right] = 0$$

W.K.T. $\omega_d = \omega_n\sqrt{1-\xi^2}$

$$\frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}}\left[(\xi\omega_n)\sin(\omega_d t_p+\theta) - \omega_n\sqrt{1-\xi^2}\cos(\omega_d t_p+\theta)\right] = 0$$

$$\frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}}\omega_n\left[\xi\sin(\omega_d t_p+\theta) - \sqrt{1-\xi^2}\cos(\omega_d t_p+\theta)\right] = 0$$

$\cos\theta = \xi \quad \therefore \sin\theta = \sqrt{1-\xi^2}$

$$\frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}}\cdot\omega_n\underbrace{\left[\underset{\cos B}{\cos\theta}\,\underset{\sin A}{\sin(\omega_d t_p+\theta)} - \underset{\sin B}{\sin\theta}\,\underset{\cos A}{\cos(\omega_d t_p+\theta)}\right]}_{\sin A-B} = 0$$

$$\frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}}\omega_n \sin(\omega_d t_p + \theta - \theta) = 0$$

$$\sin(\omega_d t_p) = 0$$

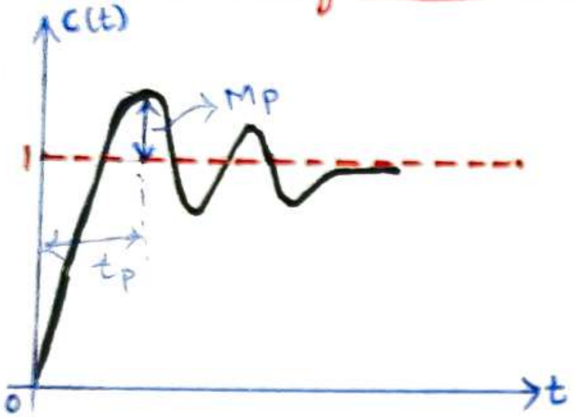
$$\omega_d t_p = n\pi \quad ; n = 1, 2, 3\ldots$$

For first peak overshoot $n = 1$

$$\omega_d t_p = \pi \Rightarrow \boxed{t_p = \pi/\omega_d}$$

$$\boxed{t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}}$$

Expression for Peak over shoot [Mp]:



2nd order control s/m which is subjected to unit step i/p

$$c(t) = 1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \cdot \sin(\omega_d t + \theta) \longrightarrow ①$$

$$t = t_p$$

$$c(t_p) = 1 - \frac{e^{-\xi \omega_n t_p}}{\sqrt{1-\xi^2}} \sin(\omega_d t_p + \theta)$$

$$t_p = \frac{\pi}{\omega_d} \quad , \quad \omega_d = \omega_n\sqrt{1-\xi^2}$$

$$\boxed{t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}}$$

$$c(t_p) = 1 - \frac{e^{-\xi \omega_n \frac{\pi}{\omega_n\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}} \sin\left(\omega_d \frac{\pi}{\omega_d} + \theta\right)$$

$$c(t_p) = 1 - \frac{e^{-\xi\pi/\sqrt{1-\xi^2}}}{\sqrt{1-\xi^2}} \underbrace{\sin(\pi+\theta)}_{(-\sin\theta)}$$

w.k.T $\sin(\pi+\theta) = -\sin\theta$

$\sin\theta = \sqrt{1-\xi^2}$

$$c(t_p) = 1 + e^{-\xi\pi/\sqrt{1-\xi^2}}$$

Final value $c(\infty) = 1$

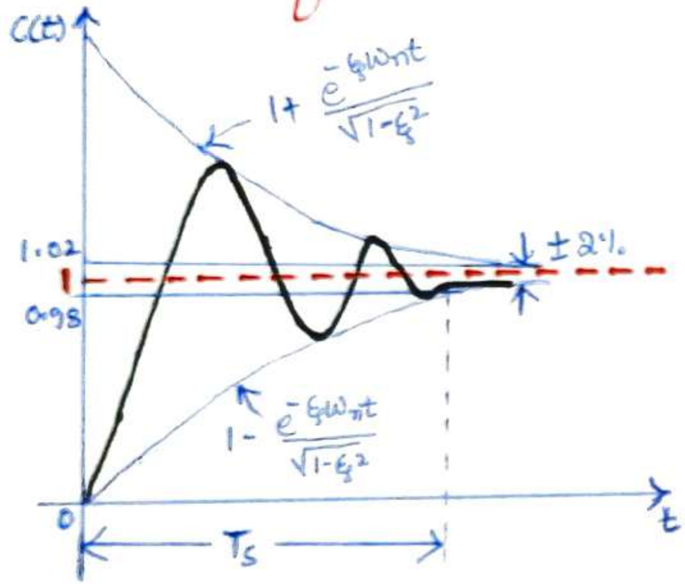
$$M_p = c(t_p) - c(\infty)$$

$$M_p = 1 + e^{-\xi\pi/\sqrt{1-\xi^2}} - 1$$

$$\boxed{M_p = e^{-\xi\pi/\sqrt{1-\xi^2}}}$$

$$\boxed{\%M_p = e^{-\xi\pi/\sqrt{1-\xi^2}} \times 100}$$

Expression for settling time: [T_s]



$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta) \longrightarrow (1)$$

At $t_s \rightarrow$ tolerance $\Rightarrow$ ±2%

$c(t)$ at $t_s = 0.98$

At $t_s \rightarrow$ there effect of exponentially decaying envelope only.

$$c(t_s) = 1 - e^{-\xi\omega_n t_s}$$

$$0.98 = 1 - e^{-\xi\omega_n t_s}$$

$$e^{-\xi\omega_n t_s} = 1 - 0.98$$

$$e^{-\xi\omega_n t_s} = 0.02$$

$$-\xi\omega_n t_s = \ln\{0.02\}$$

$$-\xi\omega_n t_s = -3.912$$

$$t_s = \frac{3.912}{\xi\omega_n}$$

$$\boxed{t_s = \frac{4}{\xi\omega_n}}$$

lly ± 5% tolerance

$$c(t_s) = 0.95$$

$$0.95 = 1 - e^{-\xi\omega_n t_s}$$

$$e^{-\xi\omega_n t_s} = 1 - 0.95$$

$$-\xi\omega_n t_s = \ln\{0.05\}$$

$$-\xi\omega_n t_s = -2.995$$

$$\boxed{t_s = \frac{3}{\xi\omega_n}}$$

Example 7.17.2 *For the system shown in the Fig. 7.17.4 obtain the closed loop T.F., damping ratio, natural frequency and expression for the output response if subjected to unit step input.*

VTU : July 17, Marks 6

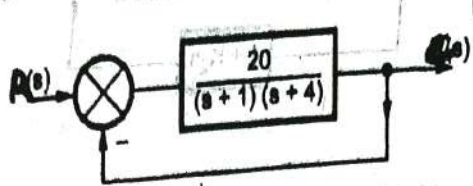


Fig. 7.17.4

$CLTF \rightarrow \dfrac{G(s)}{1+G(s)H(s)}$

$OLTF \rightarrow 1+G(s)H(s)$

Solution : $$\frac{C(s)}{R(s)} = \frac{\dfrac{20}{(s+1)(s+4)}}{1+\dfrac{20}{(s+1)(s+4)}} = \frac{20}{s^2+5s+24}$$

Key Point *Now though T.F. is not in standard form, denominator always reflect $2\xi\omega_n$ and ω_n^2 from middle term and the last term respectively.*

$\therefore$ Comparing, $s^2+5s+24$ with $s^2+2\xi\omega_n s+\omega_n^2$

$$\therefore \quad \omega_n^2 = 24 \quad \therefore \omega_n = \textbf{4.8989 rad/sec.}$$

$$2\xi\omega_n = 5 \quad \therefore \xi = \textbf{0.51031}$$

$$\omega_d = \omega_n\sqrt{1-\xi^2} = \textbf{4.2129 rad/sec.}$$

Now, for c(t) we can use standard expression for $\dfrac{C(s)}{R(s)}$ in standard form. So writing

$$\frac{C(S)}{R(S)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$\frac{C(s)}{R(s)} = \frac{20}{24}\cdot\left\{\frac{24}{s^2+5s+24}\right\}$$

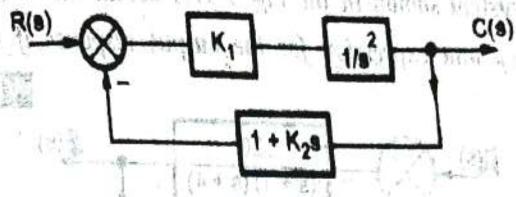
For the bracket term use standard expression, and then c(t) can be obtained by multiplying this expression by constant $\dfrac{20}{24}$.

$$\therefore \quad c(t) = \frac{20}{24}\left[1-\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t+\theta)\right]$$

$$\theta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\text{ radians} = 1.03\text{ radians}$$

$$\therefore \quad c(t) = \frac{20}{24}\left[1-1.1628\,e^{-2.5t}\sin(4.2129\,t+1.03)\right]$$

Example 7.17.3 *For a control system shown in figure, find the values of K_1 and K_2 so that $M_p = 25\ \%$ and $T_p = 4$ sec. Assume unit step input.* **VTU : Jan 08, 16, Marks 8**



Solution : $G(s) = \dfrac{K_1}{s^2}$, $H(s) = 1 + K_2\, s$

$$\text{T.F.} = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{K_1}{s^2}}{1+\frac{K_1}{s^2}(1+K_2\,s)} = \frac{K_1}{s^2+K_1 K_2\, s+K_1}$$

ω_n^2

$s^2 + 2\xi\omega_n s + \omega_n^2$

$\therefore$ Comparing with standard form,

$$\omega_n = \sqrt{K_1}\,, \quad 2\xi\,\omega_n = K_1\cdot K_2\,, \quad \therefore\ \xi = \frac{1}{2}\sqrt{K_1}\cdot K_2$$

$\xi = \dfrac{1}{2}\dfrac{K_1}{\sqrt{K_1}}\cdot K_2$

$\xi = \dfrac{1}{2}\sqrt{K_1}\cdot K_2$

Now M_p is function of ξ alone,

$$\therefore \quad \%\,M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100 \quad \text{i.e.} \quad 25 = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100$$

$$\therefore \quad 0.25 = e^{-\pi\xi/\sqrt{1-\xi^2}} \quad \text{i.e.} \quad ln\,(0.25) = \frac{-\pi\xi}{\sqrt{1-\xi^2}}$$

$$\therefore \quad -1.3862 = \frac{-\pi\xi}{\sqrt{1-\xi^2}}$$

$\therefore$ Squaring gives, $1.9218\,(1-\xi^2) = \pi^2\,\xi^2$

$$\therefore \quad \xi^2 = 0.1629 \qquad \therefore\ \xi = 0.4037$$

Now $\quad T_p = \dfrac{\pi}{\omega_d} = 4$ sec $\quad$ i.e. $\quad \dfrac{\pi}{\omega_n\sqrt{1-\xi^2}} = 4$

$$\therefore \frac{\pi}{\omega_n\sqrt{1-(0.4037)^2}} = 4 \quad \text{i.e.} \quad \omega_n = 0.8584 \ \text{rad/sec}$$

Now $\quad \omega_n = \sqrt{K_1}$ $\quad$ i.e. $\quad K_1 = \omega_n^2 = 0.7369$

and $\quad \xi = \dfrac{1}{2}\sqrt{K_1}\cdot K_2 = 0.4037 \quad$ i.e. $\quad K_2 = 0.9405$

Example 7.17.4 *A system is given by differential equation,* $\frac{d^2y}{dt^2}+4\frac{dy}{dt}+8y=8x$*, where y = output and x = input. Determine all time domain specifications for unit step input.*

VTU : July 18, 19, Marks 8

Solution : System differential equation is,

$$\frac{d^2y}{dt^2}+4\frac{dy}{dt}+8y = 8x$$

To find T.F. $\frac{Y(s)}{X(s)}$, take Laplace transform from above equation and neglect initial conditions.

$$s^2\,Y(s)+4s\,Y(s)+8\,Y(s) = 8\,X(s) \quad \text{i.e.} \quad Y(s)\left[s^2+4s+8\right] = 8\,X(s)$$

$$\therefore \quad \text{T.F. } \frac{Y(s)}{X(s)} = \frac{8}{s^2+4s+8} \text{ compare with } \frac{\omega_n^2}{s^2+2\xi\,\omega_n\,s+\omega_n^2}$$

$$\therefore \quad \omega_n^2 = 8 \quad \text{i.e.} \quad \omega_n = \mathbf{2.83\ rad/sec}$$

$$2\xi\omega_n = 4 \qquad \therefore\ \xi = \mathbf{0.7067}$$

$$\therefore \quad \omega_d = \omega_n\sqrt{1-\xi^2} = 2.83\sqrt{1-(0.7067)^2} = \mathbf{2.002\ rad/sec}$$

$$\therefore \quad T_p = \text{Time for peak overshoot} = \frac{\pi}{\omega_d} = \frac{\pi}{2.002} = \mathbf{1.57\ sec}$$

$$\%\ M_p = e^{-\pi\xi/\sqrt{1-\xi^2}}\times 100 = e^{-\pi\times 0.706/\sqrt{1-(0.706)^2}\times 100} = \mathbf{4.33\ \%}$$

$$T_s = \text{Settling time} = \frac{4}{\xi\,\omega_n} = \frac{4}{0.7067\times 2.83} = \mathbf{2\ sec}$$

$$c(t) = 1-\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d\,t+\theta) \quad \text{where } \theta = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right) = 45^\circ = \frac{\pi}{4}\text{ rad}$$

$$\therefore \quad c(t) = 1-\frac{e^{-0.7067\times 2.87\,t}}{\sqrt{1-(0.7067)^2}}\sin\left(2t+\frac{\pi}{4}\right)$$

$$\therefore c(t) = 1-\frac{e^{-0.7067\times 2.87\,t}}{\sqrt{1-(0.7067)^2}}\sin\left(2t+\frac{\pi}{4}\right) = \boxed{1-\ 1.41\,e^{-2t}\sin\left(2t+\frac{\pi}{4}\right)}$$

$$T_r = \frac{\pi-\theta}{\omega_d} = \mathbf{1.177\ sec}$$

Example 7.17.5 *A system has 30 % overshoot and settling time of 5 seconds for an unit step input. Determine : i) The transfer function ii) Peak time (t_p) iii) Output response. (Assume e_{ss} as 2 %)*

VTU : March-01, July-18, 19, Marks 8

Solution : The given values are, $M_P = 30\ \%$ and $T_s = 5$ sec

Now $\quad M_P = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100 \quad$ i.e. $\quad 30 = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100$

Solving, $\quad \xi = 0.358$

and $\quad T_s = \dfrac{4}{\xi\omega_n} \quad$ i.e. $\quad 5 = \dfrac{4}{0.358\omega_n}$

$\omega_n = 2.2346$ rad/sec

∴

i) $\quad \text{T.F.} = \dfrac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \dfrac{5}{s^2 + 1.6s + 5}$

ii) $\quad \omega_d = \omega_n\sqrt{1-\xi^2} = 2.0881$ rad/sec

∴ $\quad T_P = \dfrac{\pi}{\omega_d} = \dfrac{\pi}{2.0881} = 1.5045$ sec

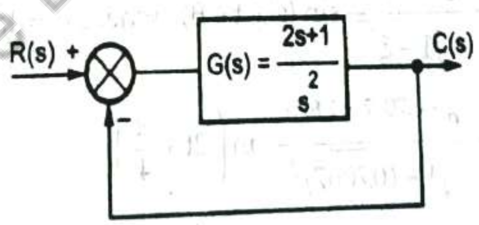
iii) $\quad \theta = \tan^{-1}\left[\dfrac{\sqrt{1-\xi^2}}{\xi}\right] = 1.205$ rad

Hence the output response is,

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$$

$$= 1 - 1.0708\, e^{-0.8\,t} \sin(2.0881\, t + 1.205)$$

Example 7.17.6 *For the system shown in given figure, obtain response to the unit step function.* **VTU : March-01, Marks 8**



Solution : The T.F. of the system is,

$$\frac{C(s)}{R(s)} = \frac{\dfrac{2s+1}{s^2}}{1+\dfrac{2s+1}{s^2}} = \frac{2s+1}{s^2+2s+1}$$

Now as numerator has term of 's' standard expression of c(t) cannot be used, though system is second order. Hence use partial fraction method substituting R(s).

$$R(s) = \frac{1}{s}, \text{ as unit step input}$$

$$\therefore \quad C(s) = \frac{1}{s} \cdot \frac{2s+1}{s^2+2s+1} = \frac{1}{s} \cdot \frac{2s+1}{(s+1)^2}$$

$$= \frac{A}{s} + \frac{B}{(s+1)^2} + \frac{C}{(s+1)} \quad \text{... Partial fraction}$$

$$\therefore \quad A(s+1)^2 + Bs + Cs(s+1) = 2s + 1$$

$$\therefore \quad As^2 + 2As + A + Bs + Cs^2 + Cs = 2s + 1$$

$$\therefore \quad A + C = 0, \quad 2A + B + C = 2, \quad A = 1$$

$$\therefore \quad C = -1, \quad B = 1$$

$$\therefore \quad c(s) = \frac{1}{s} + \frac{1}{(s+1)^2} - \frac{1}{(s+1)}$$

Taking inverse Laplace transform,

$$\therefore \quad c(t) = 1 - te^{-t} - e^{-t}$$

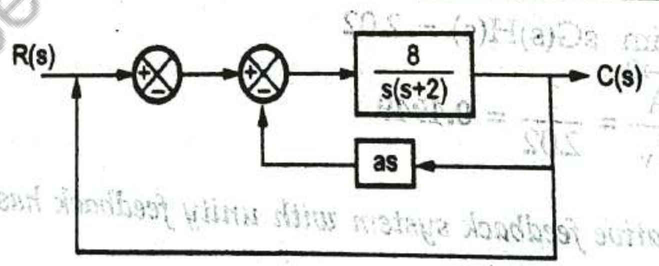
Example 7.17.7 *The system given in figure is a unity feedback system with minor feedback loop. i) In the absence of derivative feedback (a = 0), determine the damping ratio and undamped natural frequency.*

ii) Determine the constant 'a' which will increase damping ratio to 0.7.

iii) Find the overshoot in both the cases.

iv) What is the steady state error to unit ramp input for the value of 'a'.

