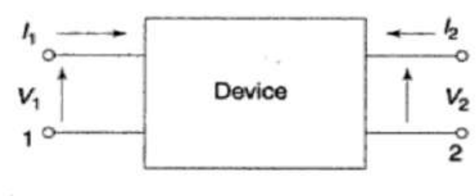
MODULE-2

MICROWAVE NETWORK THEORY AND PASSIVE DEVICES

INTRODUCTION

A microwave network is formed when several microwave devices and components such as sources, attenuators, resonators, filters, amplifiers, etc., are coupled together by transmission lines or waveguides for the desired transmission of a microwave signal. The point of interconnection of two or more devices is called a junction. For a low frequency network, a port is a pair of terminals whereas for a microwave network, a port is a reference plane transverse to the length of the microwave transmission line or waveguide. At low frequencies the physical length of the network is much smaller than the wavelength of the signal transmitted. Therefore, the measurable input and output variables are voltage and current which can be related in terms of the impedance Z- parameters, or admittance Y-parameters, or hybrid h-parameters, or ABCD parameters. For a two-port network as shown in Fig.2.1, these relationships are given by



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & -B \\ C & -D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

Fig 2.1 A two-port network

where Z_{ij}, Y_{ij} and A, B, C and D are suitable constants that characterize the junction. A, B, C and D parameters are convenient to represent each junction when a number of circuits are connected together, in cascade. Here the resultant matrix, which describes the complete cascade connection, can be obtained by multiplying the matrices describing each junction

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \cdots \begin{bmatrix} A_n & B_n \\ C_n & D_n \end{bmatrix}$$

The above parameters can be measured under short or open circuit condition for use in the analysis of the circuit.

Q1) Why conventional network parameters like Z, Y and ABCD are fail to analyse microwave network or device?

At microwave frequencies the physical length of the component or line is comparable to or much larger than the wavelength. Thus, the voltage and current are not well-defined at a given point for a microwave circuit, such as a waveguide system. Furthermore, measurement of Z, Y, h and ABCD parameters is difficult at microwave frequencies due to following reasons.

1. Non-availability of terminal voltage and current measuring equipment.
2. Short circuit and especially open circuit are not easily achieved for a wide range of frequencies.
3. Presence of active devices makes the circuit unstable for short or open circuit.
   Therefore, microwave circuits are analysed using scattering or S-parameters

Scattering or S-Parameters:

Scattering or S-parameters linearly relate the reflected(output) waves amplitude with those of incident waves. However, many of the circuit analysis techniques and circuit properties that are valid at low frequencies are also valid for microwave circuits. Thus, for circuit analysis S-parameters can be related to the Z or Y or ABCD parameters.

Scattering or S-Matrix Representation of Multiport Network

As discussed in the previous section incident and reflected amplitudes of microwaves at any port are used to characterize a microwave circuit. The amplitudes are normalised in such a way that the square of any of these variables gives the average power in that wave in the following manner

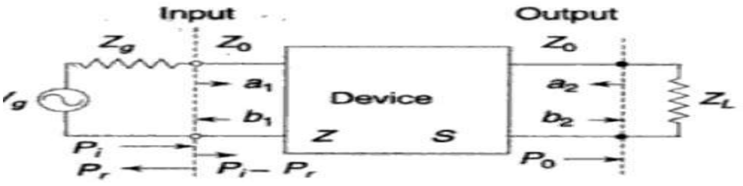


Fig 2.2 A two-port network

Input power at the nth port, $P_{in} = 1/2\ |a_n|^2$

Reflected power at the nth port, $P_{rn} = 1/2\ |b_n|^2$

where a_n and b_n represent the normalised incident wave amplitude and normalised reflected wave amplitude at the n^{th} port. The total or net power flow into any port is given by

$$P = P_i - P_r = 1/2\ (|a|^2 - |b|^2)$$

For a two-port network as shown in figure 2.2 the relation between incident and reflected waves are expressed in terms of scattering parameters S_{ij}'s

$$b_1 = S_{11}\, a_1 + S_{12}\, a_2$$
$$b_2 = S_{21}\, a_1 + S_{22}\, a_2$$

The physical significance of S-parameters can be described as follows

$S_{11} = b_1/a_1 \mid a_2 = 0$ = reflection coefficient Γ_1 at port 1 when port 2 in terminated with a matched load ($a_2 = 0$)

$S_{22} = b_2/a_2 \mid a_1 = 0$ = reflection coefficient Γ_2 at port 2 when port 1 in terminated with a matched load ($a_1 = 0$)

$S_{12} = b_1/a_2 \mid a_1 = 0$ = attenuation of wave travelling from port 2 to port 1

$S_{21} = b_2/a_1 \mid a_2 = 0$ = attenuation of wave travelling from port 1 to port 2.

In general, since the incident and reflected waves have both amplitude and phase, the S-parameters are complex numbers. For a multiport (N) networks or components, the S - parameters equations are expressed by

$$\begin{bmatrix} b_1 \\ b_2 \\ \\ \\ b_N \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & S_{22} & \cdots & S_{2N} \\ \vdots & & & \\ \vdots & & & \\ S_{N1} & S_{N2} & \cdots & S_{NN} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ \vdots \\ a_N \end{bmatrix}$$

Losses in microwave devices or circuits in terms of S-parameters

In microwave devices or circuits, it is important to express several losses in terms of S-parameters when the ports are matched terminated. In a two-port network if power fed at port l is P_i power reflected at the same port is P_r and the output power at port 2 is P_o then following losses are defined in terms of S-parameters

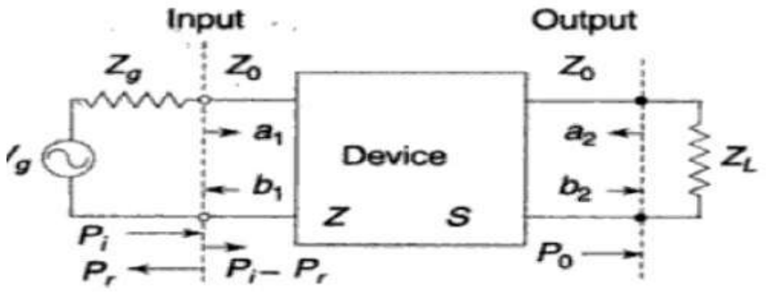


Fig 2.3 A two-port network

$$\text{Insertion loss (dB)} = 10 \log \frac{P_i}{P_o} = 10 \log \frac{|a_1|^2}{|b_2|^2}$$
$$= 20 \log \frac{1}{|S_{21}|}$$
$$= 20 \log \frac{1}{|S_{12}|}$$

$$\text{Transmission loss or attenuation (dB)} = 10 \log \frac{P_i - P_r}{P_o}$$
$$= 10 \log \frac{1 - |S_{11}|^2}{|S_{12}|^2}$$

$$\text{Reflection loss (dB)} = 10 \log \frac{P_i}{P_i - P_r}$$
$$= 10 \log \frac{1}{1 - |S_{11}|^2}$$

$$\text{Return loss (dB)} = 10 \log P_i / P_r$$
$$= 20 \log \frac{1}{|\Gamma|}$$
$$= 20 \log \frac{1}{|S_{11}|}$$

Properties of S-Parameters

In general, the scattering parameters are complex quantities having the following properties for different characteristics of the microwave network.

(a) Zero diagonal elements for perfect matched network

For an ideal N-port network with matched termination at all ports $S_{ii} = 0$, since there is no reflection from any port. Therefore, under perfect matched conditions the diagonal elements of [S] are zero.

(b) Symmetry of [S] for a reciprocal network

A reciprocal device has the same transmission characteristics in either direction of a pair of ports and is characterised by a symmetric scattering matrix, $S_{ij}=S_{ji}$ $(i \neq j)$
Which results in transpose of S-matrix $[S]_t=[S]$
Proof: This condition can be proved in the following manner. For a reciprocal network with the assumed normalisation, the impedance matrix equation is

$$[V] = [Z]\,[I] = [Z]\,([a] - [b]) = [a] + [b]$$

or, $$([Z] + [U])\,[b] = ([Z] - [U])\,[a]$$

or, $$[b] = ([Z] + [U])^{-1}\,([Z] - [U])\,[a]$$

where $[U]$ is the unit matrix. The S-matrix equation for the network is

$$[b] = [S]\,[a]$$

Comparing Eqs 6.27 and 6.28, we have

$$[S] = ([Z] + [U])^{-1}\,([Z] - [U])$$

Let $$[R] = [Z] - [U],\ [Q] = [Z] + [U]$$

For reciprocal network, the Z-matrix is symmetric. Hence

$$[R]\,[Q] = [Q]\,[R]$$

or, $$[Q]^{-1}\,[R]\,[Q]\,[Q]^{-1} = [Q]^{-1}\,[Q]\,[R]\,[Q]^{-1}$$

or, $$[Q]^{-1}\,[R] = S = [R]\,[Q]^{-1}$$

Now the transpose of $[S]$ is

$$[S]_t = ([Z] - [U])_t\,([Z] + [U])_t^{-1}$$

Since the Z-matrix is symmetrical

$$([Z] - [U])_t = [Z] - [U]$$
$$([Z] + [U])_t = [Z] + [U]$$

Therefore,

$$[S]_t = ([Z] - [U])\,([Z] + [U])^{-1}$$
$$= [R]\,[Q]^{-1} = [S]$$

Thus it is proved that $[S]_t = [S]$, for a symmetrical junction.

(c) Unitary property for a lossless junction

For any lossless network the sum of the products of each term of any one row or of any column of the S-matrix multiplied by its complex conjugate is unity.