VTU : Feb.-04, Jan.-14, 18, Marks 8



Solution : i) When a = 0,

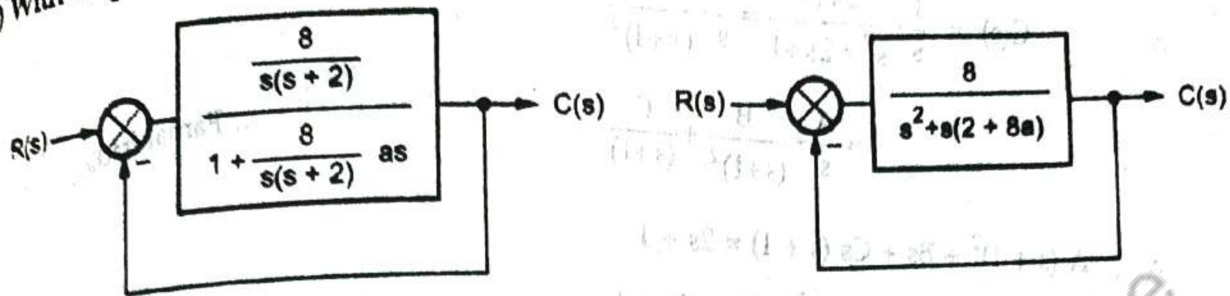
$$G(s) = \frac{8}{s(s+2)} \quad \text{and} \quad H(s) = 1$$

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{8}{s(s+2)}}{1+\frac{8}{s(s+2)}} = \frac{8}{s^2+2s+8}$$

$$\omega_n^2 = 8 \quad \text{and} \quad 2\xi\omega_n = 2$$

$$\therefore \quad \omega_n = \sqrt{8} \text{ rad/sec} \quad \text{and} \quad \xi = 0.3535$$

ii) With 'a' present, the system can be reduced as,



$$\therefore \quad \frac{C(s)}{R(s)} = \frac{\dfrac{8}{s^2+s(2+8a)}}{1+\dfrac{8}{s^2+s(2+8a)}} = \frac{8}{s^2+s(2+8a)+8}$$

$$\therefore \quad \omega_n^2 = 8 \quad \text{and} \quad 2\xi\omega_n = 2 + 8a$$

$$\therefore \quad \omega_n = \sqrt{8} \quad \text{and} \quad \xi = \frac{2+8a}{2\sqrt{8}} = 0.7 \text{ (given)}$$

$$\therefore \quad a = 0.2449 \quad \text{for} \quad \xi = 0.7$$

iii) $\% M_p = 100\, e^{-\pi\xi/\sqrt{1-\xi^2}} = 30.507\ \%$ for $\xi = 0.3535$, a = 0

and $\% M_p = 100\, e^{-\pi\xi/\sqrt{1-\xi^2}} = 4.59\ \%$ ...a = 0.2449

Thus derivative feedback reduces M_p, keeping ω_n same.

iv) For a = 0.2449, $G(s)H(s) = \dfrac{8}{s\,(s+2+8a)} = \dfrac{8}{s\,(s+3.9592)}$

$$\therefore \quad K_v = \lim_{s\to 0} sG(s)H(s) = 2.02$$

$$\therefore \quad e_{ss} = \frac{A}{K_v} = \frac{1}{2.02} = 0.4949$$

Example 7.17.8 *A negative feedback system with unity feedback has a plant* $G(s) = \dfrac{2(s+8)}{s(s+4)}$

i)Find the response of the system for a unit step input.

ii)Using the final value theorem, determine the steady-state value of the response for the same step input. **VTU : July 04, Marks 8**

Solution : H(s) = 1, $\quad G(s) = \dfrac{2(s+8)}{s(s+4)}$

i)
$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{2(s+8)}{s(s+4)}}{1+\frac{2(s+8)}{s(s+4)}} = \frac{2(s+8)}{s^2+6s+16}$$

For unit step input, $R(s) = \frac{1}{s}$

$$\therefore \quad C(s) = \frac{2(s+8)}{s(s^2+6s+16)} = \frac{A}{s}+\frac{Bs+C}{s^2+6s+16}$$

Key Point *As there is zero in the numerator, standard expression of c(t) for underdamped second order system can not be used.*

$$\therefore \quad A(s^2+6s+16)+s(Bs+C) = 2s+16$$

$$\therefore \quad A+B = 0, \quad 6A+C=2, \quad 16A=16$$

i.e. $\quad A = 1, \quad B=-1, \quad C=-4$

$$C(s) = \frac{1}{s}-\left\{\frac{s+4}{s^2+6s+16}\right\} = \frac{1}{s}-\left\{\frac{s+4}{s^2+6s+9+16-9}\right\}$$

$$= \frac{1}{s}-\left\{\frac{s+4}{(s+3)^2+(\sqrt{7})^2}\right\} = \frac{1}{s}-\left\{\frac{s+3+1}{(s+3)^2+(\sqrt{7})^2}\right\}$$

$$= \frac{1}{s}-\left\{\frac{s+3}{(s+3)^2+(\sqrt{7})^2}+\frac{1}{\sqrt{7}}\times\frac{\sqrt{7}}{(s+3)^2+(\sqrt{7})^2}\right\}$$

$$\therefore \quad c(t) = 1-\left\{e^{-3t}\cos\sqrt{7}t+\frac{1}{\sqrt{7}}e^{-3t}\sin\sqrt{7}t\right\}$$

$$\therefore \quad c(t) = 1-e^{-3t}\left\{\cos\sqrt{7}t+0.142\sin\sqrt{7}t\right\}$$ ... Unit step response

ii) Using final value theorem,

$$C_{ss} = \lim_{s\to 0} s\,C(s) = \lim_{s\to 0} s\left\{\frac{2(s+8)}{s(s^2+6s+16)}\right\} = \frac{2\times 8}{16} = 1$$

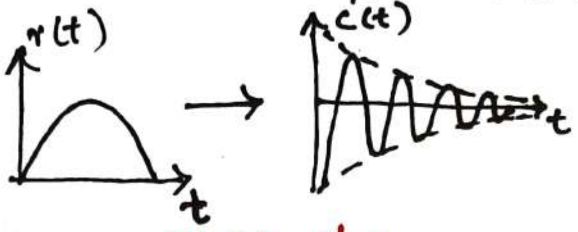
This can be cross-checked as,

$$C_{ss} = \lim_{t\to\infty} c(t) = 1-e^{-\infty}(....) = 1$$

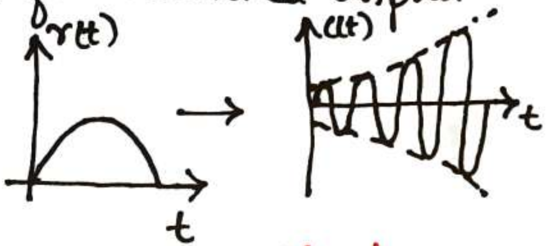
Exponential negative index term is zero as $t\to\infty$.

"Stability" Analysis of Control System:

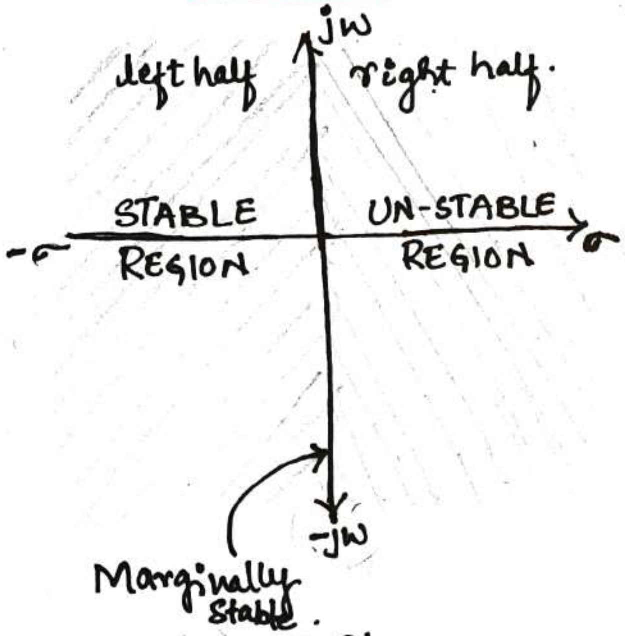
⇒ "Stable" → O/P is under control ⇒ Otherwise → "Unstable."
↳ the O/P is bounded for bounded input.



Stable s/m



Unstable s/m



S-Plane

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

$1+G(s)H(s)=0$ → gives "POLES"

Closed loop s/m
Stable → Poles → left half
Unstable → poles → right half.

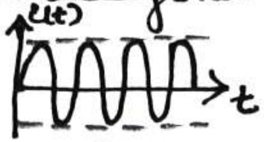
Absolute stable:

s/m response is stable for all variations.

→ LTI s/m → bounded i/p, respones → bounded o/p & controlable
→ i/p Absent → o/p → zero as time → infinity.
→ location of roots → left side - s-plane

Critical (or) Marginally stable:

→ response of s/m → neither decays nor grows. → Constant.

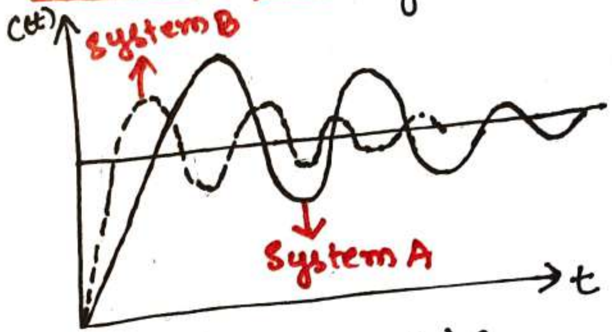


→ location of roots → jw axis (not repeated)
If repeated → Unstable s/m.

Conditional Stability:

s/m is stable → Certain bounded ranges

Relative Stability:



Relatively system 'B' is more stable than 'A'.

Poles → Faraways from jw axis
↓
s/m is relatively stable

Poles → near to jw axis → more settling time.

Hurwitz Stability Criterion:

Characteristic equation,

$$F(s) = a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n = 0$$

Condition:

$s^4 \quad s^3 \quad s^2 \quad s^1 \quad s^0$

1. All co-effecients of ch. eqn must be real and have the same sign.
2. All the powers of 's' must be present in the order of decending, None of the co-efficients must be zero.

Statement:

The condition to have all roots of ch. eqn. in left [redacted] side of s-plane is that Sub-determinents 'D_K'; $K = 1, 2, 3 \cdots n$ Obtained from Hurwitz's determinant 'H' must all be positive.

$$H = \begin{vmatrix} a_1 & a_3 & a_5 & \cdots & a_{2n-1} \\ a_0 & a_2 & a_4 & \cdots & a_{2n-2} \\ 0 & a_1 & a_3 & \cdots & a_{2n-3} \\ 0 & a_0 & a_2 & \cdots & a_{2n-4} \\ 0 & 0 & a_1 & \cdots & a_{2n-5} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & \cdots & \cdots & \cdots & a_n \end{vmatrix}_{n \times n}$$

* All co-efficients larger than 'n' & negative indicies must be replaced with zero

$$D_1 = |a_1| \quad D_2 = \begin{vmatrix} a_1 & a_3 \\ a_0 & a_2 \end{vmatrix} \quad D_3 = \begin{vmatrix} a_1 & a_3 & a_5 \\ a_0 & a_2 & a_4 \\ 0 & a_1 & a_3 \end{vmatrix} \cdots\cdots$$

$D_n = |H|$ All these subdeterminent values must be greater than zero for "Stable system"

Example:

$$F(s) = s^3 + s^2 + s^1 + 4 = 0$$

$a_0 = 1 \quad a_1 = 1 \quad a_2 = 1 \quad a_3 = 4$

$n = 3$

$$H = \begin{vmatrix} a_1 & a_3 & a_5 \\ a_0 & a_2 & a_4 \\ 0 & a_1 & a_3 \end{vmatrix} = \begin{vmatrix} 1 & 4 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 4 \end{vmatrix}$$

$$D_1 = |1| = 1 \quad D_2 = \begin{vmatrix} 1 & 4 \\ 1 & 1 \end{vmatrix} = 1 - 4 = -3$$

$$D_3 = \begin{vmatrix} 1 & 4 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 4 \end{vmatrix} = 1(4-0) - 4(4-0) + 0 = 4 - 16 = -12$$

Un Stable

Disadvantages

1. Time consuming → order of s become higher
2. the roots of RS – s-plane → not known
3. too difficult to predict Marginal stability.

Find the stability by Hurwitz Criterion for s|m.

ⓐ $s^4+8s^3+18s^2+16s+5=0$

$a_0=1,\ a_1=8,\ a_2=18,\ a_3\ 16\ \&\ a_4=5$

$n=4$ $\quad\quad$ $\underline{4\times4}$

$$H=\begin{vmatrix} a_1 & a_3 & a_5 & a_7 \\ a_0 & a_2 & a_4 & a_6 \\ 0 & a_1 & a_3 & a_5 \\ 0 & a_0 & a_2 & a_4 \end{vmatrix}=\begin{vmatrix} 8 & 16 & 0 & 0 \\ 1 & 18 & 5 & 0 \\ 0 & 8 & 16 & 0 \\ 0 & 1 & 18 & 5 \end{vmatrix}$$

$D_1=|8| \Rightarrow \boxed{D_1=8}$

$D_2=\begin{vmatrix} 8 & 16 \\ 1 & 18 \end{vmatrix}=(8\times18)-(1\times16) \Rightarrow \boxed{D_2=128}$

$D_3=\begin{vmatrix} 8 & 16 & 0 \\ 1 & 18 & 5 \\ 0 & 8 & 16 \end{vmatrix}=8[(18\times16)-(8\times5)]-16[(1\times16)-0]+0$

$\Rightarrow \boxed{D_3=1728}$

$D_4=\begin{vmatrix} 8 & 16 & 0 & 0 \\ 1 & 18 & 5 & 0 \\ 0 & 8 & 16 & 0 \\ 0 & 1 & 18 & 5 \end{vmatrix}=8\big[18[(16\times5)-0]-5[(8\times5)-0]+0\big]$
$-1\big[16[(16\times5)-0]-0+0\big] \Rightarrow \boxed{D_4=8640}$

"stable s|m"

ⓑ $s^3+s^2+s+4=0$

$a_0=1,\ a_1=1,\ a_2=1,\ a_3=4$

$n=3$ $\quad\quad$ 3×3

$$H=\begin{vmatrix} a_1 & a_3 & a_5 \\ a_0 & a_2 & a_4 \\ 0 & a_1 & a_3 \end{vmatrix}=\begin{vmatrix} 1 & 4 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 4 \end{vmatrix}$$

$D_1=|1| \Rightarrow \boxed{D_1=1}$

$D_2=\begin{vmatrix} 1 & 4 \\ 1 & 1 \end{vmatrix}=(1\times1)-(1\times4)$

$\Rightarrow \boxed{D_2=-3}$ ✓

$D_3=\begin{vmatrix} 1 & 4 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 4 \end{vmatrix}$ $\quad$ -ve

$=1[(1\times4)-0]-4[(1\times4)-0]+0$

$=4-16 \Rightarrow \boxed{D_3=-12}$ ✓

"unstable s|m"

Routh's Stability Criterion:

$$F(s) = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_n = 0$$

(i) All the co-efficients must have same sign.

(ii) All the power of 's' must be in desending order.

Routh Table:

s^n	a_0	a_2	a_4	a_6 .....
s^{n-1}	a_1	a_3	a_5	a_7 .....
s^{n-2}	b_1	b_2	b_3	...
s^{n-3}	c_1	c_2	c_3	....
$\vdots$	$\vdots$	$\vdots$	$\vdots$	
s^0	a_n			

Using 1st two rows

3rd row: $b_1 = \dfrac{a_1 \cdot a_2 - a_0 a_3}{a_1}$ $\quad b_2 = \dfrac{a_1 a_4 - a_0 a_5}{a_1}$ $\quad b_3 = \dfrac{a_1 a_6 - a_0 a_7}{a_1}$

From 2nd 3rd rows

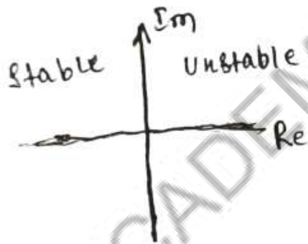
4th row: $c_1 = \dfrac{b_1 a_3 - a_1 b_2}{b_1}$ $\quad c_2 = \dfrac{b_1 a_5 - a_1 b_3}{b_1}$

This process should be continued till s^0 is obtained.

→ All the terms of columns 1 of Routh table must have same sign ⇒ "STABLE S/m."

→ If sign change exist ⇒ "UN-STABLE S/m."

The no of sign change = no of roots in right half of 's' plane.



Example: $F(s) = s^4 + 2s^3 + 4s^2 + 6s + 8 = 0$

$a_0 = 1, \; a_1 = 2, \; a_2 = 4, \; a_3 = 6, \; a_4 = 8$

s^4	1	4	8
s^3	2	6	0
s^2	1	8	0
s^1	-10	0	
s^0	8		

Unstable

two sign change

no of roots in RHS-plane = 2

Special cases of Routh's Criterion:

Case I: First element of any row of Routh's array is zero and remaining elements of row contains at least one non-zero element.

$$s^5 + 2s^4 + 3s^3 + 6s^2 + 2s + 1 = 0$$

$$\begin{array}{c|ccc}
s^5 & 1 & 3 & 2 \\
s^4 & 2 & 6 & 1 \\
s^3 & 0 & 1.5 & 0 \leftarrow \text{Special case I} \\
s^2 & \infty & &
\end{array}$$

The method will "Fail"

→ 2 Methods to overcome this problem.

METHOD I:

$$\begin{array}{c|ccc}
s^5 & 1 & 3 & 2 \\
s^4 & 2 & 6 & 1 \\
s^3 & +\varepsilon & 1.5 & 0 \\
s^2 & -\dfrac{6\varepsilon-3}{\varepsilon} & 1 & 0 \\
s^1 & +\dfrac{\dfrac{1.5(6\varepsilon-3)}{\varepsilon}-3}{\dfrac{6(\varepsilon-3)}{\varepsilon}} & 0 & \\
s^0 & 1 & &
\end{array}$$

To check sign change take $\lim_{\varepsilon \to 0}$

$$\lim_{\varepsilon\to 0} \frac{6\varepsilon-3}{\varepsilon} = \lim_{\varepsilon\to 0}\left[6-\frac{3}{\varepsilon}\right] = 6 - \lim_{\varepsilon\to 0}\frac{3}{\varepsilon} = 6-\infty$$

$\boxed{-\infty}$ −ve

$$\lim_{\varepsilon\to 0} \frac{1.5(6\varepsilon-3)-\varepsilon^2}{6\varepsilon-3} = \lim_{\varepsilon\to 0} \frac{9\varepsilon-4.5-\varepsilon^2}{6\varepsilon-3} = \frac{-4.5}{-3} = \boxed{+1.5}$$

+ve

There are two sign change ⇒ Unstable

METHOD II: replace s by $\frac{1}{z}$

$$\frac{1}{z^5} + \frac{2}{z^4} + \frac{3}{z^3} + \frac{6}{z^2} + \frac{2}{z} + 1 = 0$$

Take LCM and rearrange

$$z^5 + 2z^4 + 6z^3 + 3z^2 + 2z + 1 = 0$$

$$\begin{array}{c|ccc}
z^5 & 1 & 6 & 2 \\
z^4 & 2 & 3 & 1 \\
z^3 & 4.5 & 1.5 & 0 \\
z^2 & 2.33 & 1 & 0 \\
z^1 & -0.429 & 0 & \\
z^0 & 1 & &
\end{array}$$

There is two sign Change ⇒ Unstable

Special Cases of Routh's Criterion:

Case II: $s^6+2s^5+8s^4+12s^3+20s^2+16s+16=0$

s^6	1	8	20	16
s^5	2	12	16	0
s^4	2	12	16	0
s^3	0	0	0	0 ← Special Case II

When all the elements in any row of Routh's array are zero.
"Roots are on the imaginary axis".

Auxiliary eqn $A(s) = 2s^4+12s^2+16=0$

$$\frac{d}{ds}[A(s)] = 8s^3+24s+0=0$$

s^6	1	8	20	16
s^5	2	12	16	0
s^4	2	12	16	0
s^3	8	24	0	
s^2	6	16	0	
s^1	2.67	0		
s^0	16			

No sign change in 1^{st} column indicates the s/m might be stable.

Examine:

$2s^4+12s^2+16=0$

$s^4+6s^2+8=0$

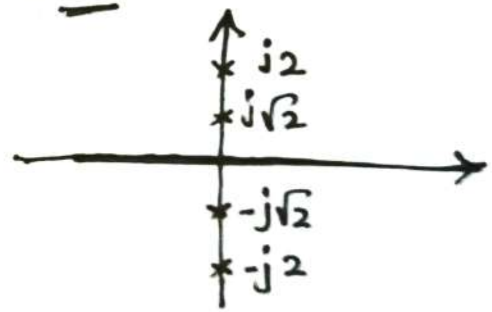
Put $s^2=x$

$x^2+6x+8=0 \qquad x=-2$ or -4

$s^2=-2$ or $s^2=-4$

$s=\sqrt{-2} \qquad s=\sqrt{-4}$

$s=\pm j\sqrt{2} \qquad s=\pm j\sqrt{4} \Rightarrow s=\pm j2$



The roots are on imaginary axis and are not repeated ⇒ "Marginally Stable"

Find the value of 'K' for the s/m to be stable.

ⓐ $G(s)H(s) = \dfrac{K(1-s)}{s(s^2+5s+9)}$

$1+G(s)H(s) = 0 \Rightarrow 1 + \dfrac{K(1-s)}{s(s^2+5s+9)} = 0$

$\Rightarrow \dfrac{s(s^2+5s+9) + K(1-s)}{s(s^2+5s+9)} = 0$

$\Rightarrow s^3 + 5s^2 + 9s + K - Ks = 0 \Rightarrow \underset{a_0}{s^3} + \underset{a_1}{5s^2} + \underset{a_2}{(9-K)s} + \underset{a_3}{K} = 0$

$a_0 = 1,\ a_1 = 5,\ a_2 = (9-K),\ a_3 = K$

$$\begin{array}{c|cc} s^3 & 1 & 9-K \\ s^2 & 5 & K \\ s^1 & \dfrac{45-5K-K}{5} & 0 \\ s^0 & K & \end{array}$$

$K > 0$

$\dfrac{45-6K}{5} > 0 \Rightarrow 45 - 6K > 0$

$45 > 6K \Rightarrow K < \dfrac{45}{6}$

$K < 7.5$

The range of K for stability is

$\boxed{0 < K < 7.5}$

ⓑ $s^3 + 3Ks^2 + (K+2)s + 4 = 0$

$a_0 = 1 \quad a_1 = 3K \quad a_2 = (K+2) \quad a_3 = 4$

$$\begin{array}{c|cc} s^3 & 1 & K+2 \\ s^2 & 3K & 4 \\ s^1 & \dfrac{3K^2+6K-4}{3K} & 0 \\ s^0 & 4 & \end{array}$$

$3K > 0 \Rightarrow K > 0$

$\dfrac{3K^2+6K-4}{3K} > 0 \Rightarrow 3K^2 + 6K - 4 > 0$

$K > 0.5275$ & $K > -2.5275$

$\boxed{0.5275 < K < 10}$

... repeated imaginary roots the system is marginally stable.]

8.11 Advantages of Routh's Criterion

VTU : July-16

Advantages of Routh's Array method are :

i) Stability of the system can be judged without actually solving the characteristic equation.

ii) No evaluation of determinants, which saves calculation time.

iii) For unstable system it gives number of roots of characteristic equation having positive real part.

iv) Relative stability of the system can be easily judged.

v) By using this criterion, critical value of system gain can be determined, hence frequency of sustained oscillations can be determined.

vi) It helps in finding out range of values of K for system stability.

vii) It helps in finding out intersection points of root locus with imaginary axis.

Review Question

1. *State the advantages of Routh's array method.* VTU : July-16, Marks 4

8.12 Limitations of Routh's Criterion

VTU : July-09,11,12,14,15,16,18, Jan.-05, 12

i) It is valid only for real coefficients of the characteristic equation.

ii) It does not provide exact locations of the closed-loop poles in left or right half of s-plane.

iii) It does not suggest methods of stabilising an unstable system.

iv) Applicable only to linear systems.

Review Question

1. *State the limitations of Routh's array.* VTU : July-09,11,12,14,15,16,18, Jan.-05,12, Marks 3

Root Locus:

→ Graphical method → the s/m performance
one of the parameter [Gain 'K'] is varied from '0 to ∞'

→ Path traced by roots of characteristic eqn.

Purpose:

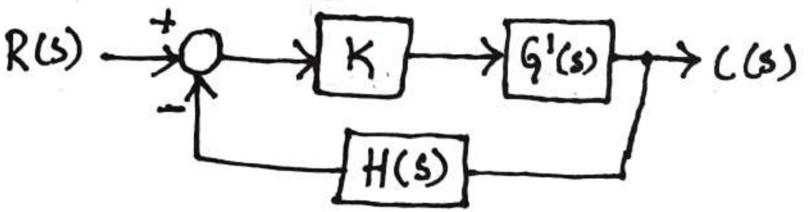
→ Find the stability of closed loop s/m.
→ Find range of 'K' for s/m to be stable.
→ Find 'ω' for marginally stable s/m.
→ Find 'K' for s/m to be overdamped, underdamped, critically damped & undamped condition.

Concept of Root locus:

The ch. eqn. of closed loop s/m

$1 + G(s)H(s) = 0 \rightarrow (1)$

Root locus → K is assumed to be a variable parameter



$G(s) = K \cdot G'(s)$; K → s/m gain
↳ Gain of forward path.

$(1) \Rightarrow \boxed{1 + K \cdot G'(s)H(s) = 0}$

From above eqn.,
→ K → Variable parameter.
→ The roots [closed loop poles] of the above eqn. depends on 'K'.
→ If 'K' is varied from −∞ to ∞ for each value of 'K' we will get seperate set of roots of ch. eqn.
→ All the roots on s-plane are joined & resulting locus ⇒ ROOT LOCUS

ROOT LOCUS: The locus of closed loop poles obtained when s/m gain 'K' is varied from −∞ to ∞.

K → 0 to ∞ ⇒ Direct root locus.
K → −∞ to 0 ⇒ Inverse root locus.
(or) Complementary root locus.

Ex:- For $G(s)H(s) = \frac{K}{s}$, Find it's root locus.

Ch. eqn. $1 + G(s)H(s) = 0$

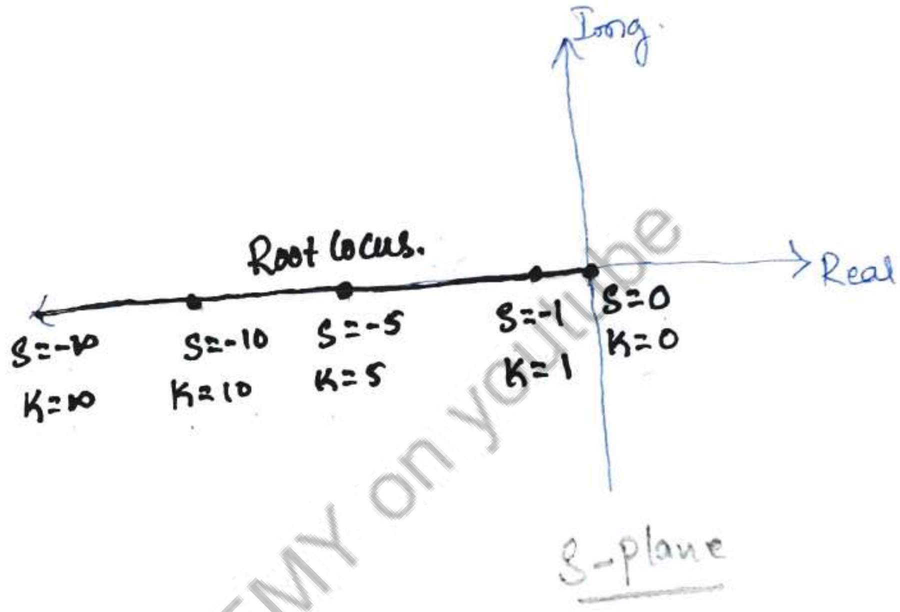
$1 + \frac{K}{s} = 0 \Rightarrow \frac{s+K}{s} = 0 \Rightarrow s + K = 0$

$\boxed{s = -K}$

root variable

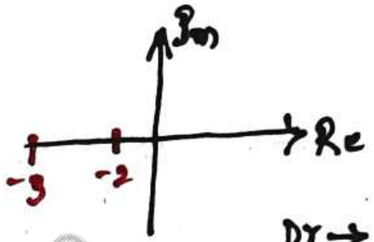
as $K \rightarrow$ varies from 0 to ∞
$s \rightarrow$ also varies.

K	$s = -K$
$K=0$	$s=0$
$K=1$	$s=-1$
$K=2$	$s=-2$
⋮	⋮
$K=\infty$	$s=-\infty$



Root locus - Rules for Construction:

Rule 1: Symmetrical about real axis.



Rule 2: Starts from open loop poles and ends at open loop zeros.

$$G(s)H(s) = \frac{K(s+3)}{(s+2)}$$

Dr → Pole

$s+2=0$

$\boxed{s=-2}$ → start

Nr → Zero

$s+3=0 \Rightarrow \boxed{s=-3}$ → end

Rule 3: Number of root locus.

N → no of Root locus

P → no of Finite poles.

Z → no of Finite zeroes.

then, $N=P$ if $P>Z$

$N=Z$ if $Z>P$

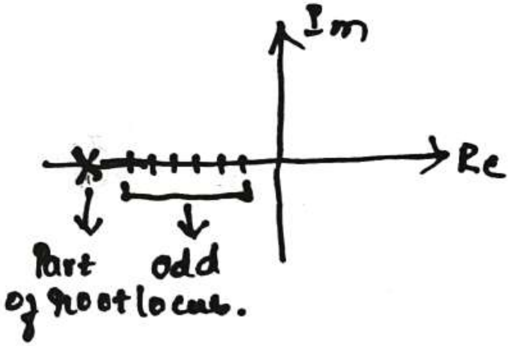
$N=Z=P$ if $P=Z$.

$P=1$ $Z=1$

$\boxed{N=1}$

Rule 4: Root locus on real axis.

Any point on real axis is a part of root locus if and only if no of poles and zeros to its right is "ODD"



Rule 5: Asymptotes → Branch of root locus that tends to infinity along a straight line.

$A = P-Z$

Rule 6: Angle of Asymptote $\phi = \dfrac{(2K+1)180^\circ}{(P-Z)}$; $K = 0, 1, 2 \ldots$

$G(s)H(s) = \dfrac{K}{s(s^2+6s+10)}$

$Z=0 \quad P=3 \quad P_1 = 0$

$K = 0, 1, 2 \qquad P_2 = -3+j$

$P_3 = -3-j$

$\underline{\underline{K=0}}$ $\phi_{K=0} = \dfrac{(2\times0+1)180^\circ}{3} = \underline{60^\circ}$

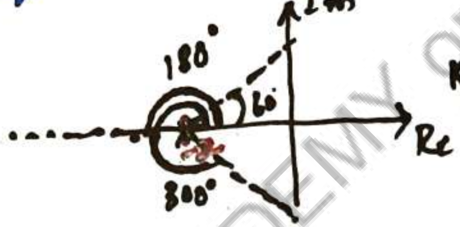
$\underline{\underline{K=1}}$ $\phi_{K=1} = \dfrac{(2\times1+1)}{3}180^\circ = \underline{180^\circ}$

$\underline{\underline{K=2}}$ $\phi_{K=2} = \dfrac{(2\times2+1)}{3}180^\circ = \underline{300^\circ}$

Centroid of Asymptotes → It is the point of intersection of asymptotes with real axis.

$\sigma_A = \dfrac{\text{Sum of poles} - \text{Sum of zeros}}{(P-Z)}$

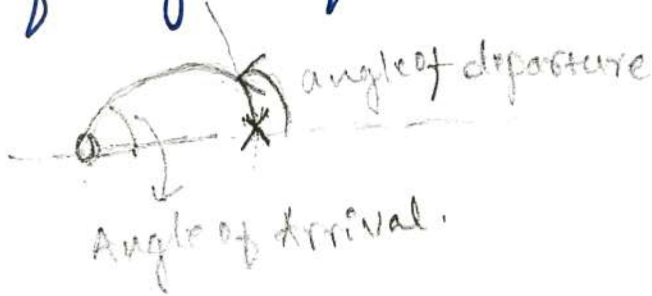
$\dfrac{-6-0}{3-0} = \underline{\underline{-2}}$



Rule 7:

Angle of departure = 180° − Sum of angles of vectors drawn to this pole from other poles + Sum of angles of vectors drawn to this poles from other zeroes.

Angle of Arival = 180° − Sum of angles of Vectors drawn to this zeros from other zeros + Sum of angles of Vectors drawn to this zeros from other poles.



Rule 8: breakaway point on real axis

Can be determined from the roots of $\frac{dK}{ds} = 0$

$$G(s)H(s) = \frac{K}{s(s^2+6s+10)}$$

$$1 + G(s)H(s) = 0 \Rightarrow 1 + \frac{K}{s(s^2+6s+10)} = 0$$

Chr $s(s^2+6s+10) + K = 0$

$$K = -s^3 - 6s^2 - 10s$$

$$\frac{dK}{ds} = -3s^2 - 12s - 10 = 0$$

$$3s^2 + 12s + 10 = 0$$

Roots

$$\boxed{\begin{aligned} s_1 &= -1.1835 \\ s_2 &= -2.815 \end{aligned}}$$

Break away points

Rule 9: Intersection of root locus braches with $j\omega$ imaginary axis can be determined through Routh-Harwitz Criteria.

$$G(s)H(s) = \frac{K}{s(s^2+6s+10)}$$

Chr eqn $\Rightarrow s^3 + 6s^2 + 10s + K = 0$

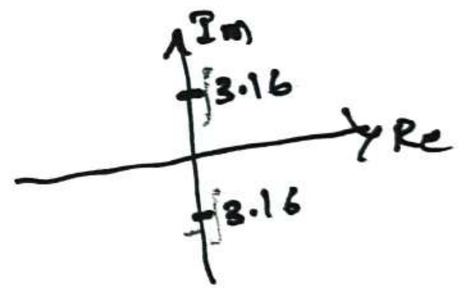
s^3	1	10
s^2	6	K
s^1	$\frac{60-K}{6}$	← assume K=60
s^0	K	

$$\frac{60-K}{6} = 0$$

$$6s^2 + K = 0$$

Put K=60

$$\boxed{s = \pm j3.16}$$



Draw root locus diagram for a s/m having $G(s)H(s) = \dfrac{K}{s(s+1)(s+3)}$.

ⓐ Root locus Starting point → Open loop pole; endpoint → Open loop zeros.

$P = 3$ $P_1 \Rightarrow s = 0$

$P_2 \Rightarrow s = -1$

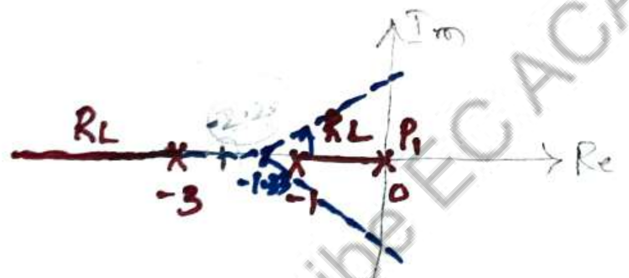
$P_3 \Rightarrow s = -3$

$Z = 0$

$s = \infty$

ⓑ no of root locus $N = P$ $\boxed{N = 3}$

ⓒ root locus on real axis



RL → Exists from 0 to −1 & to left of −3

ⓓ no of Asymtotes $A = P - Z = 3 - 0$

$\boxed{A = 3}$

ⓔ Centroid of Asymptotes

$$\sigma = \frac{\text{Sum of poles} - \text{Sum of zeros}}{\text{no of poles} - \text{no of zeros.}} = \frac{-4-0}{3-0}$$

$\sigma = -1.33$

ⓕ Angle of Asympotes

$$\phi = \frac{(2K+1)}{P-Z}180^\circ \quad ; K = 0, 1, \ldots (P-Z)-1$$

$(3-0)-1$

2

$\boxed{K = 0, 1, 2}$

$$\phi_{K=0} = \frac{(2\times 0+1)}{3}180^\circ \Rightarrow 60^\circ$$

$$\phi_{K=1} = \frac{(2\times 1+1)}{3}180^\circ \Rightarrow 180^\circ$$

$$\phi_{K=2} = \frac{(2\times 2+1)}{3}180^\circ \Rightarrow 300^\circ$$

(g) break away point $\frac{dK}{ds} = 0$

$1 + G(s)H(s) = 0$ chr eqn

$1 + \frac{K}{s(s+1)(s+3)} = 0$

$s(s+1)(s+3) + K = 0$

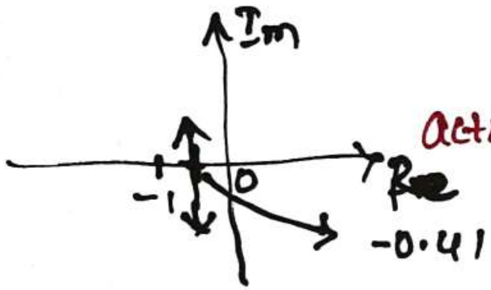
$s^3 + 4s^2 + 3s + K = 0$

$K = -s^3 - 4s^2 - 3s \rightarrow (1)$

$\frac{dK}{ds} = -3s^2 - 8s - 3 = 0$

$3s^2 + 8s + 3 = 0 \Rightarrow \boxed{s = -0.41}$

$s = -2.28$



actual breakaway point.

(h) Intersection of root locus branches with Imaginary axis.

$s^3 + 4s^2 + 3s + K = 0 \rightarrow$ chr eqn

Rough's Test

s^3	1	3
s^2	4	K
s^1	$\frac{12-K}{4}$	0
s^0	K	

$K > 0$

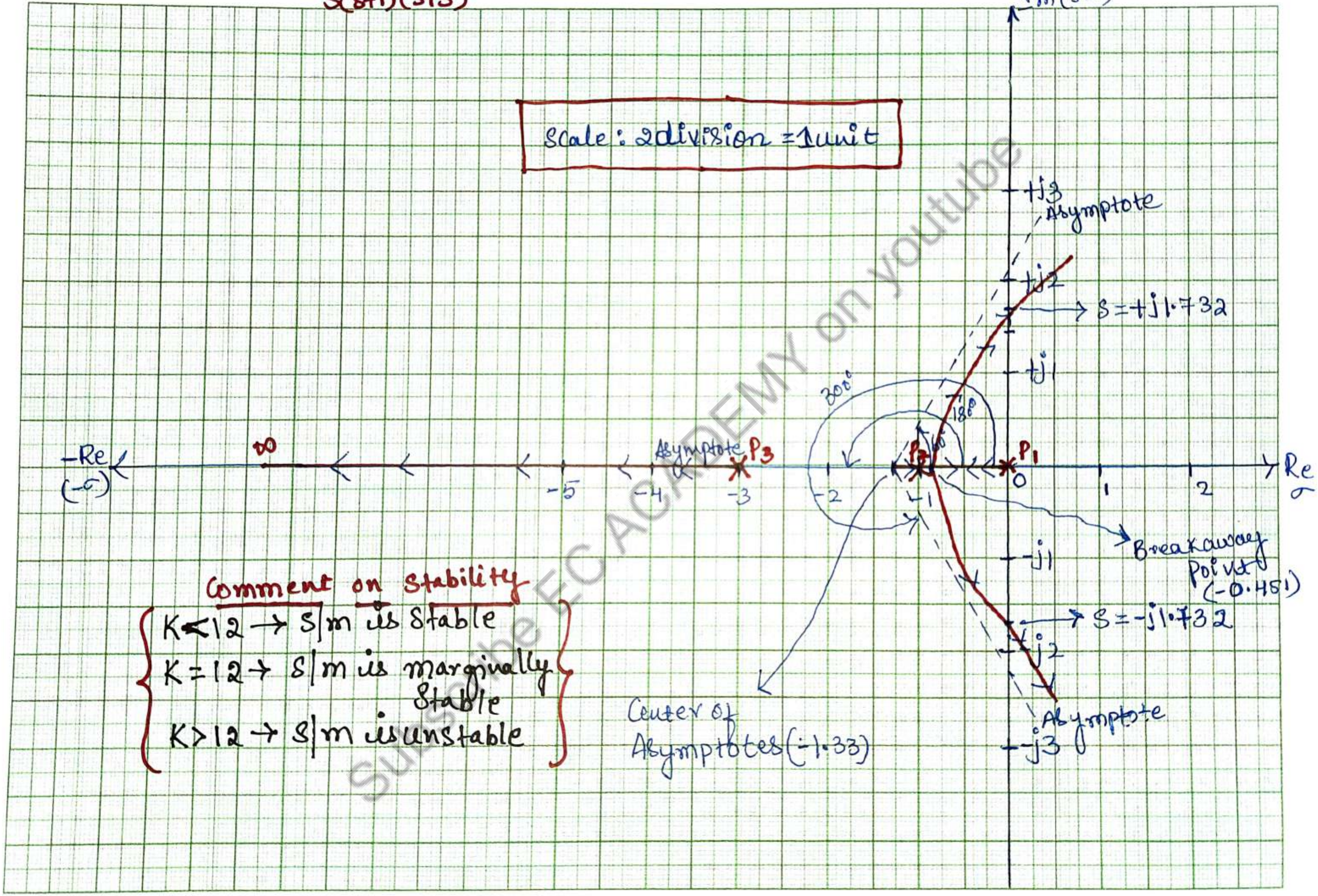
$\frac{12-K}{4} = 0$

$\Rightarrow \boxed{K = 12}$

$4s^2 + K = 0$

$4s^2 + 12 = 0 \Rightarrow s = \pm j1.732$

Root locus for $\frac{K}{S(S+1)(S+3)}$



Comment on Stability

- K < 12 → S/m is Stable
- K = 12 → S/m is marginally Stable
- K > 12 → S/m is unstable

$$G(s)H(s) = \frac{K}{s(s+3)(s^2+2s+2)}$$

ⓐ Root locus starting point

$\boxed{P=4}$ $P_1 \Rightarrow s=0$

$P_2 \Rightarrow s=-3$

$P_3 \Rightarrow s=-1+j$

$P_4 \Rightarrow s=-1-j$

end point.