Proof: For a lossless n-port device, the total power leaving N-ports must be equal to the total power input to these ports, so that

$$\sum_{n=1}^{N} |b_n|^2 = \sum_{n=1}^{N} |a_n|^2$$

or, $$\sum_{n=1}^{N} \left|\sum_{i=1}^{n} S_{ni}\, a_i\right|^2 = \sum_{n=1}^{N} |a_n|^2$$

If only i^{th} port is excited and all other ports are matched terminated, all $a_n = 0$, except a_i so that,

$$\sum_{n=1}^{N} |S_{ni}\, a_i|^2 = \sum_{n=1}^{N} |a_i|^2$$

$$\sum_{n=1}^{N} |S_{ni}|^2 = 1 = \sum_{n=1}^{N} S_{ni}\, S_{ni}^*$$

Therefore, for a lossless junction

$$\sum_{n=1}^{N} S_{ni} \cdot S_{ni}^* = 1$$

(d) Phase shift property

Complex S-parameters of a network are defined with respect to the positions of the port or reference planes. For a two-port network with unprimed reference planes I and 2 as shown in Fig. 2.4, the S-parameters have definite complex values

$$[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

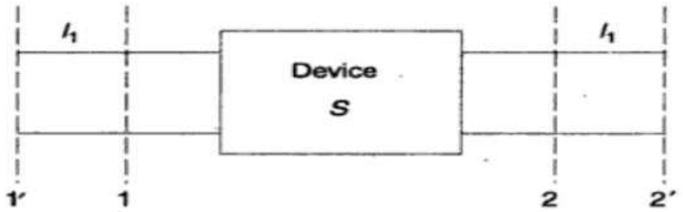


Fig.2.4 Phase Shift property of S

If the reference planes 1 and 2 are shifted outward to l'and 2' by electrical phase shifts $\phi_1 = \beta_1$ l and $\phi_2 = \beta_2$ l, respectively, then the new wave variables are $a_1 e^{j\phi_1}, b_1 e^{-j\phi_1}, a_2 e^{j\phi_2}, b_2 e^{-j\phi_2}$. The new S-matrix S' is then given by

$$[S'] = \begin{bmatrix} e^{-j\phi_1} & 0 \\ 0 & e^{-j\phi_2} \end{bmatrix} [S] \begin{bmatrix} e^{-j\phi_1} & 0 \\ 0 & e^{-j\phi_2} \end{bmatrix}$$

This property is valid for any number of ports and is called the phase shift property applicable to a shift of reference planes.

S-parameters of a Two-port Network with Mismatched Load

A two-port network or junction is formed when there is a discontinuity between the input and output ports of a transmission line. Many configurations of such junctions practically exist some of which are shown in Fig. 2.5.

During propagation of microwaves through the junction from one port to other, evanescent modes are excited at each discontinuity which contain reactive energy. Evanescent Modes decay very fast away from the junction and become negligible after a distance of the order of one wavelength. The terminal reference planes1 and 2 are chosen beyond this distance so that the equivalent voltage and currents at these positions are proportional to the total transverse electric and magnetic fields, respectively, for the propagating mode only. These circuits are analysed using S-matrix formulation

Consider a two-port network terminated by normalised load and generator

impedances $\frac{Z_L}{Z_0}$ and $\frac{Z_g}{Z_0} = 1$. Then the load reflection coefficient

$$\Gamma_2 = \frac{a_2}{b_2} = \frac{Z_L/Z_0 - 1}{Z_L/Z_0 + 1}$$

Now,

$$b_1 = S_{11} a_1 + S_{12} a_2 = S_{11} a_1 + S_{12} b_2 \Gamma_2$$
$$b_2 = S_{21} a_1 + S_{22} a_2 = S_{21} a_1 + S_{22} b_2 \Gamma_2$$

Solving for the input reflection coefficient

$$\Gamma_1 = b_1/a_1 = S_{11} + \frac{S_{12} S_{21} \Gamma_2}{1 - S_{22} \Gamma_2}$$

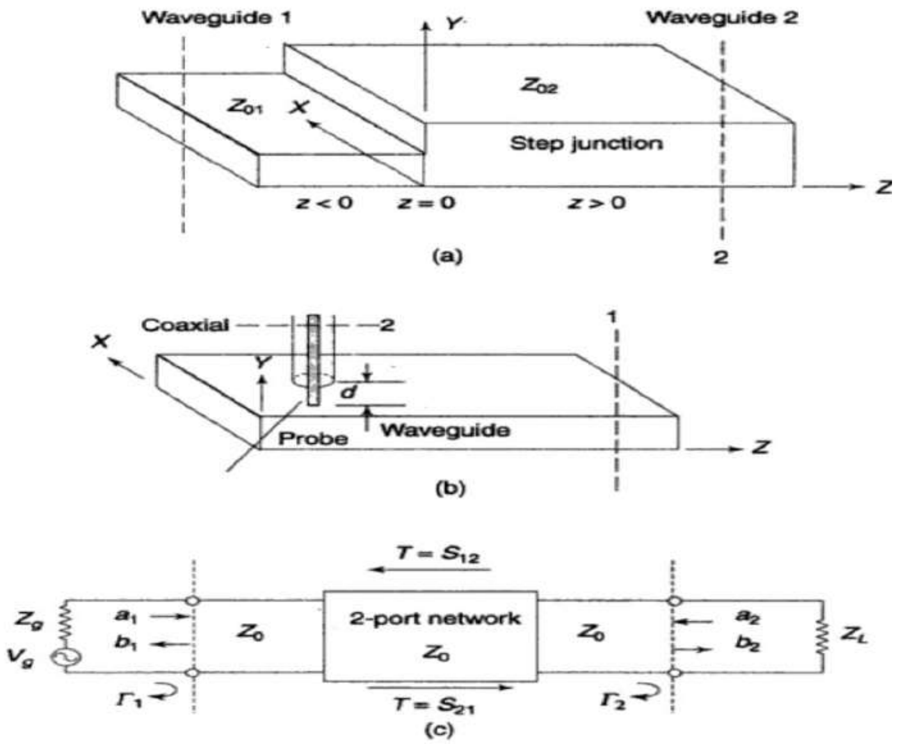


Fig 2.5 Two-port junctions (a) waveguide step junction (b) coaxial to waveguide transition (c) a two-port network

Therefore. for a mismatch load, input reflection coefficient $\Gamma_{11} \neq S_{11}$. For a reciprocal network, $S_{12} = S_{21}$, so that

$$\Gamma_1 = S_{11} + \frac{S_{12}^2 \Gamma_2}{1 - S_{22}\Gamma_2}$$

Further, if the junction is lossless, from unitary property

$$S_{11} S_{11}^* + S_{12} S_{12}^* = 1$$
$$S_{22} S_{22}^* + S_{12} S_{12}^* = 1$$
$$S_{11} S_{12}^* + S_{12} S_{22}^* = 0$$

Therefore, for a lossless, reciprocal two-port network, terminated by a mismatch load,

$$|S_{11}| = |S_{22}|$$
$$|S_{12}| = \sqrt{(1 - |S_{11}|^2)}$$

and the input reflection coefficient

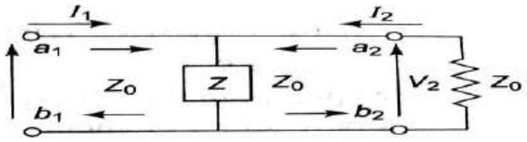
$$\Gamma_1 = S_{11} + \frac{S_{12}^2 \Gamma_2}{1 - S_{22}\Gamma_2}$$

The last equation is the working equation for the computation of the S-parameters. By measuring Γ_1 for known values of Γ_2 (0, -1 and 1) a set of simultaneous equations are obtained which will give the S-parameters of a reciprocal junction

Problem 1: A shunt Impedance Z is connected across a transmission line with characteristic impedance Z_0 Find the S-matrix of the junction.

Solution: For S-parameters, two ports are considered matched. Let the output line be match terminated. So that $a_2 = 0$. Therefore,

$$b_1 = S_{11}a_1, \quad b_2 = S_{12}a_1$$



$$S_{11} = \frac{Y_0 - Y_{in}}{Y_0 + Y_{in}} = \frac{Y_0 - (Y_0 + Y)}{Y_0 + (Y_0 + Y)}$$

$$= \frac{-Y}{2Y_0 + Y} = \frac{-1}{1 + 2Z/Z_0} = S_{22}$$

Now for the pure shunt element, the transmitted wave amplitude (for $a_2 = 0$) can be expressed by $b_2 = a_1 + b_1 = a_1 + S_{11}\,a_1 = a_1\,(1 + S_{11})$.

Therefore,

$$S_{21} = \frac{b_2}{a_1} = 1 + S_{11} = S_{12} = \frac{2Y_0}{2Y_0 + Y} = \frac{2Z/Z_0}{1 + 2Z/Z_0}$$

$$\therefore \quad [S] = \frac{1}{2Y_0 + Y}\begin{bmatrix} -1 & 2Y_0 \\ 2Y_0 & -1 \end{bmatrix} = \frac{1}{1 + 2Z/Z_0}\begin{bmatrix} -1 & 2Z/Z_0 \\ 2Z/Z_0 & -1 \end{bmatrix}$$

Alternative

$$V_1 = Z\,I_1 + Z\,I_2\,; \quad [Z] = \begin{bmatrix} Z & Z \\ Z & Z \end{bmatrix}$$

$$V_2 = ZI_1 + ZI_2;$$

$$[S] = \left(\frac{1}{Z_0}[Z] - [U]\right)\left(\frac{1}{Z_0}[Z] + [U]\right)^{-1}$$

$$= [\overline{Z} + U - 2][\overline{Z} + U]^{-1} = [U] - 2[\overline{Z} + U]^{-1}$$

$$= \frac{1}{1 + 2\overline{Z}}\begin{bmatrix} -1 & 2\overline{Z} \\ 2\overline{Z} & -1 \end{bmatrix}; \quad S_{11} - S_{12} = -1$$

Problem 2: Two transmission lines of characteristic impedance Z_1 an Z_2 are joined at plane pp'. Express S-parameters in terms of impedances when each line is matched terminated.

Solution

The incident and scattered wave amplitude are related by $[b] = [S]\,[a]$.

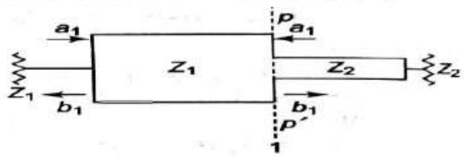
(i) Since the output line is matched ($a_2 = 0$), the input impedance Z_{in} at the junction $= Z_2 =$ load for line Z_1.