$\boxed{Z=0}$

$s=\infty$

ⓑ no. of Root locus. $\boxed{N=4}$

ⓒ R.L. on real axis exist b/w -3 & 0.

ⓓ no of Asymptotes $A=P-Z$

$\boxed{A=4}$

ⓔ Center of asymptotes

$$= \frac{\text{Sum of poles} - \text{Sum of Zeros}}{P-Z} = \frac{-5-0}{4}$$

$$= \underline{-1.25}$$

ⓕ Angle of Asymptotes.

$$\phi = \frac{(2\times K+1)}{P-Z}180^\circ \quad ; \; K = 0,1,2 \ldots (P-Z)-1$$

$(4-0)-1$

$K=0,1,2,3$

$$\phi_{K=0} = \frac{(2\times 0+1)}{4}180^\circ = \underline{45^\circ}$$

$$\phi_{K=1} = \frac{(2\times 1+1)}{4}180^\circ = \underline{135^\circ}$$

$$\phi_{K=2} = \frac{(2\times 2+1)}{4}180^\circ = \underline{225^\circ}$$

$$\phi_{K=3} = \frac{(2\times 3+1)}{4}180^\circ = \underline{315^\circ}$$

ⓖ Break away point $\frac{dK}{ds}=0$

$$1+G(s)H(s)=0 \Rightarrow 1+\frac{K}{s(s+3)(s^2+2s+2)}=0$$

$$\therefore s^4+5s^3+8s^2+6s+K=0$$

$$\Rightarrow K = -s^4-5s^3-8s^2-6s \longrightarrow ①$$

$$\frac{dK}{ds} = -4s^3-15s^2-16s-6=0$$

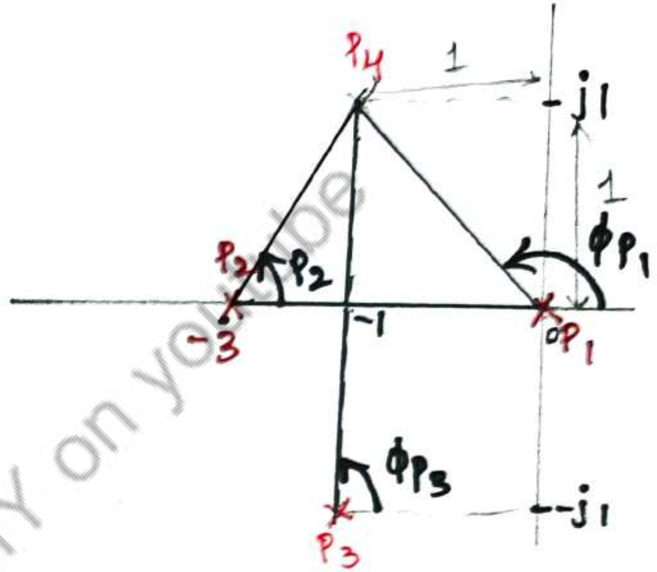
$$4s^3+15s^2+16s+6=0$$

$\boxed{s=-2.288}$ two complex root with real Part -0.7307

actual breakaway point

(h) Intersection with Imaginary axis

Ch$\underset{\cdot\cdot}{r}$ eqn $\quad s^4 + 5s^3 + 8s^2 + 6s + K = 0$



Routh Test

s^4	1	8	K
s^3	5	6	0
s^2	6.8	K	0
s^1	$\frac{40.8-5K}{6.8}$	0	
s^0	K		

$s^0 \Rightarrow K > 0$

$s^1 \Rightarrow \frac{40.8-5K}{6.8} = 0$

$K = \frac{40.8}{5} = 8.16$

$\boxed{K = 8.16}$

$s^2 \Rightarrow 6.8s^2 + K = 0$

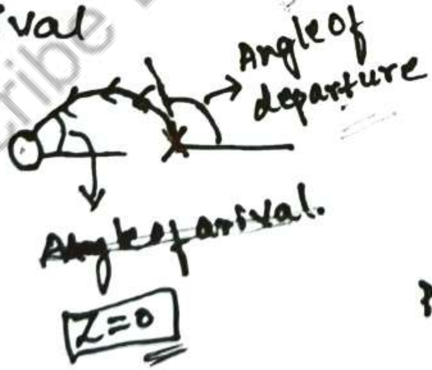
$s^2 = 1.2 \Rightarrow \boxed{s = \pm j1.095}$

$\phi_{P_1} = 180° - \tan^{-1}\left(\frac{1}{1}\right) = 135°$

$\phi_{P_3} = 90° \quad \phi_{P_2} = \tan^{-1}\left(\frac{1}{2}\right) = 26.56°$

$\Sigma\phi_p = 251.56° \quad \Sigma\phi_z = 0°$

(i) Angle of departure & Arrival



$\phi_d = 180° - \phi$

$\phi = \Sigma\phi_p - \Sigma\phi_z$

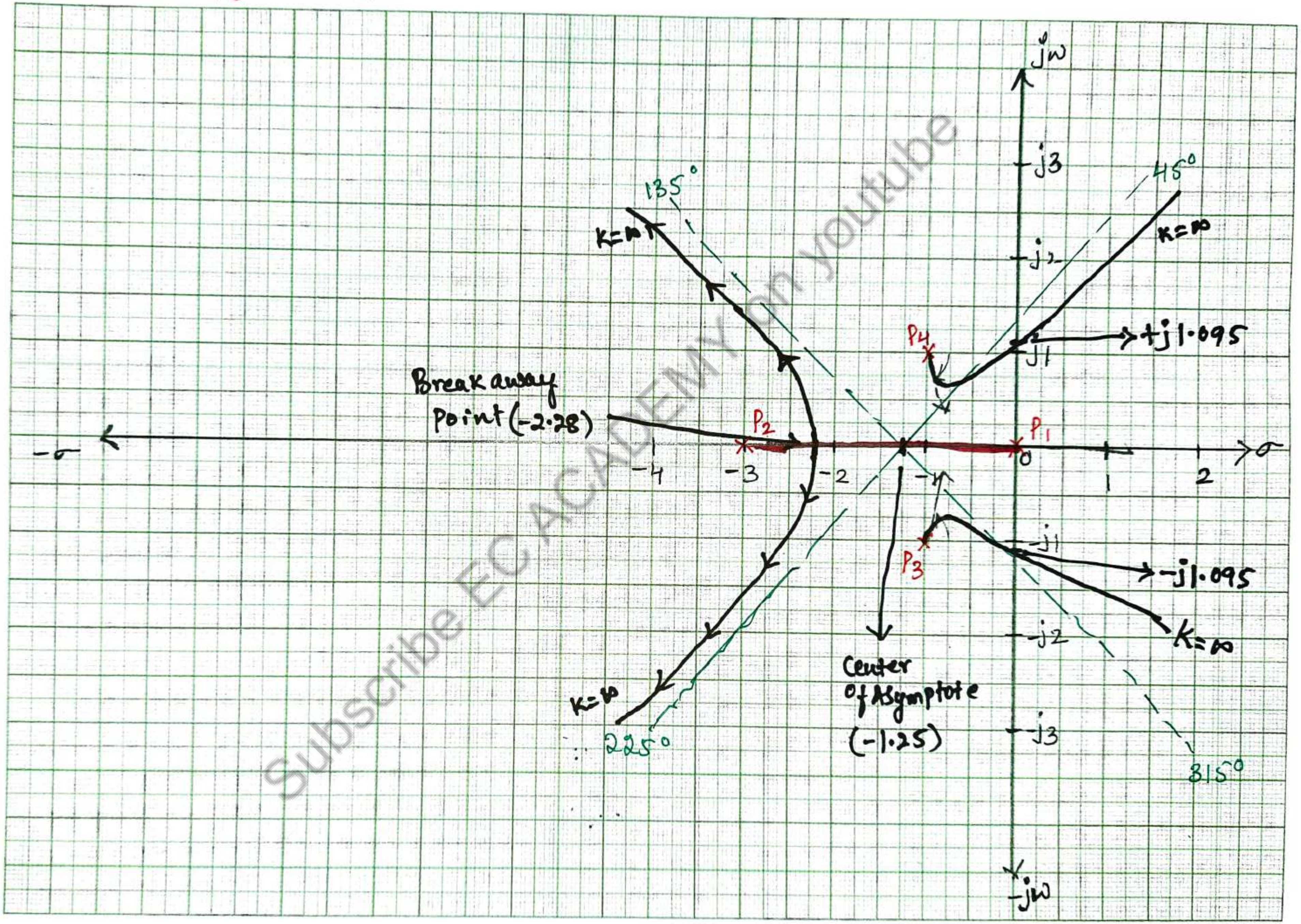
$\phi = 251.56°$

$\phi_d = 180° - 251.56°$

P4 → $\phi_d = -71.56°$ ----- at $-1+j$

P3 → $\phi_d = +71.56°$ ----- at $-1-j$

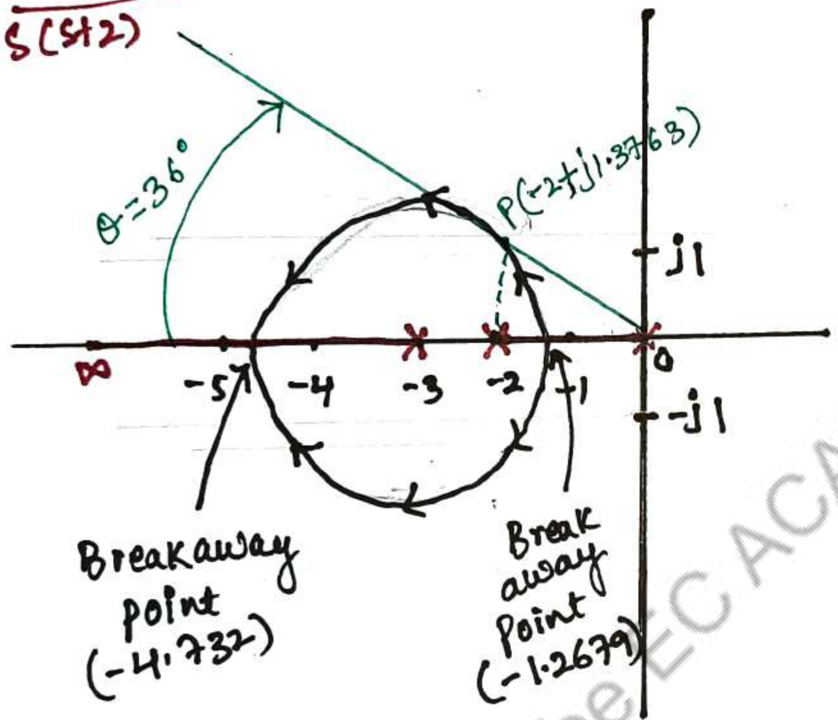
$$\frac{K}{s(s+3)(s^2+2s+2)}$$



(i) Determine the damping ratio (ξ)

(ii) Determine the value of K for max damped oscillatory response.

$$G(s)H(s) = \frac{K(s+3)}{s(s+2)}$$



(i) $\underline{\xi}$ $\boxed{\theta = 36°}$

$\therefore \cos^{-1}\xi = \theta$

$\cos^{-1}\xi = 36°$

$\Rightarrow \boxed{\xi = 0.809}$

$\cos^{-1}\xi = \theta$

Ex: $\xi = 0.5 \Rightarrow \cos^{-1} 0.5 = 60°$

(ii) P is $(-2+j1.3763)$

To find $\underline{K}$ → magnitude condition.

$$|G(s)H(s)|_{at\ P} = 1$$

$$\left|\frac{K(s+3)}{s(s+2)}\right|_{at\ s=-2+j1.3763} = 1$$

$$\therefore \frac{|K| \cdot \overbrace{|-2+j1.3763+3|}^{1.70123\angle 53.9°}}{\underbrace{|-2+j1.3763|}_{2.4278\angle -34.5°} \cdot \underbrace{|-2+j1.3763+2|}_{1.376\angle 90°}} = 1$$

$$\Rightarrow \frac{K \times 1.70123}{2.427 \times 1.376} = 1$$

$$\Rightarrow \boxed{K = 1.9641} \rightarrow \xi = 0.809$$

Advantages of "Root Locus":

1. The Absolute stability of the s/m Can be predicted from the location of roots in the s-plane.

2. Limiting range of the Value of the s/m gain 'K' Can be decided for absolute stability of the s/m.

3. Marginal Value of 'K' Can be determined

4. The Value of s/m gain 'K' for any Point on the root locus Can be determined, by using magnitude Condition.

5. Particular damping ratio of the s/m, gain 'K' Can be determined.

6. It helps in deciding the stability of the Control s/ms. with time delay.

7. Gain margin of the s/m Can be determined.

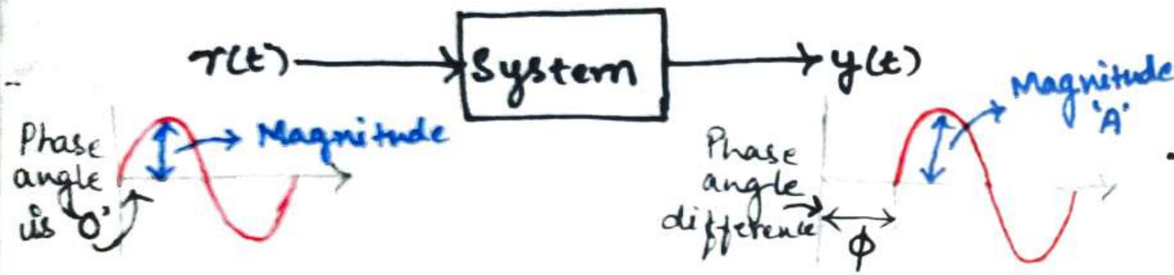
8. Phase margin of the s/m Can be determined.

9. Relative stability Can be determined.

10. Information about the settling time of the s/m Can be determined.

Frequency domain Analysis

→ defined as, "the steady state respo-nse of a s/m due to a sinusoidal input."



$r(t) = \sin(\omega t + 0)$ $\qquad$ $y(t) = A\sin(\omega t + \phi)$

Let us consider linear s/m with sinusoidal input

$$\boxed{r(t) = \sin(\omega t + 0)}$$

$y(t)$ → steady state o/p of the s/m
- ↳ sinusoidal with same frq 'ω'
- ↳ different amplitude 'A'
- ↳ different phase 'ϕ'

$$\boxed{y(t) = A\sin(\omega t + \phi)}$$

→ Freq response ⇒ both magnitude & phase varies.

Time response ⇒ only magnitude varies.

→ If o/p ~~phase~~ angle is positive → leading w.r.t. i/p ⇒ "leading Angle"

→ If o/p phase angle is negative → lagging w.r.t i/p ⇒ "lagging Angle".

NOTE: In frq domain Analysis,
'S' is replaced by 'jω'
∵ frq is variable, 'ω' is varied from '0 to ∞'

Magnitude term ⇒ $\boxed{a + jb = \sqrt{a^2 + b^2}}$

Phase term ⇒ $\boxed{a + jb = \tan^{-1}\left[\frac{b}{a}\right]}$

Advantages:

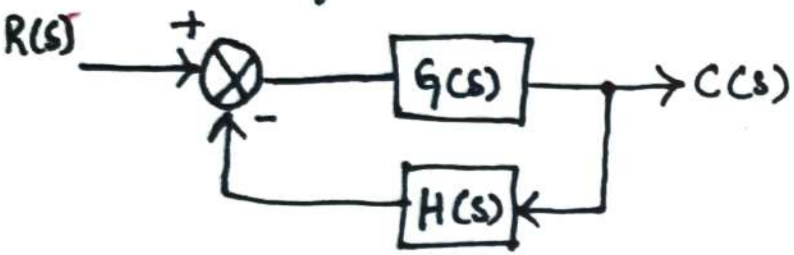
1. Frq response test are easy to perform.
2. Transfer fun can be obtained by the frq response of the s/m.
3. It is used to find the absolute & relative stability of the s/m.
4. The apparatus required for obtaining frq response is simple & easy to use.
5. Without the knowledge of the Transfer fun, the frq response of stable open loop s/m can be obtained experimentally.

Disadvantages.

1. Obtaining frq response practically is time consuming.
2. These methods are applicable to linear s/ms only.
3. For high time constant, this method is not convenient.
4. This method is considered as 'old' & 'outdated' when compared with methods developed for digital computer simulation & modelling.

Frequency Domain Specifications:

– Control System,

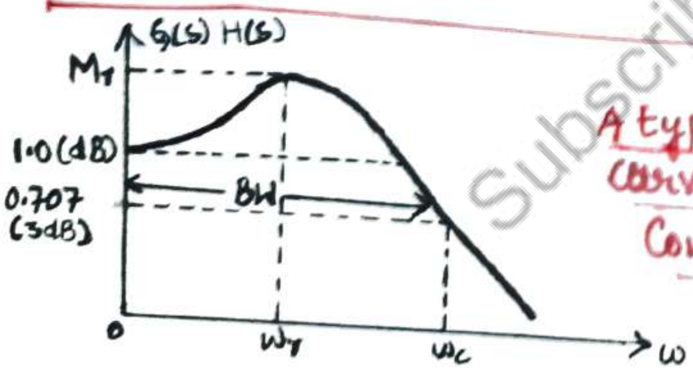


– Closed Loop Transfer fun,

$$M(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} \longrightarrow (1)$$

Put $s = j\omega$ in eqn (1)

$$\boxed{M(j\omega) = \frac{G(j\omega)}{1+G(j\omega)H(j\omega)}}$$



A typical Magnification curve of F.B. Control s/m.

(i) Resonant Peak: (M_r)

The max Value of '$M(j\omega)$' as 'ω' varied

$M_r \rightarrow$ indicates the relative stability of the Closed loop s/m.

↳ in the range 1.1 & 1.5.

(ii) Resonant frequency: (ω_r)

The frq at which peak resonance 'M_r' occurs.

(iii) Bandwidth: (BW)

The range of frequencies b/w 3dB Points.

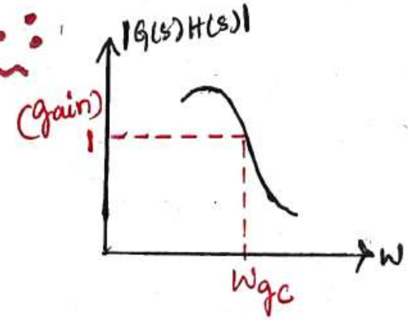
(iv) Cutoff rate:

The slope of the log magnitude curve near the cut-off frq is cut-off rate.

Cutoff rate is the ability of the s/m to distinguish a signal from noise.

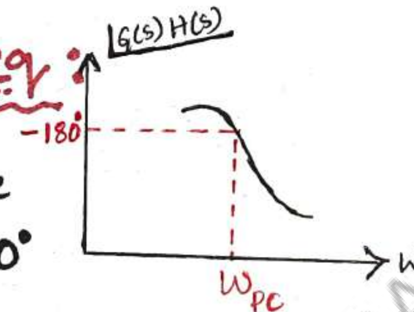
(V) Gain Cross Over frq:

The frq 'w_{gc}' at which gain is unity (1)



(Vi) Phase Crossover frq:

The frq 'w_{pc}' at which phase angle of the T.F. is $-180°$



(vii) Gain Margin: (GM)

The Gain at phase crossover frq 'w_{pc}'

$$\boxed{GM = \frac{1}{|G(j\omega)\,H(j\omega)|}}$$

in dB,

$$\boxed{GM = 20 \log \cdot \frac{1}{|G(j\omega)\,H(j\omega)|}}$$

→ GM can be calculated at 'w_{pc}' by equating phase to $-180°$

(viii) Phase Margin: (PM)

PM is the amount of additional phase lag at the gain crossover frq 'w_{gc}'.

$$\boxed{PM = 180 + \angle G(s) H(s)}$$

→ PM is calculated at 'w_{gc}' by equating gain to unity.