Therefore,

$$S_{11} = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \text{reflection coefficient on the input side.}$$

(ii) Similarly, for symmetry, assuming input side is matched ($a_1 = 0$),

$$S_{22} = \frac{Z_1 - Z_2}{Z_1 + Z_2} = -S_{11}$$



(iii) In general,

$$b_1 = S_{11}\,a_1 + S_{12}\,a_2$$

$$b_2 = S_{21}\,a_1 + S_{22}\,a_2$$

With output line matched ($a_2 = 0$) for lossless line, Z_{in} at the junction $= Z_2$, a pure shunt element.

Therefore, $b_1 = S_{11}\,a_1$. Due to different impedances of the two arms,

$$\sqrt{Z_2}\,b_2 = (a_1 + b_1)\sqrt{Z_1} = (a_1 + S_{11}\,a_1)\sqrt{Z_1} = a_1(1 + S_{11})\sqrt{Z_1}$$

or, $\quad S_{21} = b_2/a_1 = (1 + S_{11})\sqrt{Z_1/Z_2}$

or, $$S_{21} = \left(1 + \frac{Z_2 - Z_1}{Z_2 + Z_1}\right)\sqrt{\frac{Z_1}{Z_2}} = \frac{2\sqrt{Z_2 Z_1}}{Z_2 + Z_1}$$

(iv) With the input line matched ($a_1 = 0$), $b_2 = a_2 S_{22}$

$$\sqrt{Z_1}\,b_1 = (a_2 + b_2)\sqrt{Z_2} = (a_2 + S_{22}\,a_2)\sqrt{Z_2} = [a_2\,(1 + S_{22})\sqrt{Z_2}\,]$$

or, $\quad S_{12} = b_1/a_2 = (1 + S_{22})\sqrt{Z_2/Z_1}$

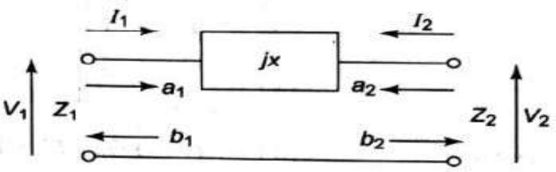
$$= \left(1 + \frac{Z_1 - Z_2}{Z_1 + Z_2}\right)\sqrt{\frac{Z_2}{Z_1}} = \frac{2\sqrt{Z_1 Z_2}}{Z_1 + Z_2}$$

Therefore, $$[S] = \begin{bmatrix} \dfrac{Z_2 - Z_1}{Z_2 + Z_1} & \dfrac{2\sqrt{Z_1 Z_2}}{Z_1 + Z_2} \\ \dfrac{2\sqrt{Z_1 Z_2}}{Z_1 + Z_2} & \dfrac{Z_1 - Z_2}{Z_1 + Z_2} \end{bmatrix}$$

Problem 3: A series reactance Z = jX is connected between two lines with different characteristic impedances $Z_1$and Z_2 Find the S-matrix of the junction.
Solution: The normalised voltages and power inputs at Port 1 and 2 are

$$a_1 = \frac{V_1^+}{\sqrt{Z_1}}, a_2 = \frac{V_2^+}{\sqrt{Z_2}}$$

$$P_1 = \frac{a_1^2}{2}, P_2 = \frac{a_2^2}{2}$$



S-matrix is determined assuming that each of the lines are terminated by its characteristic impedances.

Therefore, $$\frac{V_1^-}{V_1^+} = \frac{b_1}{a_1} = S_{11} = \frac{Z_{in} - Z_1}{Z_{in} + Z_1} = \frac{jX + Z_2 - Z_1}{jX + Z_2 + Z_1}$$

and $$\frac{V_2^-}{V_2^+} = \frac{b_2}{a_2} = S_{22} = \frac{jX + Z_1 - Z_2}{jX + Z_1 + Z_2}$$

Net input voltage for output line matched $(a_2 = 0)$

$$a_1 + b_1 = a_1 (1 + S_{11})$$

Input current $$I_1 = \frac{V_1^+}{Z_1} - \frac{V_1^-}{Z_1} = (1 - S_{11})\frac{V_1^+}{Z_1}$$

For the continuity of current in a lossless series element jX, with output port matched $(a_2 = 0)$,

$$I_2 = -I_1 = -\frac{V_1^+}{Z_1}(1 - S_{11}) = -I_2^-$$

Also $$I_2^- = \frac{V_2^-}{Z_2}$$

$$\therefore \quad \frac{V_1^+}{Z_1}(1 - S_{11}) = \frac{V_2^-}{Z_2}$$

Thus, for symmetry

$$S_{21} = S_{12} = \frac{b_2}{a_2} = \frac{V_2^-}{\sqrt{Z_2}} \Big/ \frac{V_1^+}{\sqrt{Z_1}} = \frac{V_2^-}{V_1^+}\sqrt{\frac{Z_1}{Z_2}}$$

$$= \left(\frac{Z_2}{Z_1}\right)(1 - S_{11})\sqrt{\frac{Z_1}{Z_2}} = \sqrt{\frac{Z_2}{Z_1}}(1 - S_{11})$$

or, $$S_{21} = S_{12} = \sqrt{\frac{Z_2}{Z_1}}\,\frac{2\sqrt{Z_1 Z_2}}{Z_1 + Z_2 + jX}$$

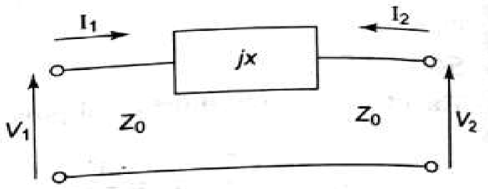
Problem 4: A series impedance Z=jX is inserted in a □ length of transmission line of characteristic impedance Z_0. Find S-parameters for the junction.

Solution

Let $$Y = \frac{1}{jX}$$

$$\therefore \quad I_1 = YV_1 - YV_2$$
$$I_2 = -YV_1 + YV_2$$

$$[Y] = \begin{bmatrix} Y & -Y \\ -Y & Y \end{bmatrix}$$



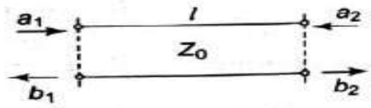
Now $$[S] = (U - Y/Y_0)(U + Y/Y_0)^{-1}$$

$$= (U - \bar{Y})(U + \bar{Y})^{-1} = \frac{1}{1 + 2\bar{Y}}\begin{bmatrix} 1 & 2\bar{Y} \\ 2\bar{Y} & 1 \end{bmatrix}; S_{11} + S_{12} = 1$$

Problem 5-: Find the S-matrix of a length l of a lossless transmission line terminated by matched impedance.

Solution: For a length l of a transmission line there is no discontinuity at the two ends, so that $S_{11} = S_{22} = 0$. The output signals arise due to input

$$b_1 = a_2\, e^{-j\beta l}$$
$$b_2 = a_1\, e^{-j\beta l}$$



$$\text{or,} \quad \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 0 & e^{-j\beta l} \\ e^{-j\beta l} & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = [S][a]$$

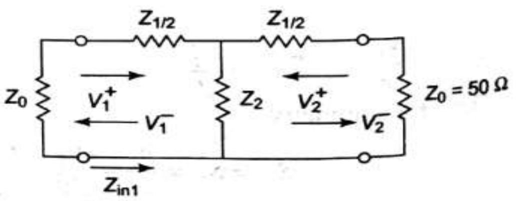
$$\therefore \quad [S] = \begin{bmatrix} 0 & e^{-j\beta l} \\ e^{-j\beta l} & 0 \end{bmatrix}$$

Problem 6: Determine the S-matrix of a 3dB T-network attenuator shown below terminated in a 50-ohm matched load with $Z_1 = 17.12$ ohms, $Z_2 = 141.78$ ohms**.**

Solution

$$\frac{Z_1}{2} = \frac{17.12}{2} = 8.56$$

$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+=0} = \frac{b_1}{a_1} = \frac{Z_{in1} - Z_0}{Z_{in1} + Z_0}$$



$$Z_{in1} = 17.12/2 + 141.78 \times (17.12/2 + 50)/(141.78 + 17.12/2 + 50)$$
$$= 50 \text{ ohms}$$
$$\therefore S_{11} = 0$$

For the symmetry, $S_{22} = S_{11} = 0$, $V_1^- = 0$, $V_2^+ = 0$

$$S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0}$$

Now $\quad V_1 = V_1^+ + V_1^- = V_1^+$

$$V_2 = V_2^+ + V_2^- = V_2^-$$

Again $\left(50 + \dfrac{17.12}{2}\right) \parallel 141.78 = \dfrac{141.78 \times 58.56}{141.78 + 58.56} = 41.44$

Therefore, $V_2^- = V_2 = V_1 \left(\dfrac{41.44}{41.44 + 8.56}\right)\left(\dfrac{50}{50 + 8.56}\right) = 0.707\, V_1$

$$\therefore \quad S_{21} = \frac{V_2^-}{V_1^+} = \frac{V_2^-}{V_1} = 0.707 \quad \text{or} \quad [S] = \begin{bmatrix} 0 & 0.707 \\ 0.707 & 0 \end{bmatrix}$$

MICROWAVE PASSIVE DEVICES

Microwave passive devices and components are designed using sections of coaxial line, waveguides, strip lines and microstrip lines for use in both, laboratory and in microwave communication and radar systems. These components can be considered as one-port or multiport networks characterised by the basic parameters, like the VSWR, reflection coefficient, and various losses, under output matched conditions. In the following sections, the basic operating principles for a number of most commonly used devices such as line sections, connectors, attenuators, phase shifters, T-junctions, isolators and circulators etc. are described.

Coaxial Connectors and Adapters

Coaxial cables are terminated or connected to other cables and components by means of shielded standard connectors. The outer shield makes a 360 degree extremely low impedance joint to maintain shielding integrity. These connectors are of various types depending on the frequency range and the cable diameter. Commonly used microwave connectors are type N (male/female), BNC (male/female), TNC (male/female), APC (sexless), etc. Adapters, having different connectors at the two ends, are also made for interconnection between two different ports in a microwave system. The basic schematic diagrams of these connectors and adapters are shown in Fig. 2.6 The type N (Navy) connector is 50-ohm and 75-ohms connector which was designed for military system applications during World War II. This is suitable for flexible or rigid cables in the frequency range of I-18 GHz. The BNC (Bayonet Navy Connector) is suitable for 0.25-inch 50 ohm or 75-ohm flexible cables used up to I GHz. The TNC (Threaded Navy Connector) is like BNC, except that, the outer conductor has thread to make firm contact in the mating surface to minimise radiation leakage at higher frequencies. These connectors are used up to 12 GHz.

The SMA (Sub-Miniature A) connectors are used for thin flexible or semirigid cables. The higher frequency is limited to 24 GHz because of generation of higher order modes beyond this limit. All the above connectors can be of male or female configurations except the APC-7(Amphenol Precision Connector-7 mm) which provides coupling without male or female configurations. The APC-7 is a very accurate 50-ohm, low VSWR connector which can operate up to 18 GHz. Another APC-3.5 connector is a high precision 50-ohm, low VSWR connector which can be either the male or female and can operate up to 34 GHz. It can mate with the oppositely sexed SM connector. Table 2.1 shows the type, dielectric in mating space and impedance of some of the above standard connectors.

Table 2.1 Coaxial Connectors

Type	Sex	Dielectric in mating space	Impedance (ohm)
N	M/F	Air	50 / 75
BNC	M/F	Solid	50 / 75
TNC	M/F	Solid	50 / 75
SMA	M/F	Solid	50
APC-7	Sexless	Air	50
APC-3.5	Sexed	Air	50

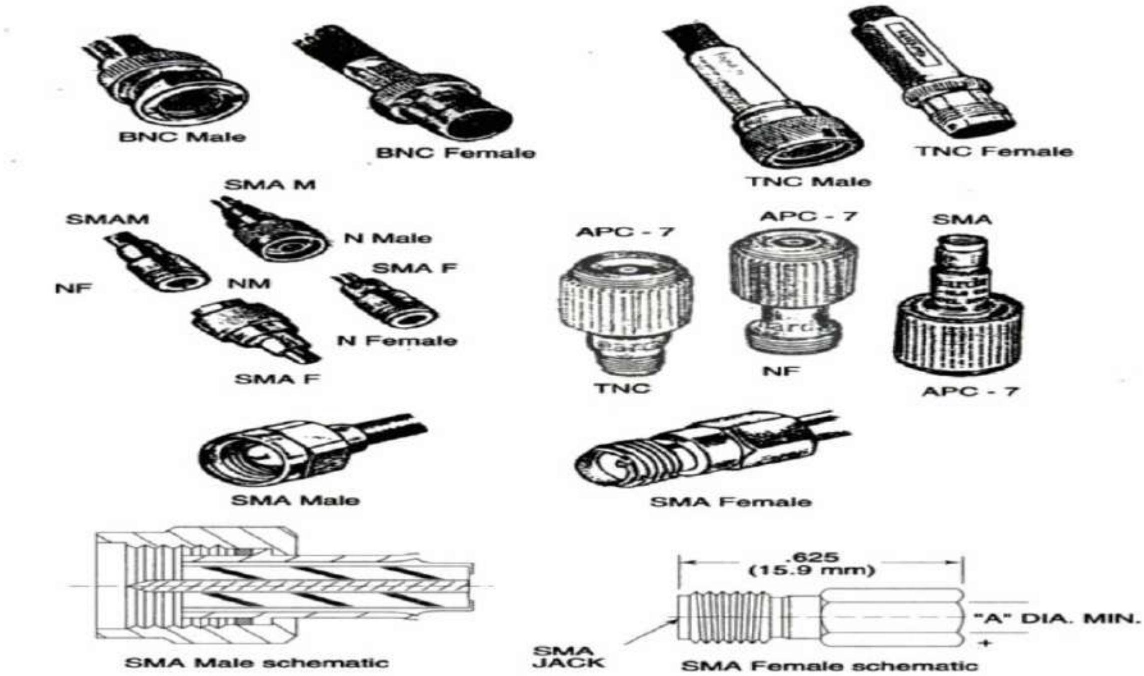


Fig 2.6 Coaxial connectors and Adapters

Attenuators

Attenuators are passive devices used to control power levels in a microwave system by partially absorbing the transmitted signal wave. Both fixed and variable attenuators are designed using resistive films (aquadag).

A coaxial fixed attenuator uses a film with losses on the centre conductor to absorb some of the power as shown in Fig. 2.7a. The fixed waveguide type consists of a thin dielectric strip coated with resistive film and placed at the centre of the waveguide parallel to the maximum E-field. Induced current on the resistive film due to the incident wave results in power dissipation, leading to attenuation of microwave energy. The dielectric strip is tapered at both ends up to a length of more than half wavelength to reduce reflections. The resistive vane is supported by two dielectric rods separated by an odd multiple of quarter wavelength and perpendicular to the electric field (Fig. 2.7b).

A variable type attenuator can be constructed by moving the resistive vane by means of micrometre screw from one side of the narrow wall to the centre where the E-field is maximum (Fig. 2.7b) or by changing the depth of insertion of a resistive vane at an E-field maximum through a longitudinal slot at the middle of the broad wall as shown in Fig. 2.7c. A maximum of 90 dB attenuation is possible with VSWR of 1.05. The resistance card can be shaped to give a linear variation of attenuation with the depth of insertion

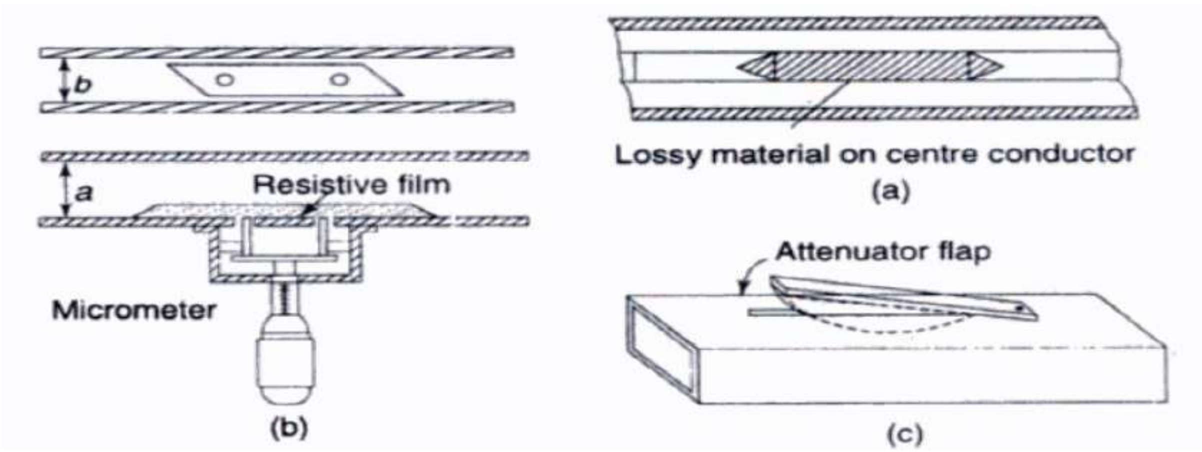


Fig 2.7 Microwave attenuator a) Coaxial line fixed attenuator b) and c) waveguide attenuator

A precision type variable attenuator

A precision type variable attenuator makes use of a circular waveguide section(C) containing a very thin tapered resistive card R2, to both sides of which are connected axisymmetric sections of circular to rectangular waveguide tapered transitions (RC1 and RC2) as shown in Fig. 2.8. The centre circular section with the resistive card can be precisely rotated by 360° with respect to the two fixed sections of circular to rectangular waveguide transitions. The induced current on the resistive card R, due to the incident signal is dissipated as heat producing attenuation of the transmitted signal. The incident TE10 dominant wave in the rectangular waveguide is converted into a dominant TE11 mode in the circular waveguide. A very thin tapered resistive card is placed perpendicular to the E-field at the circular end of each transition section so that it has a negligible effect on the field perpendicular to it but absorbs any component parallel to it Therefore, a pure TE11 mode is excited in the middle section.

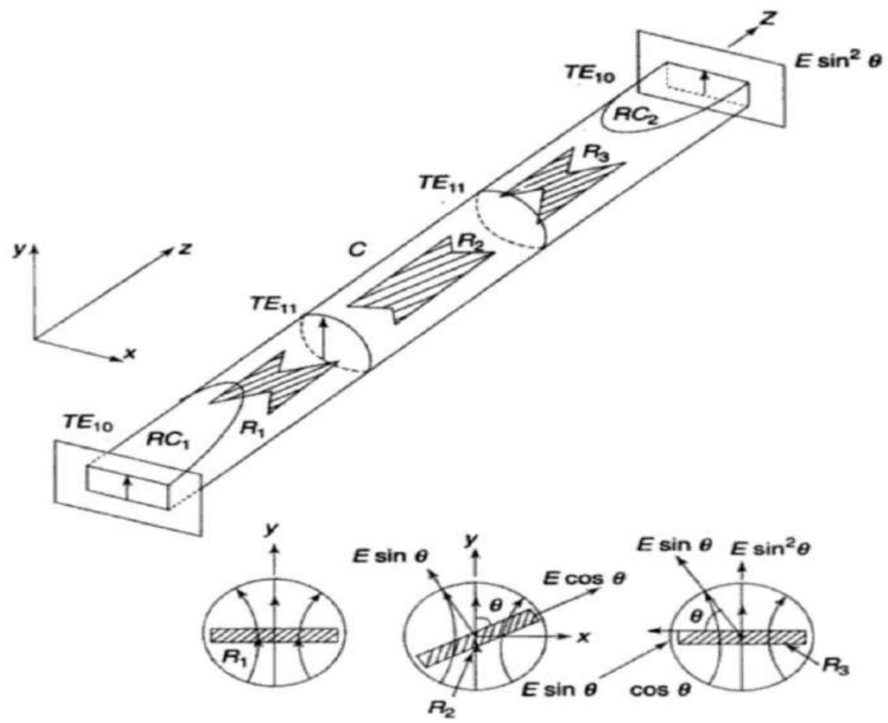


Fig. 2.8 Precision type variable attenuator R1, R2, R3 - Tapered resistive cards
RC1 & RC2 - Rectangular-to-circular waveguide transitions
C - Circular Waveguide Section