Co-relation b/w Time domain & Frequency domain for Second order s/m:

Second order s/m, $G(s) = \dfrac{\omega_n^2}{s(s+2\xi\omega_n)}$ & $H(s) = 1$

$$\frac{G}{1+GH} \Rightarrow \frac{C(s)}{R(s)} = \frac{\dfrac{\omega_n^2}{s(s+2\xi\omega_n)}}{1+\dfrac{\omega_n^2}{s(s+2\xi\omega_n)}\cdot 1} = \frac{\dfrac{\omega_n^2}{s^2+2\xi\omega_n s}}{\dfrac{s^2+2\xi\omega_n s+\omega_n^2}{s^2+2\xi\omega_n s}}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}}$$

The closed loop Transfer fun in Time domain.

freq domain $s \to j\omega$

$$\frac{C(j\omega)}{R(j\omega)} = \frac{\omega_n^2}{(j\omega)^2+2\xi\omega_n j\omega+\omega_n^2} = \frac{\omega_n^2}{-\omega^2+2\xi\omega_n j\omega+\omega_n^2}$$

$$\frac{C(j\omega)}{R(j\omega)} = \frac{\omega_n^2}{\omega_n^2-\omega^2+j2\xi\omega_n\omega}$$

divide nr. & dr by ω_n^2

$$\frac{C(j\omega)}{R(j\omega)} = \frac{1}{\left[1-\dfrac{\omega}{\omega_n}\right]^2+2\xi j\dfrac{\omega}{\omega_n}}$$

Replace $\dfrac{\omega}{\omega_n} = x$

$$\frac{C(j\omega)}{R(j\omega)} = \frac{1}{[1+x^2]+2\xi jx}$$

$a+jb$

$M = \sqrt{a^2+b^2}$

$$M = \frac{1}{\sqrt{(1-x^2)^2+(2\xi x)^2}} = \frac{1}{\sqrt{(1-x^2)^2+4\xi^2x^2}}$$

$\dfrac{1}{\sqrt{a}} \Rightarrow a^{-1/2}$

$\dfrac{dM}{dx} = 0$

$$\frac{dM}{dx} = \frac{d}{dx}\left[(1-x^2)^2+4\xi^2x^2)^{-1/2}\right]$$

$$\boxed{\frac{d}{dx}u^n = nu^{n-1}\frac{du}{dx}}$$

$$\frac{dM}{dx} = \frac{d}{dx}\left[(1-x^2)^2+4\xi^2x^2\right)^{-1/2}]$$

$$= -\frac{1}{2}\left[(1-x^2)^2+4\xi^2x^2)\right]^{-1/2-1} \times \frac{d}{dx}\left[(1-x^2)^2+4\xi^2x^2\right]$$

$-3/2$

$$= -\frac{1}{2}\frac{1}{[(1-x^2)^2+4\xi^2x^2]^{3/2}} \cdot *$$

$$2(1-x^2)\cdot(0-2x)+8\xi^2x$$

$$= -\frac{1}{2}\frac{-4x+4x^3+8\xi^2x}{[(1-x^2)^2+4\xi^2x^2]^{3/2}} = 0$$

$$-4x+4x^3+8\xi^2x=0$$

$$4x-4x^3-8\xi^2x=0$$

$$4x[-x^2-2\xi^2+1]=0 \qquad 4x=0 \quad \boxed{x=0} \times$$

$$x^2+2\xi^2-1=0 \Rightarrow x^2=1-2\xi^2$$

$$x=\sqrt{1-2\xi^2}$$

$$\because x=\frac{\omega}{\omega_n} \qquad \frac{\omega}{\omega_n}=\sqrt{1-2\xi^2}$$

Resonent frq which maximize $M \to \omega_r$

$$\boxed{\omega_r = \omega_n\sqrt{1-2\xi^2}}$$ Resonent frq

$$M_r = \frac{1}{\sqrt{[1-(\sqrt{1-2\xi^2})^2]^2+4\xi^2(\sqrt{1-2\xi^2})^2}}$$

$$= \frac{1}{\sqrt{4\xi^4+4\xi^2(1-2\xi^2)}}$$

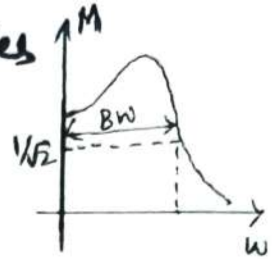
$$= \frac{1}{2\xi\sqrt{\xi^2+1-2\xi^2}} = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

$$\boxed{M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}}$$ Resonent Peak.

$\xi=0 \qquad \omega_r=\omega_n$ & $M_r=\infty$

Bandwidth of Second order System:

Bandwidth ⇒ The range of frequencies over wich 'M' is equal or greater than $1/\sqrt{2}$



$$M = \frac{1}{\sqrt{(1-x^2)^2 + (2\xi x)^2}} \qquad x = \frac{\omega}{\omega_n} \Rightarrow x_b = \frac{\omega_b}{\omega_n}$$

$$M\Big|_{x=x_b} = \frac{1}{\sqrt{(1-x_b^2)^2 + (2\xi x_b)^2}} = \frac{1}{\sqrt{2}}$$

$$\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{2}}, \quad \sqrt{a} = \sqrt{2}$$

$$(1-x_b^2)^2 + (2\xi x_b)^2 = 2$$

$(a-b)^2 = a^2+b^2-2ab$

$$\Rightarrow 1 + x_b^4 - 2x_b^2 + 4\xi^2 x_b^2 = 2$$

$$\Rightarrow x_b^4 - 2x_b^2 + 4\xi^2 x_b^2 - 1 = 0$$

$$\Rightarrow x_b^4 - 2x_b^2(1-2\xi^2) - 1 = 0$$

Put $x_b^2 = y$

$$y^2 - 2y(1-2\xi^2) - 1 = 0 \longrightarrow (1)$$

$$ay^2 + by + c = 0 \longrightarrow (2)$$

$$y = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$a = 1,\ b = -2(1-2\xi^2)\ \&\ c = -1$

$$y = \frac{-(-2(1-2\xi^2)) \pm \sqrt{(-2(1-2\xi^2))^2 - 4(1)(-1)}}{2(1)}$$

$$y = \frac{+2(1-2\xi^2) \pm \sqrt{4(1-2\xi^2)^2 + 4}}{2}$$

$\to (a-b)^2 = a^2+b^2-2ab$

$$y = \frac{2(1-2\xi^2) \pm \sqrt{4(1^2+4\xi^4-4\xi^2)+4}}{2}$$

$$y = \frac{2(1-2\xi^2) \pm \sqrt{4[(1^2+4\xi^4-4\xi^2)+1]}}{2}$$

$$y = \frac{2(1-2\xi^2) \pm \sqrt[2]{4}\cdot\sqrt{4\xi^4-4\xi^2+2}}{2}$$

$$y = (1-2\xi^2) \pm \sqrt{4\xi^4-4\xi^2+2} \qquad \because y = x_b^2$$

$$x_b^2 = 1-2\xi^2 + \sqrt{2-4\xi^2+4\xi^4}$$

$$\therefore x_b = \sqrt{(1-2\xi^2 + \sqrt{2-4\xi^2+4\xi^4}}$$

$\because x_b = \frac{\omega_b}{\omega_n}$

$\Rightarrow \omega_b = x_b \omega_n$

$$\Rightarrow \boxed{\omega_b = \omega_n \cdot \sqrt{(1-2\xi^2) + \sqrt{2-4\xi^2+4\xi^4}}}$$

Bandwidth.

① For unity feedback s/m $G(s) = \frac{100}{s(s+5)}$ determine (i) Resonance peak (M_r) (ii) Resonance frequency (ω_r)

$H(s) = 1$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{100}{s(s+5)}}{1+\frac{100}{s(s+5)}\cdot 1} = \frac{100}{s^2+5s+100}$$

$$= \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$\omega_n^2 = 100 \Rightarrow \boxed{\omega_n = 10}$

$2\xi\omega_n = 5 \Rightarrow \boxed{\xi = 0.25}$

(i) $M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} = \frac{1}{2\times 0.25\times\sqrt{1-0.25)^2}}$

$\boxed{M_r = 2.0655}$

(ii) $\omega_r = \omega_n\sqrt{1-2\xi^2} = 10\sqrt{1-2\times(0.25)^2}$

$\boxed{\omega_r = 9.3541 \text{ rad/sec}}$

② Find the OLTF of a unity F.B. 2nd order s/m for which $M_r = 1.1$ units & $\omega_r = 11.2$ rad/sec

$M_r = 1.1$ & $\omega_r = 11.2$ rad/sec.

$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} = 1.1 \Rightarrow \frac{1}{\xi\sqrt{1-\xi^2}} = 2.2$

$\frac{1}{\xi^2(1-\xi^2)} = 4.84 \Rightarrow \xi^2 - \xi^4 = 0.2066$

$\xi^4 - \xi^2 + 0.2066 = 0$ Let $x = \xi^2$

$x^2 - x + 0.2026 = 0$ $x = 0.7083, 0.2916$ $\Downarrow$ ξ^2

$\xi = 0.8416, 0.54$

* M_r Cannot exist for $\xi > 0.707$ *

$\boxed{\xi = 0.54}$

$\omega_r = \omega_n\sqrt{1-2\xi^2} = 11.2$

$11.2 = \omega_n 0.6456$

$\boxed{\omega_n = 17.348 \text{ rad/sec}}$

OLTF $\Rightarrow \boxed{\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s(s+2\xi\omega_n)} = \frac{300.960}{s(s+18.7858)}}$

③ For a closed loop Control s/m $G(s) = \dfrac{100}{s(s+8)}$ & $H(s)=1$. Determine the resonent peak & resonent frq

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{100}{s(s+8)}}{1+\frac{100}{s(s+8)}\cdot 1} = \frac{100}{s^2+8s+100} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$\omega_n^2 = 100 \Rightarrow \omega_n = 10 \text{ rad/sec}$

$2\xi\omega_n = 8 \quad \therefore \xi = 0.4$

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} \Rightarrow \boxed{M_r = 1.3638}$$

$$\omega_r = \omega_n\sqrt{1-2\xi^2} \Rightarrow \boxed{\omega_r = 8.2462 \text{ rad/sec}}$$

④ (i) Find the Value of 'K' and 'a' so that $M_r = 1.04$ & $\omega_r = 11.55$ rad/sec. (ii) Also for the value of 'K' & 'a' found in (i) Calculate Settling time & B.W. for OLTF $G(s) = \dfrac{K}{s(s+a)}$

$H(s) = 1$

(i) $$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{K}{s(s+a)}}{1+\frac{K}{s(s+a)}\cdot 1} = \frac{K}{s^2+as+K} \doteq \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$\omega_n^2 = K \Rightarrow \omega_n = \sqrt{K}$

$2\xi\omega_n = a \Rightarrow \xi = \dfrac{a}{2\sqrt{K}}$

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} = 1.04 \Rightarrow \frac{1}{\xi\sqrt{1-\xi^2}} = 2.08$$

$\xi\sqrt{1-\xi^2} = 0.4807$ .... Square b.s.

$\xi^2(1-\xi^2) = 0.231$

$\xi^2 - \xi^4 = 0.231 \Rightarrow \xi^4 - \xi^2 + 0.231 = 0$

Put $\xi^2 = x$ $\quad x^2 - x + 0.23 = 0 \Rightarrow x = 0.637, 0.362 \;\downarrow\; \xi^2$

$\xi = \cancel{0.798}, 0.601$ ✓

* M_r Cannot exist for $\xi > 0.707$

$\omega_r = \omega_n\sqrt{1-2\xi^2} = 11.55 \Rightarrow \omega_n = 11.55$

$$\Rightarrow \boxed{K = 485.1861 \text{ \& } a = 26.5248}$$

(ii) $T_s = \dfrac{4}{\xi\omega_n} = 0.3016$ sec

$BW = \omega_n\sqrt{1-2\xi^2+\sqrt{2-4\xi^2+4\xi^4}} = 25.23$ rad/sec

Bode Plot :

Frq domain Analysis

$s \to jw$

i/P Sinwt → [LTI] → o/P A Sin(wt+φ)

T.F. G(S)

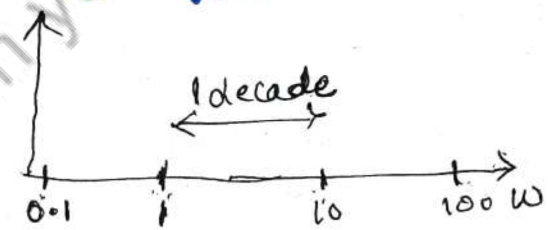
→ Provide the vizualization of Amplification & phase Shift which varies with 'w'.

$$|G(jw)| = \sqrt{Re[G(jw)]^2 + Im[G(jw)]^2} \quad \text{Magnitude}$$

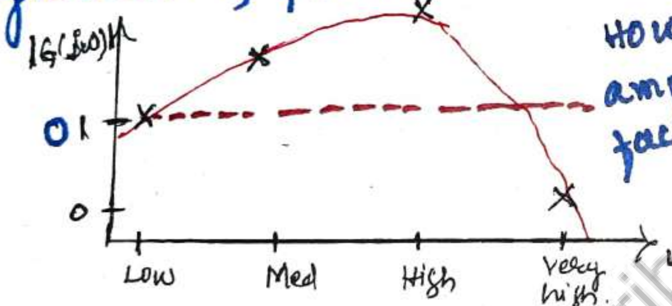
$$\angle G(jw) = \tan^{-1}\left[\frac{Im[G(jw)]}{Re[G(jw)]}\right] \quad \text{Phase}$$

$a+ib \quad M = \sqrt{a^2+b^2} \quad \phi = \tan^{-1}[b/a]$

→ x-axis → 'w' is drawn in semilog scale ($\log_{10}$)



Bode plot → graphical representation of Magnitude & phase



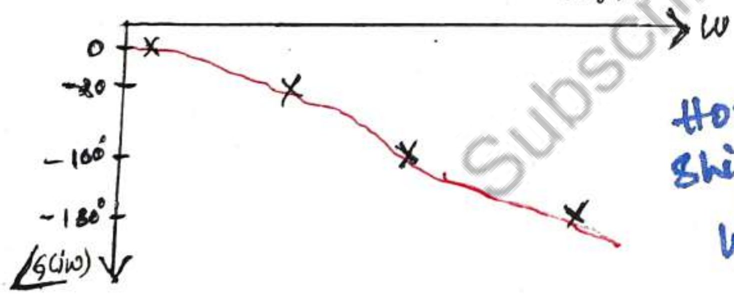
How the amplification factor varies with frq

→ y-axis → 'M' is expressed in units of dB. $\boxed{20 \log |G(jw)|}$

0 dB ⇒ no Amplification

dB > 0 ⇒ Amplification

dB < 0 ⇒ Attenuation

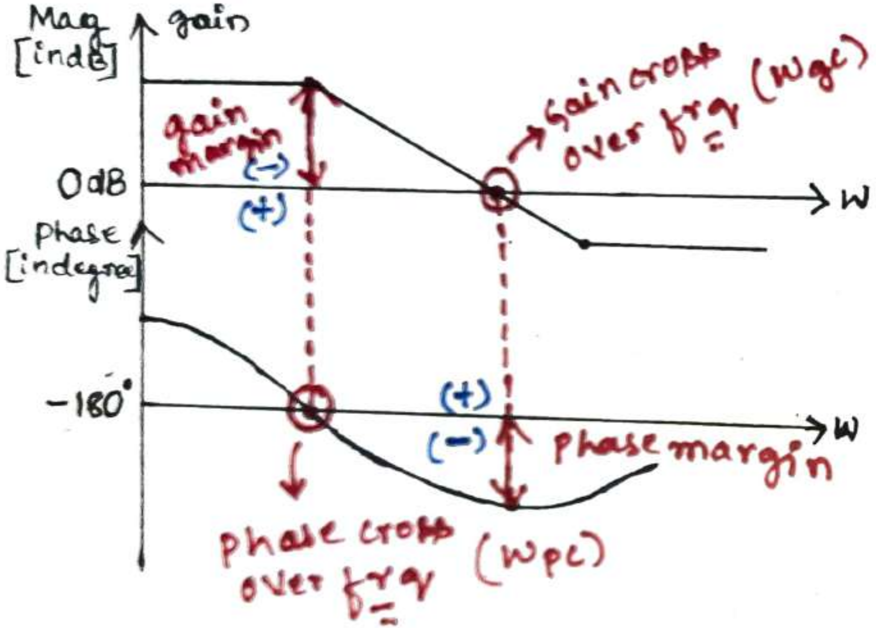


How does phase shift varies with frq

Procedure :

1. Replace $s \to jw$
2. Find M in dB
3. Find phase of s/m in degrees/rad
4. plot M & φ for increasing value of 'w' $w \to 0$ to ∞.

Gain Margin [GM] & Phase Margin [PM] in Bode Plot:



$\omega_{pc} > \omega_{gc} \rightarrow$ s/m is stable

$\omega_{pc} < \omega_{gc} \rightarrow$ s/m is unstable

$\omega_{pc} = \omega_{gc} \rightarrow$ s/m is marginally stable

Mathematically,

$$GM = \frac{1}{|G(j\omega)H(j\omega)|}\bigg|_{\omega=\omega_{pc}}$$

in dB,

$$\boxed{\begin{aligned} GM &= 20\log_{10}\frac{1}{|G(j\omega)H(j\omega)|}\bigg|_{\omega=\omega_{pc}} \\ GM &= -20\log_{10}|G(j\omega)H(j\omega)|\Big|_{\omega=\omega_{pc}} \end{aligned}}$$

$$PM = \left[\angle G(j\omega)H(j\omega)\Big|_{\omega=\omega_{gc}}\right] - [(-180°)]$$

$$\boxed{PM = 180° + \angle G(j\omega)H(j\omega)\Big|_{\omega=\omega_{gc}}}$$

$G(s) = \dfrac{80}{s(s+2)(s+20)}$. Draw the Bode plot. Determine GM, PM, ω_{gc} & ω_{pc}

Comment on the Stability.

$$G(s) = \frac{80^{2}}{s\times 2(1+s/2)\,20(1+\frac{s}{20})} = \frac{2}{s\,(1+\frac{s}{2})(1+\frac{s}{20})}$$

Simple pole Simple Pole

$K = 2$

$\frac{1}{s} \rightarrow$ 1 pole at origin. $\rightarrow$ -20 dB/dec

2 pole at origin $\rightarrow$ -40 dB/dec

3 poles —"— $\rightarrow$ -60 dB/dec

$s = +20$ dB

$\frac{1}{s} = -20$ dB

$s^2 = +40$ dB

$\frac{1}{s^2} = -40$ dB

$\dfrac{1}{1+\frac{s}{2}}$; $T_1 = \frac{1}{2}$ $\therefore \omega_{c_1} = \frac{1}{T_1} = 2$ rad/sec

$\dfrac{1}{(1+sT)}$ $\dfrac{1}{1+\frac{s}{20}}$ $T_2 = \frac{1}{20}$ $\therefore \omega_{c_2} = \frac{1}{T_2} = 20$ rad/sec.

Magnitude plot Analysis: $K = 2 \Rightarrow 20\log K \Rightarrow 20\log 2 \Rightarrow 6$ dB

Term	corner freq [Ascending order]	Slope	change in slope.
$\frac{1}{s}$	—	-20 dB/dec	-20 dB/dec.
$\frac{1}{1+s/2}$	2 rad/sec	-20 dB/dec	-40 dB/dec.
$\frac{1}{1+s/20}$	20 rad/sec	-20 dB/dec	-60 dB/dec.