If the resistive card in the centre section is kept at an angle Θ relative to the E-field direction of the TE11 mode, the component EcosΘ parallel to the card get absorbed while the component EsinΘ is transmitted without attenuation. This later component finally appears as electric field component $E\sin^2\Theta$ in rectangular output guide. Therefore. the attenuation of the incident wave is

$$\alpha = \frac{E}{E\sin^2\theta} = \frac{1}{\sin^2\theta} = \frac{1}{|S_{21}|}$$

or,

$$\alpha\,(\mathrm{dB}) = -40\log(\sin\theta) = -20\log|S_{21}|$$

Therefore, the precision rotary attenuator produces attenuation which depends only on the angle of rotation Θ of the resistive card with respect to the incident wave polarisation. Attenuators are normally matched reciprocal devices

The S-matrix of an ideal precision rotary attenuator is

$$[S] = \begin{bmatrix} 0 & \sin^2\theta \\ \sin^2\theta & 0 \end{bmatrix}$$

Phase Shifters

A phase shifter is a two-port passive device that produces a variable change in phase of the wave transmitted through it. A phase shifter can be realised by placing a lossless dielectric slab within a waveguide parallel to and at the position of maximum E-field. A differential phase change is produced due to the change of wave velocity through the dielectric slab compared to that through an empty waveguide. Two ports are matched by reducing the reflections of the wave from the dielectric slab tapered at both ends as shown in Fig. 2.9.

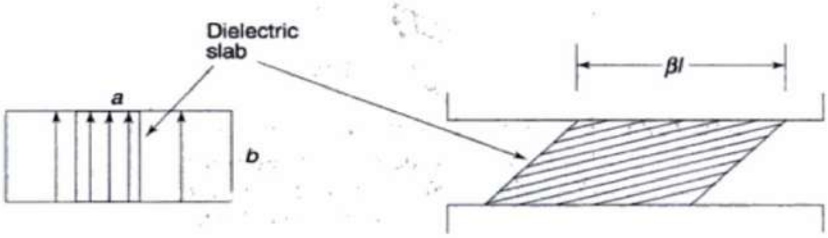


Fig. 2.9 Phase Shifters

The propagation constant through a length l of a dielectric slab and of an empty guide are, respectively,

$$\beta_1 = \frac{2\pi}{\lambda_{g1}} = \frac{2\pi\sqrt{[1-(\lambda_o/(2a\sqrt{\varepsilon_r}))^2]}}{\lambda_o/\sqrt{\varepsilon_r}}$$

$$\beta_2 = \frac{2\pi}{\lambda_{g2}} = \frac{2\pi\sqrt{[1-(\lambda_o/2a)^2]}}{\lambda_o/\sqrt{\varepsilon_r}}$$

Thus the differential phase shift produced by the phase shifter is $\Delta\phi = (\beta 1 = \beta 2)$ l. By adjusting the length l, different phase shifts can be produced. The S-matrix of an ideal phase shifter can be expressed by

$$[S] = \begin{bmatrix} 0 & e^{-j\Delta\phi} \\ e^{-j\Delta\phi} & 0 \end{bmatrix}$$

Precision phase shifter

A precision phase shifter can be designed as a rotary type as shown in Fig. 2.10. This uses a section of circular waveguide containing a lossless dielectric plate of length 2l called halfwave (180°) section. This section can be rotated over 360° precisely between two sections of circular to rectangular waveguide transitions each containing lossless dielectric plates of length l called quarter wave (90°) sections oriented at an angle of 45° with respect to the broad wall of the rectangular waveguide ports at the input and output. The incident TE, wave in the rectangular guide becomes a TE, wave in the circular guide. The halfwave section produces a phase shift equal to twice its rotation angle Θ with respect to the quarter wave section. The dielectric plates are tapered through a length of quarter wavelength at both ends for reducing reflection due to discontinuity.

The principle of operation of the rotary phase shifter can be explained as follows. The TE11 mode incident field E_i in the input quarter wave section can be decomposed into two transverse components, one E1 polarised parallel and other, E2 perpendicular to quarter wave plate. After propagation through the quarter wave plate these components are

$$E_1 = E_i \cos 45^\circ \, e^{-j\beta_1 l} = E_o \, e^{-j\beta_1 l}$$
$$E_2 = E_i \sin 45^\circ \, e^{-j\beta_2 l} = E_o \, e^{-j\beta_2 l}$$

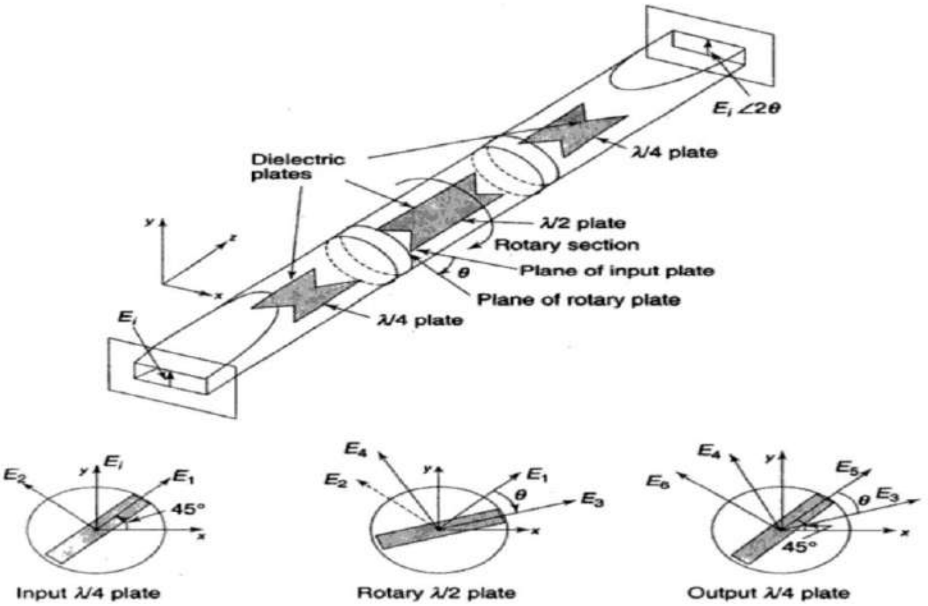


Fig 2.10 Precision Rotary Phase shifter

where, E0 = Ei/√2. The length l is adjusted such that these two components will have equal magnitude but a differential phase change of (β1=β2) l=90°. Therefore, after propagation through the quarter wave plate these field components become

$$E_1 = E_o\, e^{-j\beta_1 l}$$
$$E_2 = jE_o\, e^{-j\beta_1 l} = jE_1 = E_1\, e^{j\pi/2}$$

Thus, the quarter wave sections convert a linearly polarised TE_{11} wave to a circularly polarised wave and vice-versa. After emergence from the halfwave section, the field components parallel and perpendicular to the halfwave plate can be represented as

$$E_3 = (E_1 \cos\theta - E_2 \sin\theta)\, e^{-j2\beta_1 l} = E_o\, e^{-j\theta}\, e^{-j3\beta_1 l}$$
$$E_4 = (E_2 \cos\theta + E_1 \sin\theta)\, e^{-j2\beta_2 l} = E_o\, e^{-j\theta}\, e^{-j3\beta_1 l}\, e^{-j\pi/2}$$

since $2(\beta_1 - \beta_2)l = \pi \quad \text{or} \quad -2\beta_2 l = \pi - 2\beta_1 l$

After emergence from the halfwave section the field components E3and E4 may again be decomposed into two TE11 modes, polarised parallel and perpendicular to the output quarter wave plate. At the output end of this quarter wave plate the field components parallel and perpendicular to the quarter wave plate can be written as

$$E_5 = (E_3 \cos\theta + E_4 \sin\theta)\, e^{-j\beta_1 l} = E_o\, e^{-j2\theta}\, e^{-j4\beta_1 l};$$
$$E_6 = (E_4 \cos\theta - E_3 \sin\theta)\, e^{-j\beta_2 l} = E_o\, e^{-j2\theta}\, e^{-j4\beta_1 l};$$

Therefore, the parallel component E, and perpendicular component E, at the output end of the quarter wave plate are equal in magnitude and in phase to produce a resultant field which is a linearly polarised TE_{11} wave

$$E_{out} = \sqrt{2}\, E_o\, e^{-j2\theta}\, e^{-j4\beta_1 l}$$
$$= E_i\, e^{-j2\theta}\, e^{-j4\beta_1 l}$$

having the same direction of polarisation as the incident field Ei with a phase change of $2\Theta + 4\beta_1 l$. Since Θ can be varied and $4\beta_1 l$ is fixed at a given frequency and structure, a phase shift of 2 Θ can be obtained by rotating the halfwave plate precisely through an angle of Θ with respect to the quarter wave plates.

Waveguide Tees

Waveguide tees are three-port components. They are used to connect a branch or section of the waveguide in series or parallel with the main waveguide transmission line for providing means of splitting, and also of combining power in a waveguide system. The two basic types of waveguide tees are E-plane (series) T and H-plane (shunt) T. These are named according to the axis of the side arm which is parallel to the E-field or the H-field in the collinear arms, respectively.

Because of the junction, waveguide tees are poorly matched devices. Adjustable matching reactance can be introduced by means of a tuning screw at the centre. Because of symmetry and absence of non-linear elements in the junction, the S-matrix is symmetric: Sij = Sji; i= 1,2; j= 1, 2. The general S-matrix for a tee junction is

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{11} & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix}$$

E-Plane tee

An E -plane tee is a waveguide tee in which the axis of its side arm is parallel to the E field of the main guide If the collinear arms are symmetric about the side arm, there are two different transmission characteristics

Fig. 2.11 E-Tee or Series-T

- When the waves are fed into the side arm (port 3), the waves appearing at port 1 and port 2 of the collinear arm will be in opposite phase and in the same magnitude. $S_{13}= -S_{23}$
- If two input waves are fed into ports1 and 2 of the collinear arms, the output wave at port 3 will be opposite in phase and subtractive. Sometimes this third port is called the difference arm.