Phase plot Analysis: $s \rightarrow j\omega$

$-ve \rightarrow$ pole

$+ve \rightarrow$ zeros

$\frac{1}{j\omega} \Rightarrow -\tan^{-1}(\frac{\omega}{0})$

$\tan^{-1}(\infty) \Rightarrow 90^\circ$

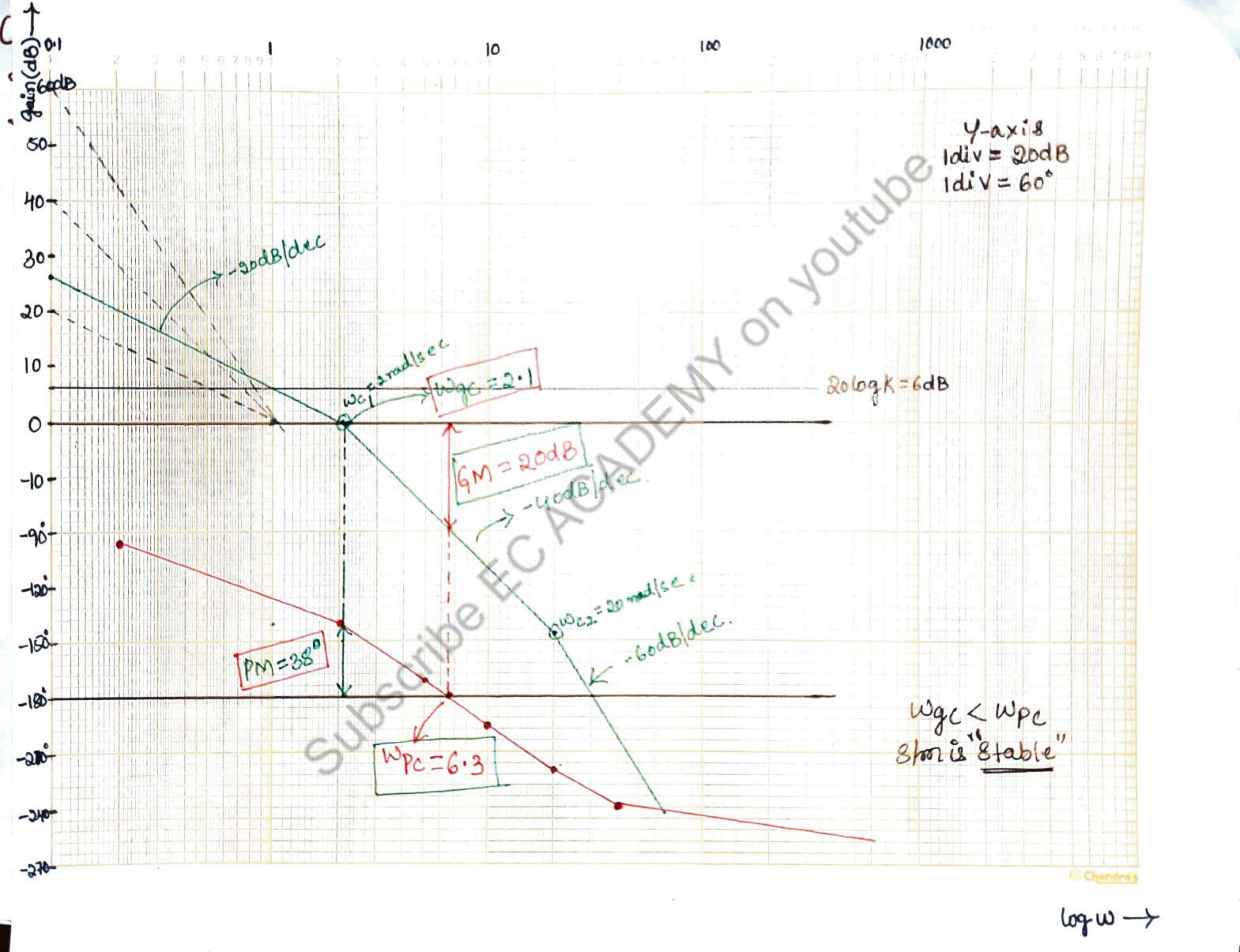
-90°

$\dfrac{1}{1+\frac{j\omega}{2}} \Rightarrow -\tan^{-1}\left(\dfrac{\frac{\omega}{2}}{1}\right) = -\tan^{-1}\left(\frac{\omega}{2}\right)$

$\dfrac{1}{1+j\frac{\omega}{20}} \Rightarrow -\tan^{-1}\left(\dfrac{\frac{\omega}{20}}{1}\right) = -\tan^{-1}\left(\frac{\omega}{20}\right)$

0.2 2 5 10 20 40 ∞

ω (rad/s)	-90°	$-\tan^{-1}(\frac{\omega}{2})$	$-\tan^{-1}(\frac{\omega}{20})$	ϕ_R
0.2	-90°	-5.71°	-0.57°	-96.28°
2	-90°	-45°	-5.71°	-140.71°
5	-90°	-68.19°	-14.03°	-172.22°
10	-90°	-78.69°	-26.56°	-195.29°
20	-90°	-84.28	-45°	-219.28°
40	-90°	-87.13	-63.43°	-240.56°
∞	-90°	-90	-90	-270°



gain (dB)
0.1
1
10
100
1000
60dB
50
40
30
20
10
0
-10
-90°
-120°
-150°
-180°
-210°
-240°
-270°
Y-axis
1div = 20dB
1div = 60°
-20dB/dec
ω_{c1} = 2 rad/sec
ω_{gc} = 2.1
20 log K = 6dB
GM = 20dB
-40dB/dec
ω_{c2} = 20 rad/sec
-60dB/dec
PM = 38°
ω_{pc} = 6.3
$\omega_{gc} < \omega_{pc}$
Sm is "Stable"
log ω
Subscribe EC ACADEMY on youtube

Construct bode plot for a unity feedback System with $G(s) = \frac{10(s+10)}{s(s+2)(s+5)}$ & find gain Margin, phase margin. Also comment on Stability.

Magnitude: $\underline{20\log K} \Rightarrow 20\log 10 = \underline{20\,dB/dec}$

$G(s) = \frac{10\,[redacted]\,(s+10)}{s(s+2)(s+5)} = \frac{10\times10(1+s/10)}{s\times2(1+s/2)\,5(1+s/5)}$

$G(s) = \frac{\overset{K}{10}\,\overbrace{(1+s/10)}^{zero}}{s\underbrace{(1+s/2)}_{pole}\underbrace{(1+s/5)}_{pole.}}$ $\boxed{K = 10}$

$\frac{1}{s} \rightarrow$ 1 pole at origin $\rightarrow -20\,dB/dec$

$\frac{1}{1+\frac{s}{2}}$; $T_1 = \frac{1}{2}$ $\therefore \omega_{c1} = \frac{1}{T_1} = 2$ rad/sec

$(1+sT)$ $\frac{1}{1+s/5}$; $T_2 = \frac{1}{5}$ $\omega_{c2} = \frac{1}{T_2} = 5$ rad/sec.

$(1+s/10)$; $T_3 = \frac{1}{10}$ $\therefore \omega_{c3} = \frac{1}{T_3} = 10$ rad/sec.

Terms	Corner freq [Ascending]	Slope	change in slope
$\frac{1}{s}$	—	$-20\,dB/dec$	$-20\,dB/dec$
$\frac{1}{1+\frac{s}{2}}$	$\omega_{c1} = 2$ rad/sec	$-20\,dB/dec$	$-40\,dB/dec$
$\frac{1}{1+\frac{s}{5}}$	$\omega_{c2} = 5$ rad/sec	$-20\,dB/dec$	$-60\,dB/dec$
$(1+s/10)$	$\omega_{c3} = 10$ rad/sec	$+20\,dB/dec$	$-40\,dB/dec$

Phase plot : $s \to j\omega$ $\quad -ve \to$ poles

$+ve \to$ zeros.

$$\frac{1}{j\omega} \Rightarrow -\tan^{-1}\left(\frac{\omega}{0}\right) = -90^\circ$$

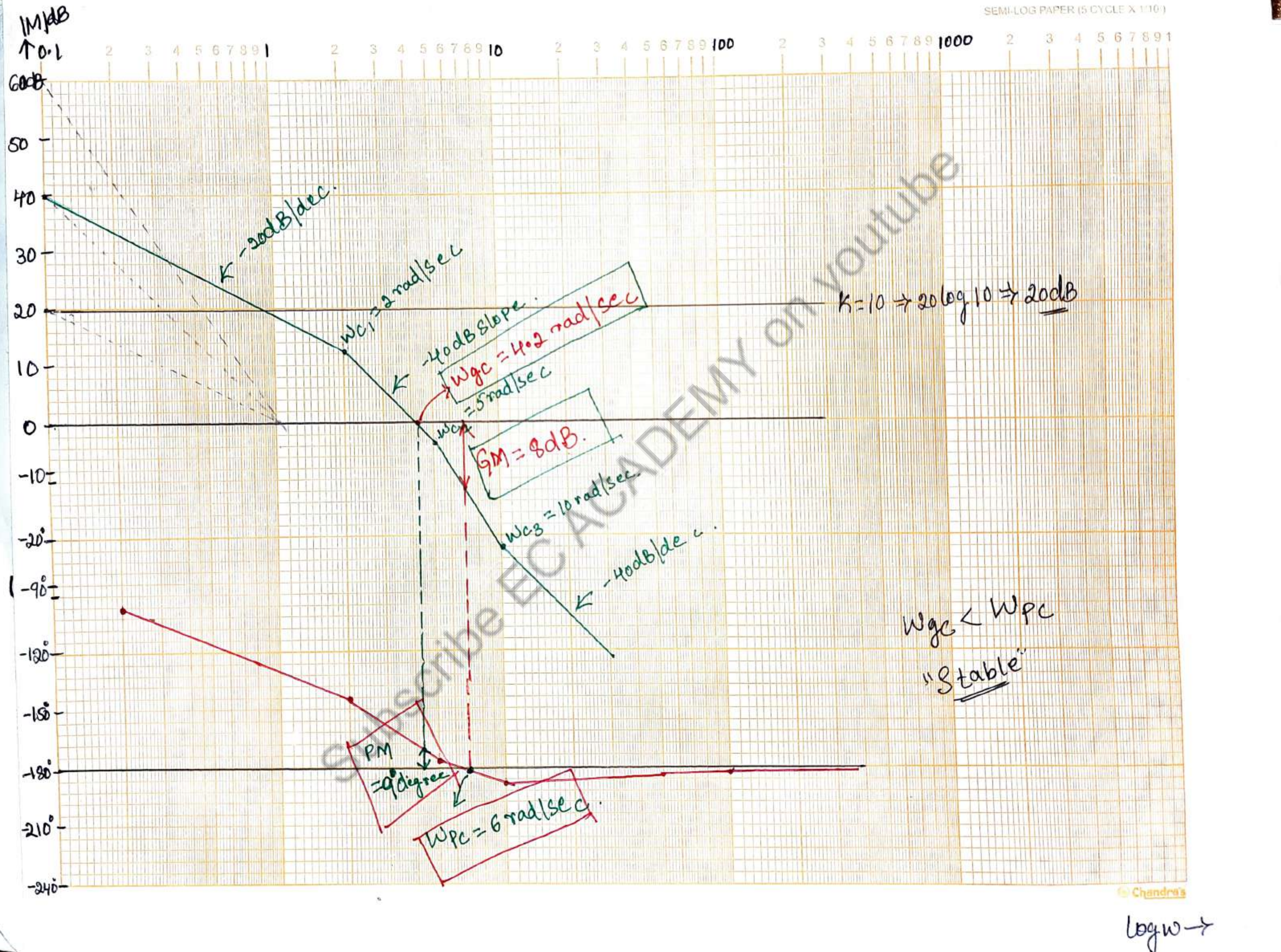
$$\frac{1}{1+\frac{j\omega}{2}} \Rightarrow -\tan^{-1}\left(\frac{\omega/2}{1}\right) = -\tan^{-1}\left(\frac{\omega}{2}\right)$$

$$\frac{1}{1+\frac{j\omega}{5}} \Rightarrow -\tan^{-1}\left(\frac{\omega/5}{1}\right) = -\tan^{-1}\left(\frac{\omega}{5}\right)$$

$$1+j\omega/10 \Rightarrow \tan^{-1}\left(\frac{\omega/10}{1}\right) = \tan^{-1}\left(\frac{\omega}{10}\right)$$

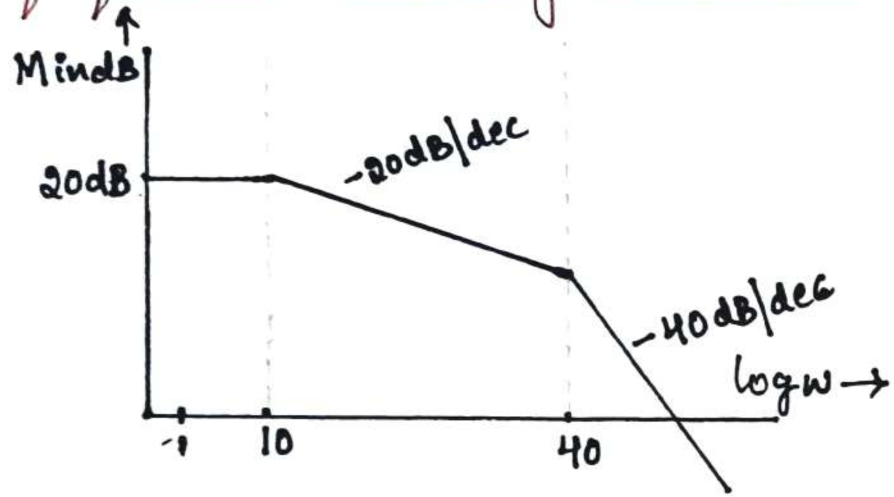
$0.2 \quad \boxed{2 \quad 5 \quad 10} \quad 50 \quad 100 \quad \infty$

ω	$-90^\circ - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{5}\right) + \tan^{-1}\left(\frac{\omega}{10}\right)$
0.2	-96.85°
2	-145.49°
5	-176.63°
10	-187.12°
50	-183.30°
100	-181.70°
∞	$-90^\circ - 90^\circ - \cancel{90^\circ} + \cancel{90^\circ} = -180^\circ$



|M|dB
0.1
1
10
100
1000
SEMI-LOG PAPER (5 CYCLE X 1/10)
60dB
50
40
30
20
10
0
-10
-20
-90°
-120°
-150°
-180°
-210°
-240°
-20dB/dec.
ωc1 = 2 rad/sec
-40dB slope.
ωgc = 4.2 rad/sec
ωc2 = 5 rad/sec
GM = 8dB.
ωc3 = 10 rad/sec.
-40dB/dec.
K=10 ⇒ 20log10 ⇒ 20dB
PM = 9° degree
ωpc = 6 rad/sec
ωgc < ωpc
"Stable"
Subscribe EC ACADEMY on youtube
Chandra's
log ω →

Transferfunction from Magnitude plot.



Find 'K'

$20 \log K = 20$

$\log K = 1$

$\therefore K = \text{anti}\log(1) = 10^1$

$\boxed{K = 10}$

(i) $w = 10$ rad/sec. → 0 to $-20dB$ $\quad \frac{1}{(1+S/T)}$

↳ pole $\quad 0 + [-20dB]$

$T = 1/10 \quad \therefore \left(\frac{1}{1+S/10}\right)$

* +20dB → zero present
-20dB → pole present
0dB → no pole & no zeros *

(ii) $w = 40$ rad/sec $\quad -20dB \to -40dB$

↓ $\quad -20dB + [-20dB]$

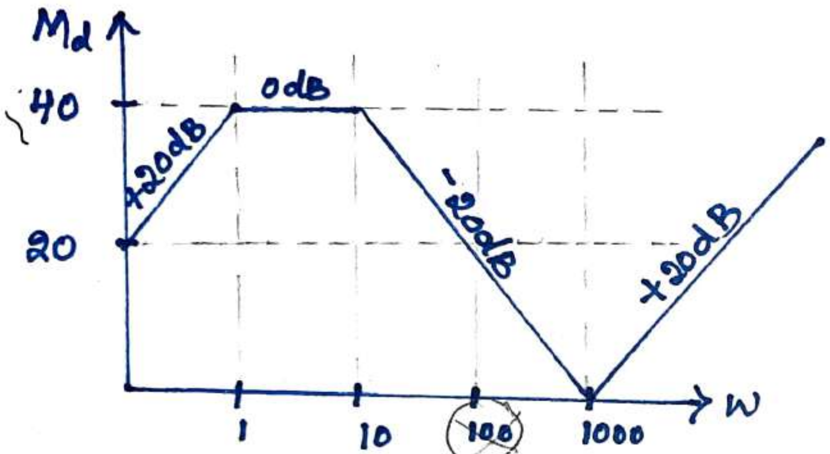
pole

$T = 1/40 \quad \therefore \left(\frac{1}{1+S/40}\right)$

$G(s)H(s) = \frac{K\ (\text{zeros})}{\text{Poles}}$

$$\boxed{G(S)H(S) = \frac{10}{\left(1+\frac{S}{10}\right)\left(1+S/40\right)}}$$

Find the T.F. of the s/m whose Bode diagram is given in fig.



→ Corner freq

Find K: $20\log K = 40 \Rightarrow \log K = 2$

$K = \text{antilog}(2) = 10^2 \Rightarrow \boxed{K=100}$

(i) $\omega = 1$ rad/sec +20dB → 0dB

↓ 20dB+[−20dB]

Pole

−20dB → pole
+20dB → zero
0dB → no pole & no zero

$T = \frac{1}{1} = 1$

$\therefore \left(\frac{1}{1+s}\right)$

(ii) $\omega = 10$ rad/sec 0dB → −20dB

↓ 0dB+[−20dB]

Pole

$T = \frac{1}{\omega} = \frac{1}{10}$ $\therefore \left(\frac{1}{1+s/10}\right)$

(iii) $\omega = 1000$ rad/sec −20dB → +20dB

↓ −20dB + [+40dB]

Zero of second order $(1+s/T)^2$

$T = \frac{1}{1000}$ $\therefore (1+s/1000)^2$

$G(s)H(s) = \frac{K\,(\text{zeros})}{(\text{poles})}$

$$\boxed{G(s)H(s) = \frac{100\,(1+s/1000)^2}{(1+s)(1+s/10)}}$$

Minimum and Non Minimum Phase System & All pass system

Minimum phase system all finate poles & Zeros are in the left side of S-plane.

$$G(s) = \frac{(s+2)}{(s+4)(s+6)}$$

$Z \to s = -2$

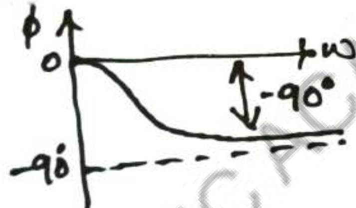
$P \to s = -4$

$s = -6$

$$\angle G(j\omega) = -\tan^{-1}(\omega/4) - \tan^{-1}(\omega/6) + \tan^{-1}(\omega/2)$$

$\omega = 0$ $\quad \angle G(j\omega) = 0°$

$\omega = \infty$ $\quad \angle G(j\omega) = -90°$



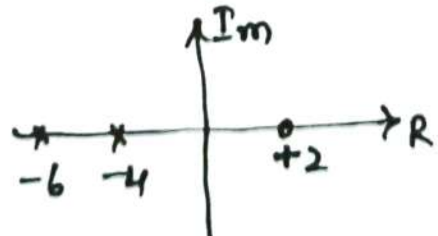
Non Minimum Phase System atleast one pole or one zero or both are present in right side of S-plane.

$$G(s) = \frac{(s-2)}{(s+4)(s+6)}$$

$Z \to s = +2$

$P \to s = -4$

$s = -6$

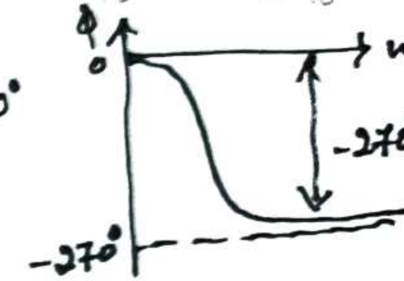


$$\angle G(j\omega) = -\tan^{-1}(\omega/4) - \tan^{-1}(\omega/6) - \tan^{-1}(\omega/2)$$

$-90°$ $\quad -90°$ $\quad -90°$

$\boxed{\omega = 0}$ $\quad \angle G(j\omega) = 0°$

$\boxed{\omega = \infty}$ $\quad \angle G(j\omega) = -270°$



All pass system [All pass filters]

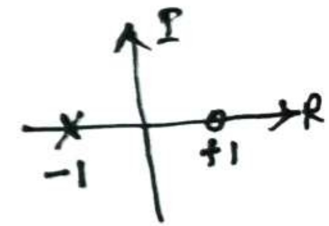
→ symmetrical pattern of poles & Zeros

Mirror image

$$G(s) = \frac{s-1}{s+1}$$

$Z \to s = +1$

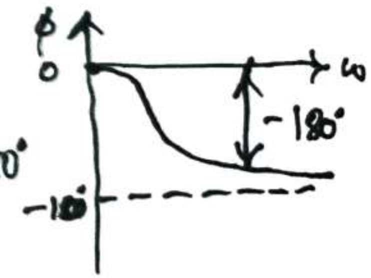
$P \to s = -1$



$$\angle G(j\omega) = -\tan^{-1}(\omega) - \tan^{-1}(\omega)$$

$\boxed{\omega = 0}$ $\quad \angle G(j\omega) = 0°$

$\boxed{\omega = \infty}$ $\quad \angle G(j\omega) = -180°$



Compensating Networks

→ physical device

→ electrical network, mechanical unit, hydrolic (or) combination.