By analogy with the voltage relationship in the series circuit, E-plane junction is also called a series junction. All diagonal elements of the S-matrix of a E-plane T junction cannot be zero simultaneously since the tee junction cannot be matched to all the three arms simultaneously. Considering as matched port 3, the S-matrix of a E-plane T can be derived as follows.

Denoting the incident and outgoing signal variables at the ith port by a_i and b_i, respectively, for an input power at port 3, the net input power to port 3 is $|a_3|^2 - |b_3|^2 = |a_3|^2 (1 - |S_{33}|^2)$, and the output power is $|b_1|^2 + |b_2|^2 = 2 |a_3|^2 |S_{13}|^2$, since $|S_{31}| = |S_{32}|$ by symmetry. Since the junction is assumed lossless, the input power must be equal to the output power i.e.,

$$(1 - |S_{33}|^2) = 2 |S_{13}|^2$$

By a suitable matching element we can make $S_{33} = 0$, so that $|S_{13}| = 1/\sqrt{2}$. From the symmetry characteristics described above,

$$S_{13} = S_{31} = 1/\sqrt{2}, \; S_{23} = S_{32} = -1/\sqrt{2}$$

After matching the port 3, if one attempts to match either port 1 or 2 by similar method, the matching elements, such as irises or tuning screws will interact with each other and matching at port 3 would be disturbed. Based on power consideration it can also be shown that $S_{11}=S_{22} = 1/2$ and $S_{12}= S_{21} = 1/2$ for $S_{33} = 0$. Therefore, with matching at port 3, the S-matrix of a E-plane T can be expressed by real values with proper choice of reference plane:

$$[S] = \begin{bmatrix} 1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & 1/2 & -1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \end{bmatrix}$$

H-plane tee (shunt tee):

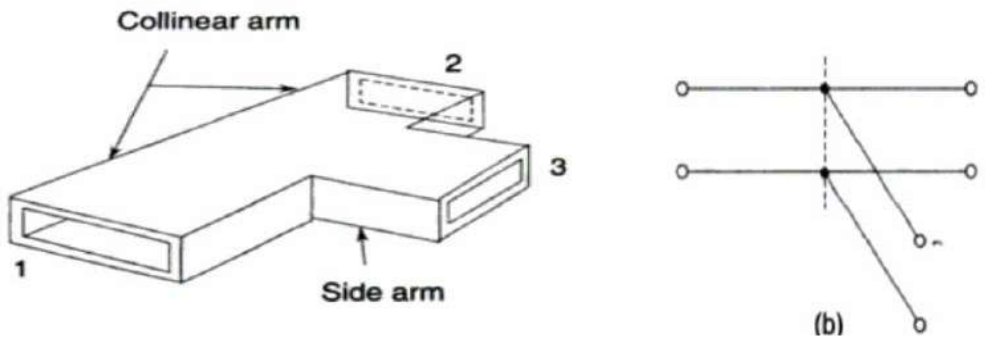


Fig. 2.12 H-Tee or Shunt-T

An H -plane tee is a waveguide tee in which the axis of its side arm is "shunting" the E field or parallel to the H field of the main guide as shown in above Fig. It can be seen that if two input waves are fed into port 1 and port 2 of the collinear arm, the output wave at port 3 will be in phase and additive. Because of this, the third port is called the sum arm. On the other hand, if the input is fed into port 3, the wave will split equally into port 1 and port 2 in phase and in the same magnitude. i.e. $S_{13}=S_{23}$

H-plane junction is also called a shunt junction. For a symmetrical and lossless junction, in absence of non-linear elements at the H-plane junction, the S-parameters are obtained in similar manner as in the case of E-plane junction:

$$[S] = \begin{bmatrix} 1/2 & -1/2 & 1/\sqrt{2} \\ -1/2 & 1/2 & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}$$

Problem 7: A 20 mW signal is fed into one of collinear port 1of a lossless H-plane T-junction. Calculate the power delivered through each port when other ports are terminated in matched load.

Solution

Since ports 2 and 3 are matched terminated, $a_2 = a_3 = 0$, $S_{11} = 1/2$. The total effective power input to port 1 is

$$P_1 = |a_1|^2 (1 - |S_{11}|^2)$$
$$= 20 (1 - 0.5^2) = 15 \text{ mW}$$

The power transmitted to port 3 is

$$P_3 = |a_1|^2 |S_{31}|^2$$
$$= 20 \times (1/\sqrt{2})^2 = 10 \text{ mW}$$

The power transmitted to port 2 is

$$P_2 = |a_1|^2 |S_{21}|^2$$
$$= 20 \times (1/2)^2 = 5 \text{ mW}.$$

Therefore, $P_1 = P_3 + P_2$

Problem 8: In a H-plane T-junction, compute power delivered to the loads 40 ohm and 60 ohms connected to arms I and 2 when 10 mW power is delivered to matched port 3.

Solution

With port 3 matched, the scattering matrix for *H*-plane *T* is

$$[S] = 1/2 \begin{bmatrix} 1 & 1 & \sqrt{2} \\ 1 & 1 & \sqrt{2} \\ \sqrt{2} & \sqrt{2} & 0 \end{bmatrix}$$

Therefore, input power at port 3 is equally divided in arms 1 and 2. Since input at port 3 = 10 mW = 0.01 W, power towards ports 1 and 2 = 0.005 W = $1/2\, |b_1|^2 = 1/2\, |b_2|^2$. Considering 1st order reflection, reflected power from ports 1 and 2 are

$$1/2\, |\Gamma_1 b_1|^2 \text{ and } 1/2\, |\Gamma_2 b_2|^2$$

Therefore, power delivered to load $Z_1 = 40$ ohm and $Z_2 = 60$ ohm are

$$P_1 = 1/2\, |b_1|^2 - 1/2\, |\Gamma_1 b_1|^2 = 1/2\, |b_1|^2 (1 - |\Gamma_1|^2)$$

and $$P_2 = 1/2\, |b_2|^2 - 1/2\, |\Gamma_2 b_2|^2 = 1/2\, |b_2|^2 (1 - |\Gamma_2|^2)$$

Now taking the characteristic impedance of the line = 50 ohm

$$|\Gamma_1| = |40 - 50| / |40 + 50| = 1/9 ; |\Gamma_1|^2 = 0.01234$$
$$|\Gamma_2| = |60 - 50| / |60 + 50| = 1/11; |\Gamma_2|^2 = 8.2694 \times 10^{-3}$$

Therefore,

$$P_1 = 0.005 (1 - 0.01234) = 4.938 \times 10^{-3} = 4.9383 \text{ mW}$$
$$P_2 = 0.005 (1 - 8.2694 \times 10^{-3}) = 4.9586 \times 10^{-3} \text{ W}$$
$$= 4.9586 \text{ mW}$$

Hybrid or Magic-T

A combination of the E-plane and H-plane tees forms a hybrid tee, called a magic-T, having 4 ports as shown in Fig. 2.13

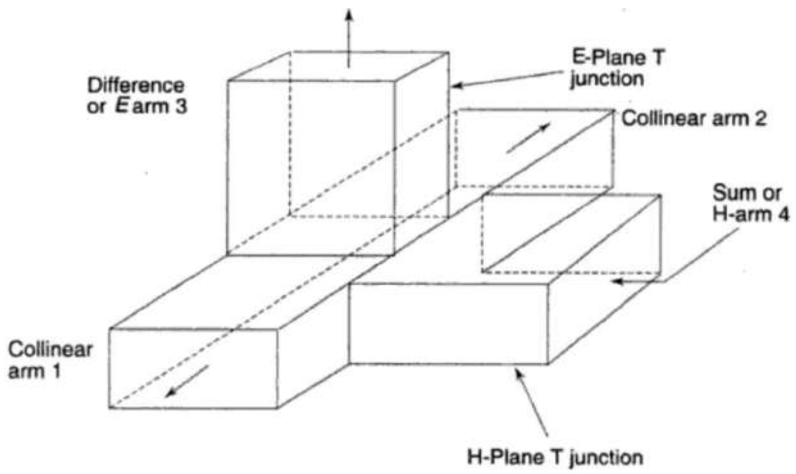


Fig 2.13 Magic Tee

The magic-T has the following characteristics when all the ports are terminated with matched load.

1. If two in phase waves of equal magnitude are fed into ports, 1 and 2, the output at port 3 is subtractive and hence zero and total output will appear additively at port 4. Hence port 3 is called the difference or E-arm and 4 the sum or H-arm.
2. A wave incident at port 3 (E-arm) divides equally between ports1 and 2 but opposite in phase with no coupling to port 4 (H-arm). Thus $S_{13}= -S_{23}, S_{43}=0$
3. A wave incident at port 4 (H-arm) divides equally between ports1 and 2 in phase with no coupling to port 3 (E-arm). Thus $S_{14}= S_{24}, S_{34}=0$
4. A wave fed into one collinear port 1 or 2, will not appear in the other collinear ports 2 or 1. Hence two collinear ports 1 and 2 are isolated from each other, making $S_{12}= S_{21}=0$

A magic-T can be matched by putting tuning screws suitably in the E and H arms without destroying the symmetry of the junction. Therefore, for an ideal lossless magic-T matched at ports 3 and 4, $S_{33}= S_{44}=0$. The procedure of derivation of the S-matrix considers the symmetry property at the junction for which $S_{13} =S_{31} =-S_{23} =-S_{32}$ and $S_{14} =S_{41} =S_{24} =S_{42}$

Therefore, the S-matrix for a magic-T, matched at ports 3 and 4 is given by

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & -S_{13} & S_{14} \\ S_{13} & -S_{13} & 0 & 0 \\ S_{14} & S_{14} & 0 & 0 \end{bmatrix}$$

From the unitary property applied to rows 1 and 2, we get

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1$$
$$|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1$$

Subtracting these two equations:

$$|S_{11}|^2 - |S_{22}|^2 = 0$$

or, $$|S_{11}| = |S_{22}|$$

From the unitary property applied to rows 3 and 4

$$2\,|S_{13}|^2 = 1, \text{ or } |S_{13}| = 1/\sqrt{2}$$

$$2\,|S_{14}|^2 = 1, \text{ or } |S_{14}| = 1/\sqrt{2}$$

Substituting these values in above equation

$$|S_{11}|^2 + |S_{12}|^2 + 1/2 + 1/2 = 1$$

or,

$$|S_{11}|^2 + |S_{12}|^2 = 0$$

which is valid if $S_{11}=S_{12}= 0$ also $S_{22}= 0$ Therefore

$$[S] = \begin{bmatrix} 0 & 0 & S_{13} & S_{13} \\ 0 & 0 & -S_{13} & S_{13} \\ S_{13} & -S_{13} & 0 & 0 \\ S_{13} & S_{13} & 0 & 0 \end{bmatrix}$$

where $|S_{13}| = 1/\sqrt{2} = |S_{14}|$

By proper choice of reference planes in arms 3 and 4, it is possible to make both S13and S14 real, resulting in the final form of S-matrix of magic-T

$$[S] = 1/\sqrt{2}\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \\ 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

Problem 9: A magic-T is terminated at collinear ports 1 and 2 and difference port 4 by impedances of reflection coefficients $\Gamma_1= 0.5$, $\Gamma_{12}= 0.6$ and $\Gamma_4= 0.8$, respectively. If 1W power is fed at sum port 3, calculate the power reflected at port 3 and power transmitted to other three ports.

Solution

S-matrix for a matched magic-T with collinear ports 1 and 2 and sum and difference ports 3 and 4, respectively, is given by

$$[S] = 1/\sqrt{2}\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix}$$

If a_1, a_2, a_3 and a_4 be the normalised input voltages and b_1, b_2, b_3 and b_4 are the corresponding output voltage at ports 1, 2, 3 and 4 respectively, then

$$a_1 = \Gamma_1 b_1,\ a_2 = \Gamma_2 b_2,\ a_3 = \text{input applied voltage, and}$$

$$a_4 = \Gamma_4 b_4.$$

Now, $\quad P_i = |a_3|^2 = 1\text{W}, \quad \text{or,} \quad a_3 = 1\text{ V}$

Therefore,

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = 1/\sqrt{2}\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} .5b_1 \\ .6b_2 \\ 1.0 \\ .8b_4 \end{bmatrix}$$

or,

$$b_1 + 0 + 0 - 0.8\, b_4/\sqrt{2} = 1/\sqrt{2}$$
$$0 + b_2 + 0 + 0.8\, b_4/\sqrt{2} = 1/\sqrt{2}$$
$$-0.5b_1/\sqrt{2} - 0.6\, b_2/\sqrt{2} + b_3 + 0 = 0$$
$$-0.5b_1/\sqrt{2} + 0.6\, b_2/\sqrt{2} + 0 + b_4 = 0$$

The unknown quantities b's may be solved by Cramer's rule

$$b_1 = \frac{\begin{bmatrix} 1 & 0 & 0 & -0.8 \\ 1 & \sqrt{2} & 0 & 0.8 \\ 0 & -0.6 & \sqrt{2} & 0 \\ 0 & 0.6 & 0 & \sqrt{2} \end{bmatrix}}{\begin{bmatrix} \sqrt{2} & 0 & 0 & -0.8 \\ 0 & \sqrt{2} & 0 & 0.8 \\ -0.5 & -0.6 & \sqrt{2} & 0 \\ -0.5 & 0.6 & 0 & \sqrt{2} \end{bmatrix}} = \sqrt{2}\ \frac{1 - 0.6 \times 0.8}{2 - 0.8\,(0.5 + 0.6)}$$

$$= \sqrt{2}\ \frac{1 - 0.48}{2 - 0.88} = 0.6566 \text{ V}$$

Similarly,

$$b_2 = \sqrt{2}\ \frac{1 - 0.5 \times 0.8}{2 - 0.8\,(0.5 + 0.6)} = 0.7576 \text{ V}$$

$$b_3 = \frac{0.5 + 0.6 - 2 \times 0.5 \times 0.6 \times 0.8}{2 - 0.8\,(0.5 + 0.6)} = 0.5536 \text{ V}$$

$$b_4 = \sqrt{2}\ \frac{0.5 - 0.6}{2 - 0.8\,(0.5 + 0.6)} = -\,0.0893 \text{ V}$$

Therefore,
Power transmitted at port 1 = $|b_1|^2 = 0.4309$ W
Power transmitted at port 2 = $|b_2|^2 = 0.5738$ W
Power transmitted at port 4 = $|b_4|^2 = 0.00797$ W
Power reflected at port 3 = $|b_3|^2 = 0.3065$ W

Note: Power absorbed at port $i = 1/2\ (|b_i|^2 - |a_i|^2)$; $i = 1, 2, 4$. Total power absorbed by the system = $1/2\ (|a_3|^2 - |b_3|^2)$.

Application of Magic Tee:

1. **E-H Tuner:** In an E-H tuner (Fig. 2.14.) both the E and H arms are terminated by movable shorts which act as E-plane and H-plane stubs. The position of the shorts can be adjusted so that a wide range of load impedance may be matched to reduce the VSWR of a waveguide system connected through the collinear arms.

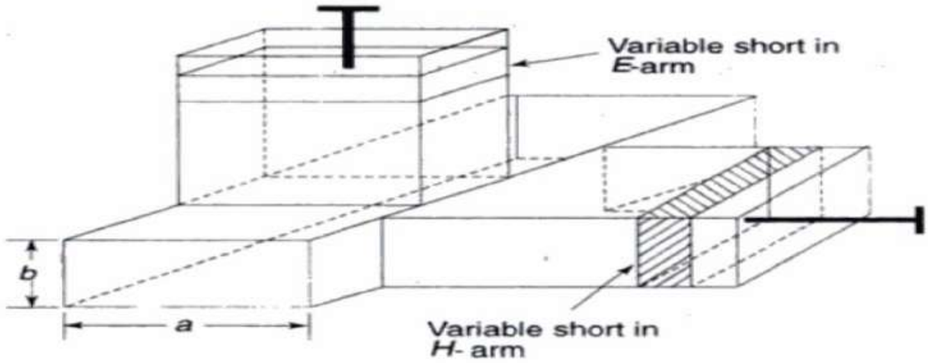


Fig 2.14 E-H Tuner

2. **Balanced Mixer:** In a balanced microwave mixer configuration, an incoming signal is fed to the E-arm and a local oscillator signal is fed to the H-arm as shown in Fig. 2.15. These two signals when enter the collinear arms, the crystal diodes placed in these arms produce the IF signal or difference signal in the following manner.

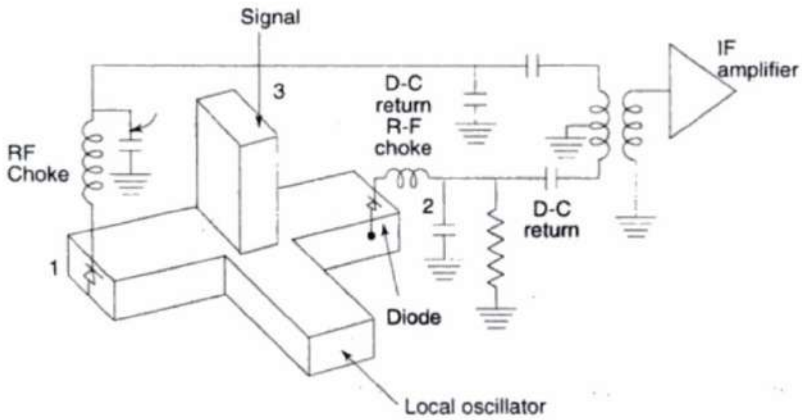


Fig 2.15 Microwave Mixer

The local oscillator signal from the H-arm will arrive at the diodes in phase. whereas the incoming signal from E-arm will arrive at the diodes out-of-phase. These signals are mixed in the nonlinear diodes and produce IF signals in the collinear arms which are out-of-phase by 180°. Since local oscillator noise will be in phase at the diodes. this gets cancelled at the balanced IF input whereas. the IF signals are added up in phase for amplification in IF amplifier. Moreover. LO and RF signals are uncoupled due to magic-T properties of E and H arms.

Circulators and Isolators

Ferromagnetic materials (ferrite: Mg + Mn, Ni + Zn alloys) when placed in dc magnetic field electromagnetic wave propagation becomes non-reciprocal. This property is used for construction of circulators and isolators.

A. Circulators

A circulator is a multiport junction in which the wave can travel from one port to the next immediate port in one direction only as shown in Fig.2.16(a). Commonly used circulators are three-port or four-port passive devices although a greater number of ports is possible.

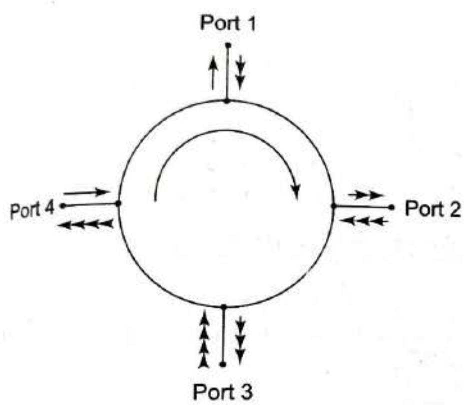


Fig 2.16 (a)Schematic diagram of a four-port circulator

A. Four-port circulator

A four-port circulator can be constructed from two magic-Ts and a non-reciprocal 180° phase shifter a combination of two 3 dB side hole directional couplers with two nonreciprocal phase shifters as shown in Figs 2.16 (b) and (c).