Electrical n/w

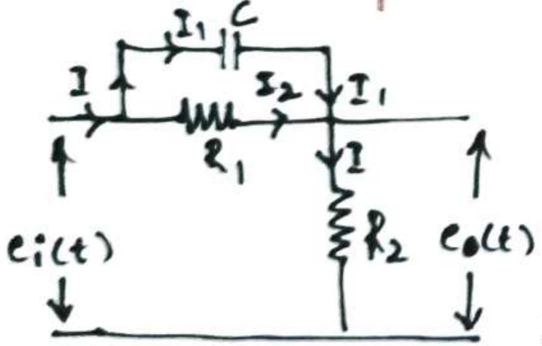
1. Lead n/w (or) Lead compensator
2. Lag n/w (or) Lag compensator
3. Lag-lead n/w (or) lag-lead compousator.

Sin i/p → [n/w] → Sin steady state o/p

Effect:

1. Increase damping of C.L. s/m.
2. Less overshoot, less rise time, less settling time → Improvement in transient response.
3. Improves phase margin of C.L. s/m
4. Impoves GM & PM → Improves relative stability.
5. Increases B.W. → Faster response
6. Steady State error doesnot get affected.

1. Lead Compensator:

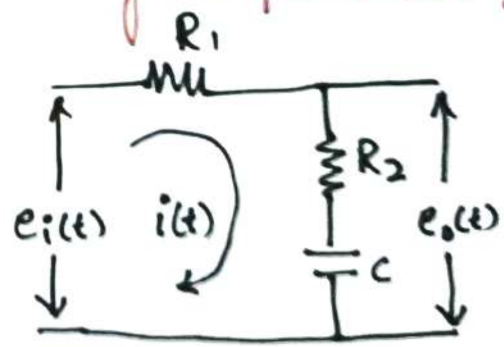


$$\boxed{\frac{E_o(s)}{E_i(s)} = \frac{\left[s+\frac{1}{T}\right]}{s+\frac{1}{\alpha T}}}$$

where: $T = R_1 C$ & $\alpha = \frac{R_2}{R_1+R_2} < 1$

$Z \rightarrow s = -\frac{1}{T}$ & $P \rightarrow s = -\frac{1}{\alpha T}$

2. Lag Compensator:



$$\boxed{\frac{E_o(s)}{E_i(s)} = \frac{1}{\beta} \cdot \frac{\left[s+\frac{1}{T}\right]}{\left[s+\frac{1}{\beta T}\right]}}$$

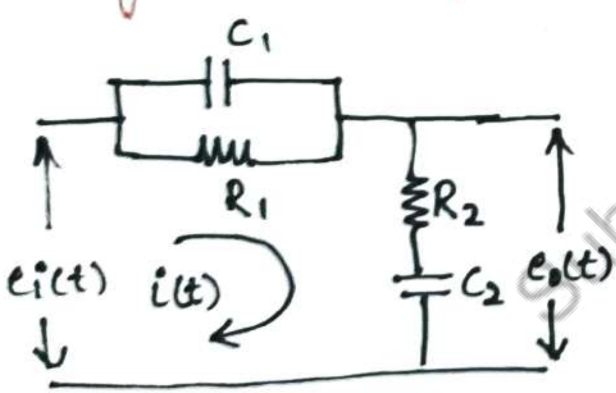
$T = R_2 C$ & $\beta = \frac{R_1+R_2}{R_2} > 1$

$Z \rightarrow s = -\frac{1}{T}$ & $P \rightarrow s = -\frac{1}{\beta T}$

Effect :

1. High gain at low frq → improves Performance
2. attenuation characteristics
3. Shifts gain Crossover frq to a lower frq Point → Reduces B.W.
4. more Rise time & Settling time → slow Response.
5. More Sensitive
6. S/m is less Stable.

3. Lag - lead Compensator:



$$\boxed{\frac{E_o(s)}{E_i(s)} = \frac{(1+T_1 s)(1+T_2 s)}{\left[1+\frac{T_1}{\beta}s\right](1+T_2\beta s)}}$$

$\beta > 1$

$T_1 = R_1C_1 \quad T_2 = R_2C_2$

$$\frac{\beta}{T_1} + \frac{1}{\beta T_2} = \frac{1}{R_1C_1} + \frac{1}{R_2C_2}$$

$\alpha\beta = 1$

T_1 → phase lead portion

T_2 → phase lag portion.

→ Provides Attenuation near & above the gain crossover frq

$P \rightarrow s = -\frac{\beta}{T_1}, -\frac{1}{\beta T_2}$

$Z \rightarrow s = -\frac{1}{T_1}, -\frac{1}{T_2}$

Effect :

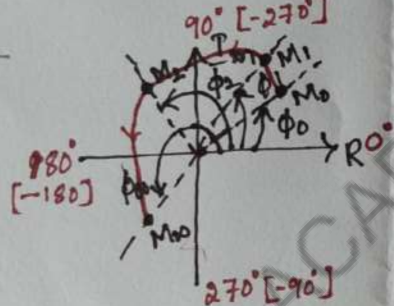
1. used for Fast response & good static accuracy
2. Increases low frq gain → improves the steady state
3. B W increases → s/m Fast.

Polar Plot:

→ graphical representation to find the stability of the s/m. in frq domain.

→ polar plot → $M = |G(j\omega)H(j\omega)|$ is plotted against the phase $\phi = \angle G(j\omega)H(j\omega)$ by varying i/p frq ω from 0 to ∞

ω	M	ϕ
0	M_0	ϕ_0
ω_1	M_1	ϕ_1
ω_2	M_2	ϕ_2
⋮	⋮	⋮
∞	M_∞	ϕ_∞



Polar plane

Rules:

1. Substitute $S = j\omega$ in open loop Transfer fun
2. write the expression for Magnitude and phase for $G(j\omega)H(j\omega)$
3. find Magnitude & phase by varying ω from 0 to ∞
4. Identify starting M & P at $\omega = 0$ → polar plot starts.
5. Identify ending M & P at $\omega = \infty$ → polar plot ends.
6. Sketch the ploar plot with the help of above information.

Polar Plot of Type 0, Type 1 & Type 2 s/ms

(i) Type 0 System:

$$G(s)H(s) = \frac{1}{1+Ts} \qquad s \to j\omega$$

$$a+jb$$

$$G(j\omega)H(j\omega) = \frac{1}{1+j\omega T} \qquad M = \sqrt{a^2+b^2}$$

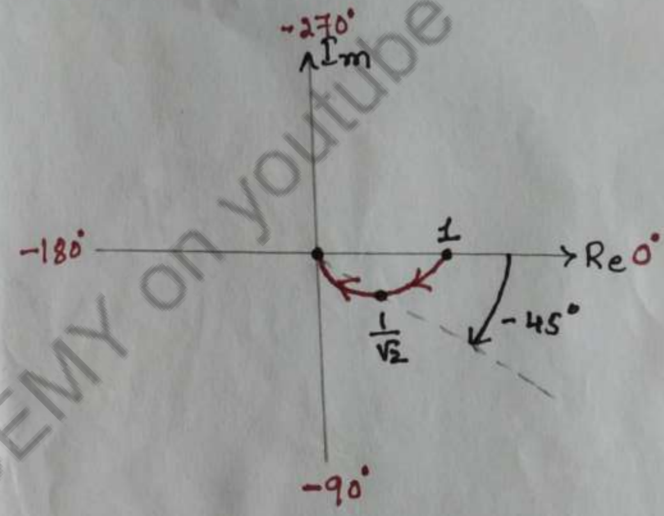
$$\phi = \tan^{-1}\left(\frac{b}{a}\right)$$

$$= \frac{1+j0}{1+j\omega T}$$

$$M = \frac{\sqrt{1^2+0^2}}{\sqrt{1^2+\omega^2T^2}} \Rightarrow \boxed{M = \frac{1}{\sqrt{1+\omega^2T^2}}}$$

$$\phi = 0 - \tan^{-1}\left(\frac{\omega T}{1}\right) \Rightarrow \boxed{\phi = -\tan^{-1}(\omega T)}$$

ω	M	ϕ
0	1	$0^\circ \to 1\angle 0^\circ$ Starting point
$\frac{1}{T}$	$\frac{1}{\sqrt{2}}$	-45°
∞	0	$-90^\circ \to 0\angle -90^\circ$ end point



$$-90^\circ - 0^\circ \Rightarrow -90^\circ$$

Clockwise 90°

(ii) Type 1 System:

$$G(s)\,H(s) = \frac{1}{s(1+Ts)} \qquad s \rightarrow j\omega$$

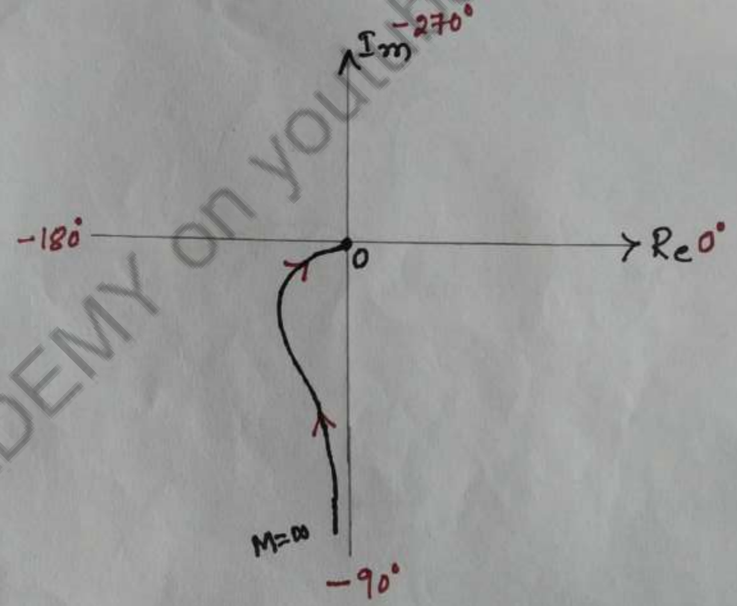
$$G(j\omega)\,H(j\omega) = \frac{1}{j\omega(1+j\omega T)}$$

$$M = \frac{\sqrt{1^2+0^2}}{\sqrt{0^2+\omega^2}\,\sqrt{1^2+\omega^2T^2}} \Rightarrow \boxed{M = \frac{1}{\omega\sqrt{1+\omega^2T^2}}}$$

$$\phi = 0 - 90^\circ - \tan^{-1}\left(\frac{\omega T}{1}\right)$$

$$\boxed{\phi = -90^\circ - \tan^{-1}(\omega T)}$$

w	M	ϕ
0	∞	-90° → starting
∞	0	-180° → end



$$-180^\circ - (-90^\circ) = -90^\circ$$

clockwise 90°

(iii) Type 2 system:

$$G(s)H(s) = \frac{1}{s^2(1+Ts)} \qquad s \to j\omega$$

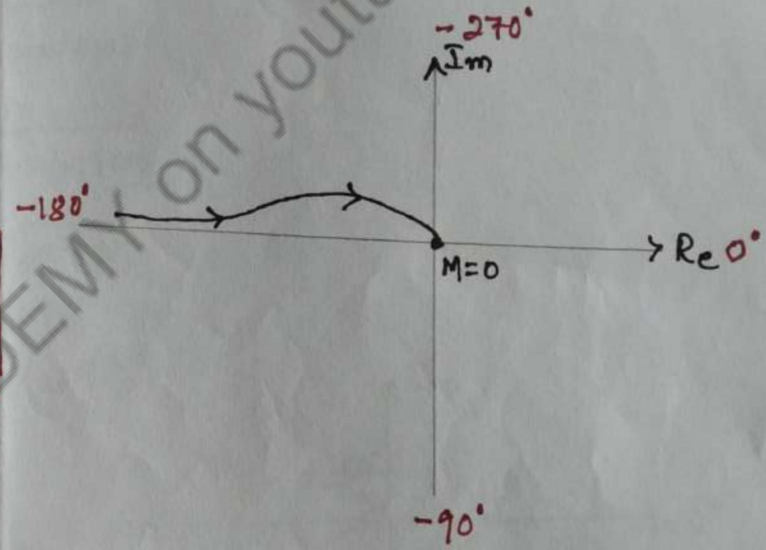
$$G(j\omega)H(j\omega) = \frac{1}{j\omega\, j\omega(1+j\omega T)}$$

$$M = \frac{\sqrt{1^2+0^2}}{\sqrt{0^2+(\omega^2)^2}\sqrt{1^2+\omega^2T^2}} = \boxed{\frac{1}{\sqrt{1+T^2\omega^2}\cdot \omega^2}}$$

$$\phi = 0 - 180^\circ - \tan^{-1}\left(\frac{\omega T}{1}\right)$$

$$\boxed{\phi = -180^\circ - \tan^{-1}(\omega T)}$$

ω	M	ϕ
0	∞	$-180^\circ \to$ start
∞	0	$-270^\circ \to$ end



$-270^\circ - (-180^\circ) = -90^\circ$

clockwise 90°

Draw Polar Plot of $G(s)H(s) = \dfrac{100}{s^2+10s+100}$

$s \to j\omega$

$$G(j\omega)H(j\omega) = \frac{100}{(100-\omega^2)+j10\omega}$$

$$\frac{100}{j^2\omega^2+10j\omega+100}$$

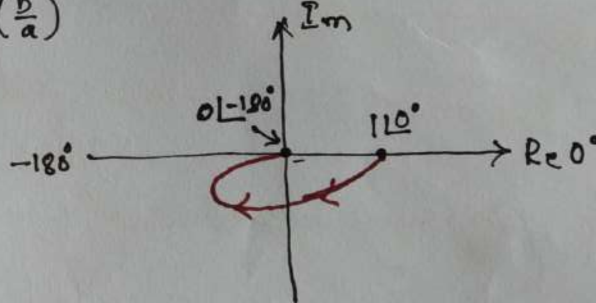
$$\frac{100}{-\omega^2+10j\omega+100}$$

$$M = \frac{\sqrt{100^2-0}}{\sqrt{(100-\omega^2)^2+(10\omega)^2}}$$

$$\left\{\begin{array}{l} M=\sqrt{a^2+b^2} \\ a+jb \\ \phi=\tan^{-1}\left(\frac{b}{a}\right) \end{array}\right.$$

$$\phi = 0 - \tan^{-1}\left(\frac{10\omega}{100-\omega^2}\right)$$

$\omega \to 0$	$M=1$	$\phi=0^\circ$
$\omega \to \infty$	$M=0$	$\phi=-180^\circ$

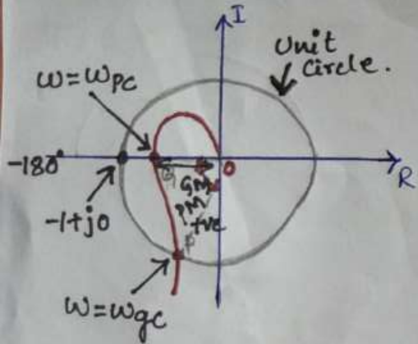


$\therefore$ Rotation of plot $= -180^\circ - 0^\circ$

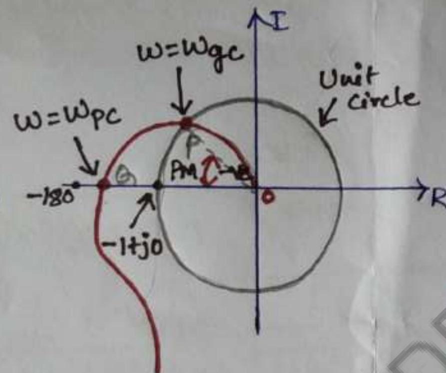
$= -180^\circ$

Clockwise 180°

ω_{gc} & ω_{pc} in Polar plot



$\omega_{pc} > \omega_{gc}$



$\omega_{pc} < \omega_{gc}$

$\omega_{pc} > \omega_{gc} \rightarrow$ Stable

$\omega_{pc} < \omega_{gc} \rightarrow$ Unstable

$\omega_{pc} = \omega_{gc} \rightarrow$ Marginally stable

GM & PM :

$$\boxed{GM = \frac{1}{d(OQ)}}$$

dB

$$\boxed{[GM]_{dB} = 20 \log_{10}\left[\frac{1}{d(OQ)}\right] dB}$$

$$\boxed{PM = 180^\circ + \angle G(j\omega)H(j\omega)\Big|_{\omega=\omega_{gc}}}$$

Stable s/m $\rightarrow$ GM & PM $\rightarrow$ +ve

Unstable s/m $\rightarrow$ GM & PM $\rightarrow$ -ve

Marginally stable s/m $\rightarrow$ GM & PM = 0

$G(s)H(s) = \frac{1}{s(s+1)(s+1/2)}$. Sketch the polar plot and find (i) Phase crossover frq (ii) Gain crossover frq (iii) GM & (iv) PM.

$s \to j\omega$

$G(j\omega)H(j\omega) = \frac{1}{j\omega(1+j\omega)(0.5+j\omega)}$ $a+ib$ $\sqrt{a^2+b^2}$

$M = \frac{\sqrt{1^2+0^2}}{\sqrt{0^2+\omega^2}\sqrt{1^2+\omega^2}\sqrt{0.5^2+\omega^2}} \Rightarrow \boxed{M = \frac{1}{\omega\sqrt{1+\omega^2}\sqrt{0.25+\omega^2}}}$

$\phi = 0 - 90^\circ - \tan^{-1}\left(\frac{\omega}{1}\right) - \tan^{-1}\left(\frac{\omega}{0.5}\right)$

$\boxed{\phi = -90^\circ - \tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{0.5}\right)}$ $\omega \to 0$ to ∞

ω	M	ϕ
0	∞	-90° → start
0.5	2.5298	-161.5°
1	0.6324	-198.43°
2	0.1084	-229.39°
∞	0	-270° → end

$-270^\circ - (-90^\circ)$

-180°

clockwise 180°

Phase crossover frq: ω_{pc}

× & ÷ by complex conjugate of dr.

$$\frac{-j\omega(1-j\omega)(0.5-j\omega)}{j\omega(-j\omega)(1+j\omega)(1-j\omega)(0.5+j\omega)(0.5-j\omega)}$$

$(a+jb)(a-jb) = a^2+b^2$

$$\frac{(-j\omega + j^2\omega^2)(0.5-j\omega)}{-j^2\omega^2(1+\omega^2)(0.5^2+\omega^2)}$$

$j^2 = -1$

$$\frac{(-j\omega-\omega^2)(0.5-j\omega)}{(\omega^2)(1+\omega^2)(0.25+\omega^2)}$$

$$\frac{-0.5j\omega + [j^2\omega^2 - 0.5\omega^2] + j\omega^3}{(\omega^2)(1+\omega^2)(0.25+\omega^2)}$$

$$\frac{\omega^2[-1.5] - j\omega[0.5-\omega^2]}{(\omega^2)(1+\omega^2)(0.25+\omega^2)}$$

$$\frac{\cancel{w^2}[-1.5]}{(\cancel{w^2})(1+w^2)(0.25+w^2)} - \frac{jw(0.5-w^2)}{(w^2)(1+w^2)(0.25+w^2)}$$

$$= \frac{-1.5}{(1+w^2)(0.25+w^2)} - \frac{jw(0.5-w^2)}{(w^2)(1+w^2)(0.25+w^2)}$$

equate imaginary part to zero

$$0.5-w^2=0 \Rightarrow w^2=0.5$$

$$\therefore \boxed{w = 0.707}$$

$w_{pc} = 0.707$ rad/sec

Gain crossover frq: w_{gc} $\quad M=1$

$$\frac{1}{w\sqrt{1+w^2}\sqrt{0.25+w^2}} = 1$$

$$w\sqrt{1+w^2}\sqrt{0.25+w^2} = 1$$

Square on b.s.

$$w^2(1+w^2)(0.25+w^2) = 1$$

$$(w^2+w^4)(0.25+w^2) = 1$$

$$0.25w^2 + \underline{w^4 + 0.25w^4} + w^6 = 1$$

$$\Rightarrow w^6 + 1.25w^4 + 0.25w^2 = 1$$

Put $w^2 = x$

$$x^3 + 1.25x^2 + 0.25x - 1 = 0$$

$$x_1 = 0.6609 \qquad x_2 = x_3 = -0.9554$$

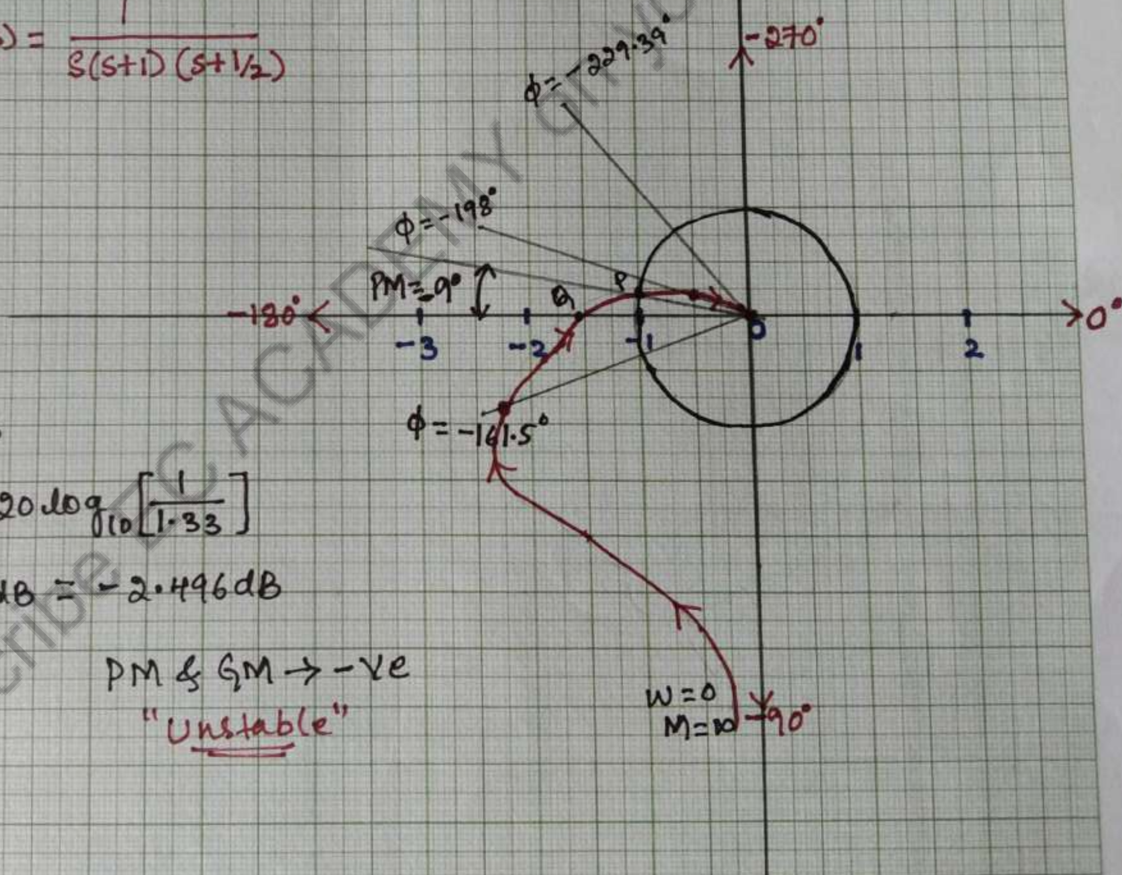
$$w^2 = 0.6609 \Rightarrow \boxed{w = 0.812}$$

$w_{gc} = 0.812$ rad/sec.

$$G(s)\,H(s) = \frac{1}{s(s+1)(s+1/2)}$$

Scale :

2div = 1unit



$OQ = 1.33$

$$[GM]_{dB} = 20\log_{10}\left[\frac{1}{1.33}\right]$$

$$GM_{dB} = -2.496\,dB$$

PM & GM → −ve

"Unstable"

Nyquist Plot Analysis:

Following steps to understand Nyquist Plot.

1] Pole - Zero Configuration

2] Encirclement & Counting the number of encirclement.

3] Analytic function and its singularities

4] Mapping theorem (or) Principle of argument

5] Nyquist Stability criterion.

1] Pole - Zero Configuration:

$G(s)H(s) \rightarrow$ Open loop transfer fun

poles $\rightarrow$ open loop poles

Zeros $\rightarrow$ open loop Zeros

Consider $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$

Poles $\rightarrow$ roots of the ch. eqn

$1+G(s)H(s)=0 \rightarrow$ Closed loop Poles of the s/m

Ex:- $G(s)H(s) = \frac{10}{s(s+2)}$

then open loop poles are $s=0,-2$

open loop zeros are absent

$1+G(s)H(s)=0 \Rightarrow 1+\frac{10}{s(s+2)}=0 \Rightarrow 1+\frac{10}{s^2+2s}=0$

$\frac{s^2+2s+10}{s^2+2s}=0 \Rightarrow \boxed{s^2+2s+10=0}$ $\underbrace{s=-1,-1}_{\text{Closed loop Poles}}$

Let, $F(s) = 1+G(s)H(s) = \frac{P(s)}{Q(s)}$

Roots of $P(s)=0 \rightarrow$ zeros of $1+G(s)H(s)$

Roots of $Q(s)=0 \rightarrow$ poles of $1+G(s)H(s)$

LCM of $1+G(s)H(s)$ will be denominator of $G(s)H(s)$

So, $Q(s)=0$ gives the roots $\rightarrow$ open loop poles

$\boxed{\text{Poles of } 1+G(s)H(s) = \text{open loop poles}}$

$P(s)=0 \rightarrow$ zeros of $1+G(s)H(s)$.

$\hookrightarrow$ gives the eqn $\Rightarrow$ Ch. eqn of s/m

$\boxed{\text{Zeros of } 1+G(s)H(s) = \text{Closed loop poles}}$

Ex:- $G(s)H(s) = \dfrac{10}{s(s+2)}$

$$F(s) = 1 + G(s)H(s) \Rightarrow 1 + \frac{10}{s(s+2)} = \frac{s^2+2s+10}{s(s+2)} = \frac{P(s)}{Q(s)}$$

$\therefore$ $Q(s) = 0 \rightarrow$ Poles of $1+G(s)H(s)$

i.e., $s = 0, -2 \Rightarrow$ open loop poles

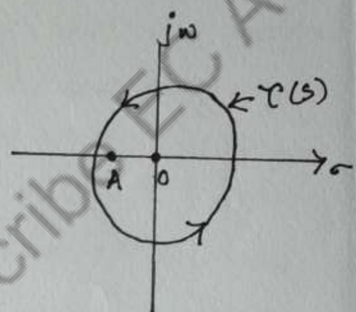
$P(s) = 0 \rightarrow$ Zeros of $1+G(s)H(s)$

i.e., $s = -1, -1 \Rightarrow$ closed loop poles

The s/m is absolutely stable if all zeros of $1+G(s)H(s)$ i.e., closed loop poles of the s/m are located in left half of s-plane.

2] Encirclement:

A point is said to be encircled by a closed path if it is found to lie inside that closed path.



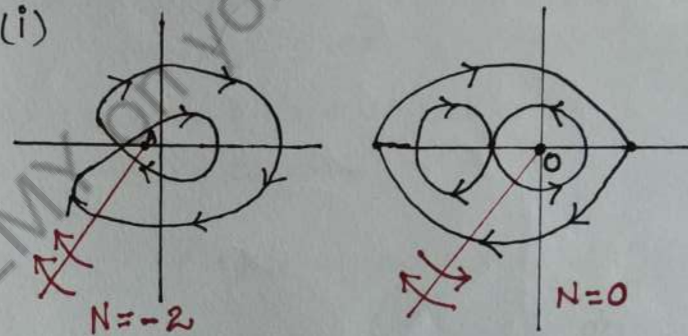
The points O, A $\rightarrow$ encircled by the colsed path.

Counting Number of encirclements:

$\rightarrow$ no of encirclement of point O & A is one in Anticlockwise direction.

For complicated cases.

(i)



3] Analytic Fun and Singularities:

A mathematical fun is said to be analytic at a point in a plane if its value & its derivative has finate existance at that point.

$$F(s) = \frac{25}{s(s+1)}$$

F(s) is analytic at all points in s-plane except $s=0$ & $s=-1$ $\because$ $F(s) = \infty$

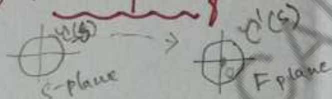
Poles of the $f^{\underline{n}}$ ~~are~~ it's singularities, if it is having only one value of S

$\underline{Ex}$:- $F(S) = \sqrt{S}$ & $S = 9$

$\therefore S = +3$ & $-3 \rightarrow$ not single valued.

We will assume the transfer $f^{\underline{n}}$ of s/m are single valued.

4] Mapping theorem (or) Principle of Argument



Mapping Theorem satates that the mapped locus $\gamma'(s)$ encircles the new origin of F-plane as many times as the difference between the number of zeros & poles of F-plane which are encircled by $\gamma(s)$ path in S-plane.

$$\boxed{N = Z - P}$$

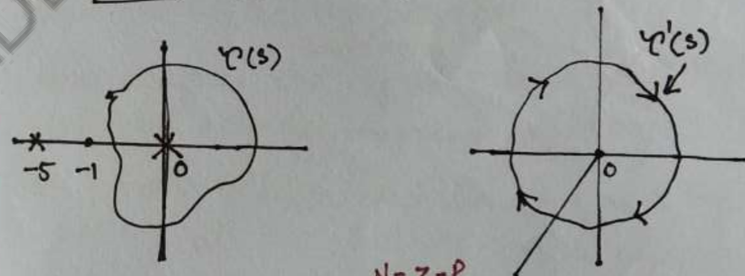
$N \rightarrow$ Encirclements of origin of F-plane.

$P \rightarrow$ no of poles of $F(s)$ encircled by $\gamma(s)$ path in splane.

$Z \rightarrow$ no of zeros of $F(s)$ encircled by $\gamma(s)$ path in S-plane.

The statement is also called as Principle of Argument.

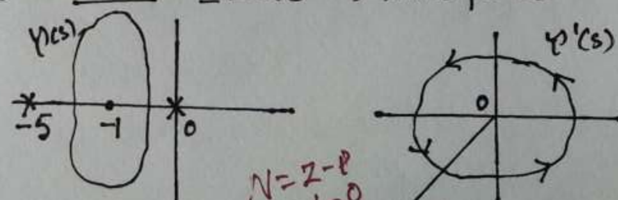
(i) $P > Z$: 1 pole & no zeros.



S-plane

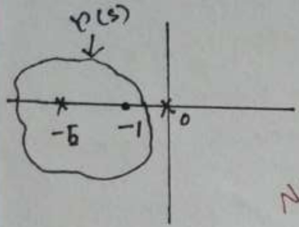
$N = Z - P$
$N = 0 - 1$
$N = -1$ F-plane

(ii) $P < Z$: 1 zero & zero poles

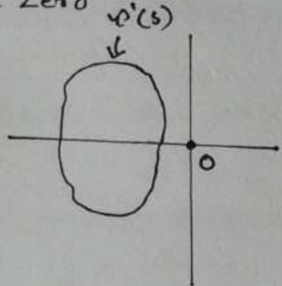


$N = Z - P$
$N = 1 - 0$
$N = 1$

iii) P=Z : 1 Pole & 1 Zero



$N = Z - P$
$1 - 1$
$N = 0$



6] Nyquist Stability Criterion :

Select a single valued fun F(s) as $1+G(s)H(s)$; $G(s)H(s) \rightarrow$ open loop T.F.

$\therefore F(s) = 1 + G(s)H(s)$

Now,

$\boxed{\text{Poles of } 1+G(s)H(s) = \text{Poles of } G(s)H(s) = \text{open loop poles}}$

$\boxed{\text{Zeros of } 1+G(s)H(s) = \text{Closed loop poles}}$

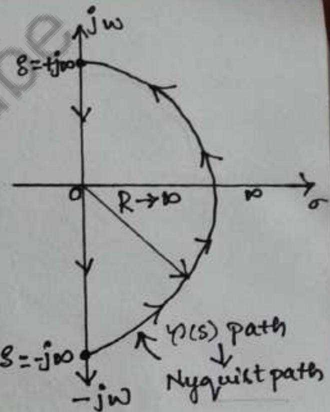
For stability, all Zeros of $1+G(s)H(s)$ must be in left half of S-plane.

location of Zeros $\Rightarrow$ "Unknown".

→ Nyquist Suggested,

→ rather analizing Presence of zeros in left half,

→ better to examine the presence of any one zero in right half.



→ We Know the poles of G(s)H(s), which are encircled by the Nyquist path.

→ Now map all the points of Nyquist path into F-plane with the help of mapping fun to get r'(s)

This mapping obtained in F-plane by mapping all the points on Nyquist Path is Called Nyquist plot.

→ We can determine the no of encirclements of Origin by Nyquist plot in F-plance (N)

→ $\therefore N = Z - P$

as N & P are Known $\Rightarrow$ We can find Z

Z = no. of zeros of $1+G(s)H(s)$ encircled by Nyquist path in s-plane.

→ but Nyquist path encircles only right half of s-plane.

∴ Z = no. of zeros of $1+G(s)H(s)$ which are located in right half of s-plane.

→ Stability → no zeros of $1+G(s)H(s)$ must be in right half of s-plane.
i.e., $Z = 0$ for stability.

$N = Z - P$

∴ $\boxed{N = -P}$ for Nyquist stability.

Nyquist Stability Criterion

For absolute stability of the s/m, the no. of encirclements of new origin of F-plane by Nyquist plot must be equal to no. of poles of $1+G(s)H(s)$ i.e. poles of $G(s)H(s)$ which are in right half of s-plane in clockwise direction.

Now for mapping Nyquist path from s-plane to F-plane, insted of considering mapping fun $1+G(s)H(s)$, it is considered as $G(s)H(s)$ only.

→ but stability criterion remains the same $\boxed{N = -P}$

→ but N changes as,

N = no. of encirclements of a critical point $-1+j0$ of F-plane by Nyquist plot.

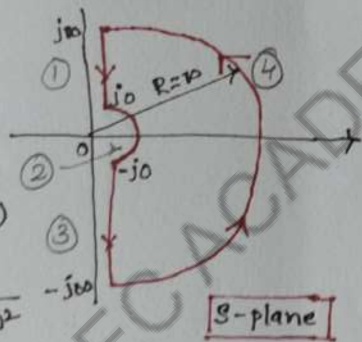
For a control s/m $G(s)H(s) = \frac{K}{s(s+2)(s+10)}$

Sketch the Nyquist plot. & hence calculate the range of 'K' for stability.

Step 1: $P = 0$

Step 2: $N = -P \Rightarrow \boxed{N=0}$ $\quad -1+j0$

Step 3: Nyquist path.



Step 4:

$$G(j\omega)H(j\omega) = \frac{K}{j\omega(2+j\omega)(10+j\omega)}$$

$$M = \frac{K}{\omega\sqrt{4+\omega^2} \times \sqrt{100+\omega^2}}$$

$$\phi = \frac{\tan^{-1}(0/K)}{\tan^{-1}\left(\frac{\omega}{0}\right)\tan^{-1}\left(\frac{\omega}{2}\right)\tan^{-1}\left(\frac{\omega}{10}\right)}$$

$$= -90° - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{10}\right)$$

① $s = +j\infty$ to $s = +j0$ $\quad s = j\omega$

$\omega \to \infty$ to 0

Starting point $\omega \to \infty$ $\quad 0\angle -270°$ $\quad \underline{+180°}$

end point $\omega \to 0$ $\quad 0\angle +90°$

② $s = +j0$ to $s = -j0$ $\quad \omega \to 0$ to -0

Starting point $\omega \to 0$ $\quad \infty\angle -90°$ $\quad +180°$

End point $\omega \to -0$ $\quad \infty\angle +90°$

③ Mirror image of section ①

④ is an origin.

Step 5:

$$G(j\omega)H(j\omega) = \frac{K(-j\omega)(10-j\omega)(2-j\omega)}{(j\omega)(-j\omega)(10+j\omega)(10-j\omega)(2+j\omega)(2-j\omega)}$$

$$= \frac{-Kj\omega[20-12j\omega-\omega^2]}{\omega^2(4+\omega^2)(100+\omega^2)} = \frac{-12K\omega^2}{D} - \frac{Kj\omega(20-\omega^2)}{D}$$

$$\omega(20-\omega^2) = 0 \Rightarrow \cancel{20\omega} - \cancel{20}$$

$$\omega^2 = 20 \quad \omega = \sqrt{20} \Rightarrow \omega_{pc}$$

For feedback control s/m $G(s)H(s) = \dfrac{40}{(s+4)(s^2+2s+2)}$

$$\frac{-12K\omega^2}{\omega^2(4+\omega^2)(100+\omega^2)} \qquad \omega = \sqrt{20}$$

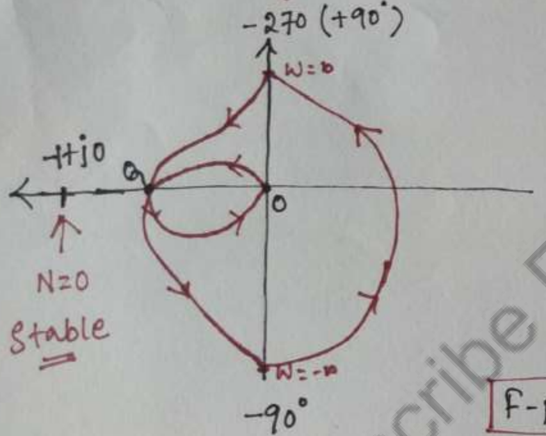
$$\text{Point } a = \frac{-12K \times 20}{20(20+4)(100+20)} \Rightarrow \frac{-K}{240}$$

For stability $N=0$ $|0a|<1$

$$\left|\frac{-K}{240}\right| < 1 \quad \therefore K < 240$$

$\therefore$ For stability $\boxed{0 < K < 240}$

Step 6: The Nyquist plot.



$\boxed{\text{F-plane}}$

P, PI, PD and PID Controllers

(a) P- Controller: [Proportional Controller]

→ Device → o/p signal → proportional i/p signal.
→ Improves → SSE, Stability.
→ Decreases → sensitivity → parameter variations.
→ Disadvantage → Constant SSE.

T.F. $\boxed{G_c(s) = K_p}$

(b) P-I Controller: [Proportional Integral]

→ Device → O/P signal → Proportional to i/p
└→ integral of i/p.
→ Increases → order of the S/m. by one
→ reduces → SSE
→ Disadvantage → less stable.

T.F. $\boxed{G_c(s) = K_p + \frac{K_i}{s}}$

(c) P-D Controller: [Proportional Derivative]

→ Device → o/p signal → Proportional i/p signal.
└→ Derivative of i/p.
→ Increases → Damping
→ reduction in peak overshoot

T.F. $\boxed{G_c(s) = K_p + K_d s}$

(d) P-I-D Controller: [Proportional Integral Derivative]

→ Device → o/p signal → Proportional to i/p.
└→ integral of i/p
└→ Derivative of i/p
→ Stabilizes gain
→ reduces SSE & Peak overshoot

T.F. $\boxed{G_c(s) = K_p + \frac{K_i}{s} + K_d s}$