In Fig. 2.16 (b), an input signal at Port 1 is split into two in-phase and equal amplitude waves in the collinear arms b and d of the magic-tee, T1 and added up to emerge from Port 2 in the magic tee, T2. On the other hand, a signal at Port 2 will be splitted into two equal amplitude and equiphase waves in the collinear arms of the magic-tee, T2 and appears at point b and d out of phase due to presence of the nonreciprocal 180° phase shifter. These out-of-phase waves add up and appear from Port 3 in the magic-tee, T1. In a similar manner, an input signal at Port 3 will emerge from 4, an input at Port 4 will appear at Port 1. Thus, the circulator property is exhibited.

In Fig. 2.16 (c), each of the two 3 dB couplers introduces a 90° phase shift. An input signal at Port 1 is splitted into two components by the coupler l and the coupled signals are again splitted into two components by the coupler 2 with a 90° phase shift in each. Each of the two-phase shifters produces additional phase shift so that the signal components at Port 2 are in phase, and at Port 4 they are out of phase. Since Port 3 is the decoupled port for the directional coupler, the input signal at Port 1 appears in Port 2. Similarly, signals from Port 2 to Port 3, from Port 3 to Port 4 and from Port 4 to Port 1.

A perfectly matched, lossless, and non-reciprocal four-port circulator has S-matrix:

$$[S] = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

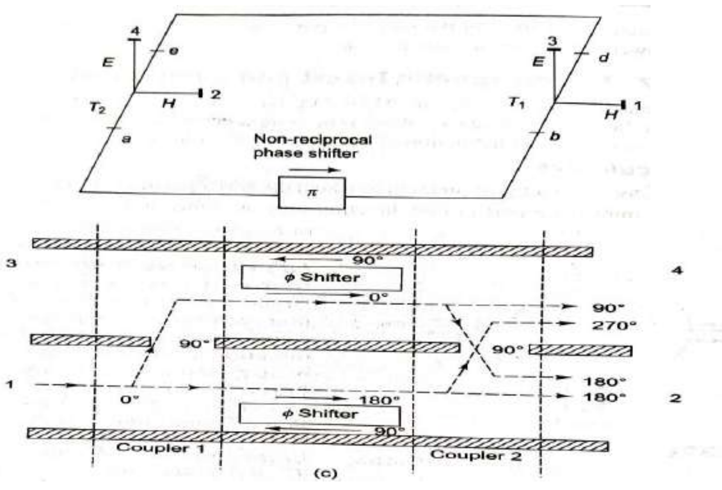


Fig 2.16 (b) and (c) Four-port circulators

B. Three-port circulator

A three-port circulator is formed by a 120° H-plane waveguide or strip line symmetrical Y-junction with a central ferrite post or disc. A steady magnetic field H_O is applied along the axis of the disc as shown in Fig. 2.17. Depending on the polarisation of the incident wave and the direction of H_O the microwave signal travels from one port to the immediate next one only.

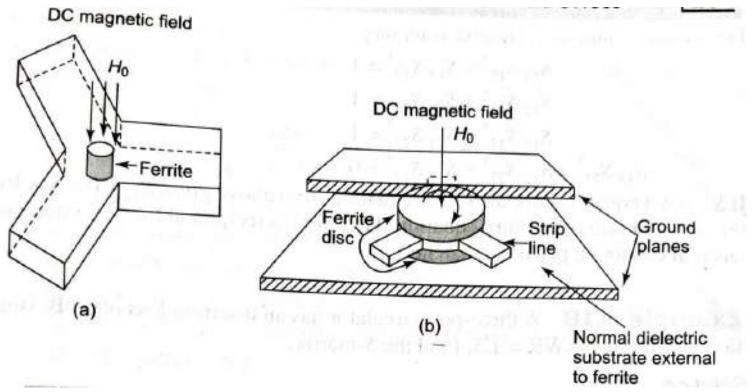


Fig. 2.17 Three-port circulator: (a) waveguide type (b) strapline type

For a perfectly matched, lossless, non-reciprocal three-port circulator, the S-matrix is

$$[S] = \begin{bmatrix} 0 & 0 & S_{13} \\ S_{21} & 0 & 0 \\ 0 & S_{32} & 0 \end{bmatrix}$$

If the terminal planes are properly chosen to make the phase angles of S_{13}, $S_{21}$and S_{32} zero, so that S_{13}=S_{21}= S_{32} =1

$$[S] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

The matching of the junction can be achieved by placing suitable tuning elements in each arm. Since in practice losses are always present, the performance is limited by finite isolation and non-zero insertion loss. Typical characteristics can be represented by
Insertion loss < 1 dB
Isolation = 3040 dB
VSWR < 1.5

Problem 10: Prove that it is impossible to construct a perfectly matched, lossless, reciprocal three-port junction
Solution: A perfectly matched three-port junction has a scattering matrix:

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

For a lossless junction S-matrix is unitary

$$S_{12} S_{12}^* + S_{13} S_{13}^* = 1$$
$$S_{12} S_{12}^* + S_{23} S_{23}^* = 1$$
$$S_{13} S_{13}^* + S_{23} S_{23}^* = 1$$
$$S_{13} S_{23}^* = S_{12} S_{23}^* = S_{12} S_{13}^* = 0$$

If S_{12} is not equal to zero, the fourth equation from above gives $S_{13} = 0 = S_{23}$. But this does not satisfy the third equation. Therefore, a reciprocal lossless three-port junction cannot be perfectly matched.

Problem 11: A three-port circulator has an insertion loss of 1 dB, isolation 30 dB and VSWR = 1.5. Find the S-matrix.
Solution: The S-matrix of a three-port circulator is

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

Insertion loss =1 dB = – 20 log $|S_{21}|$

or, $|S_{21}| = 10^{-1/20} = 0.89$

For the same insertion loss between ports 1 and 2, 2 and 3, 3 and 1, $|S_{21}| = |S_{32}| = |S_{13}| = 0.89$.

The isolation between the ports is 30 dB = – 20 log $|S_{31}|$

or, $|S_{31}| = 10^{-30/20} = 10^{-1.5} = 0.032$
$= |S_{23}| = |S_{12}|$

Since VSWR $S = 1.5$, reflection coefficient

$$|\Gamma| = \frac{S-1}{S+1} = \frac{1.5-1}{1.5+1} = 0.2 = |S_{11}|$$
$$= |S_{22}| = |S_{33}|$$

By placing reference planes suitably to make the phase of S-parameters zero,

$$[S] = \begin{bmatrix} 0.200 & 0.032 & 0.890 \\ 0.890 & 0.200 & 0.032 \\ 0.032 & 0.890 & 0.200 \end{bmatrix}$$

B. Isolators

An isolator is a two-port non-reciprocal device which produces a minimum attenuation to wave propagation in one direction and very high attenuation in the opposite direction. Thus, when inserted between a signal source and load almost all the signal power can be transmitted to the load and any reflected power from the load is not fed back to the generator output port. This eliminates variations of source power output and frequency pulling due to changing loads.

An isolator can be constructed in a rectangular waveguide (a* b) operating in dominant mode as shown in Fig. 2.18. The non-reciprocal characteristics are obtained by establishing a steady magnetic field H_O in y direction and placing a ferrite slab at any of the longitudinal plane's x = x1 near and parallel to the narrow waveguide wall, where the magnetic field exhibits circular polarisation. This occurs at x1 = a/4 or, 3a/4.

For the propagation of waves in +z direction, direction of rotation of H in the planes at x1 = a/4 and 3a/4 are opposite to each other. The non-reciprocal characteristic is achieved by placing a ferrite slab at any one of these two planes. The required steady state magnetic field H_O, in the y-direction is established by placing permanent magnetic poles between the two broad walls.

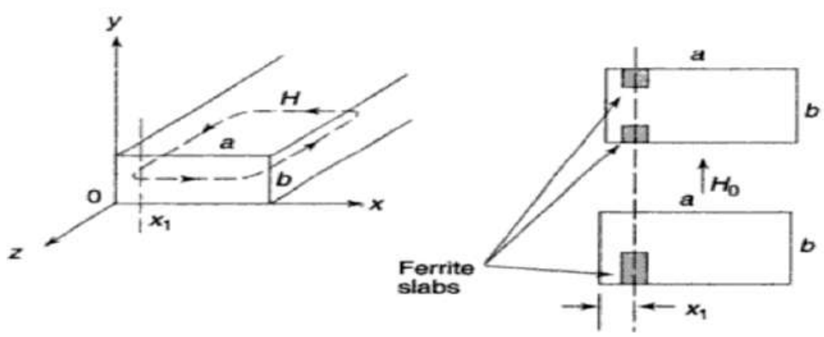


Fig 2.18 Waveguide Isolator

It is known that the attenuation in ferrite for negative/clockwise circular polarisation is very small whereas for positive/counter clockwise circular polarisation is very large at and near the resonance frequency $f = f0$ Therefore, the ferrite slab is placed in such a way that while transmission it encounters negative circular polarisation in the reverse direction. The steady magnetic field is set to be equal to the resonant value. The isolation of the order of 20 -30 dB in the backward direction and a transmission loss of 0.5 dB in the forward direction can be achieved with a VSWR of the order of 1.1 Since the reverse power is absorbed in the ferrite and dissipated as heat, the maximum power handling capability of an isolator is limited. To increase the capacity of heat dissipation, two ferrite slabs of smaller heights are used instead of one with a larger height.

A Faraday rotation Isolator

A Faraday rotation Isolator is a circular waveguide section axially loaded with a ferrite rod of smaller diameter as shown in Fig. 2.19. The ferrite rod is subjected to a steady axial magnetic field H_0, of strength much smaller than the resonant intensity so that dissipative loss in the ferrite is neglected. The dominant TE_{11}, mode in the circular section can be decomposed into two oppositely rotating circularly polarised waves of equal magnitude. These waves encounter different permeabilities μ'_+ and μ'_- for the clockwise and anticlockwise directions of field rotation and exhibit changes in the phase velocities. This will result in a change in the plane of polarisation of the main mode TE_{11} which will experience gradual rotation Θ during propagation to the other end. The rotation angle Θ is proportional to the length of the ferrite rod. In this case for the reverse wave the direction of rotation remains the same confirming the non-reciprocal characteristics of the ferrite.

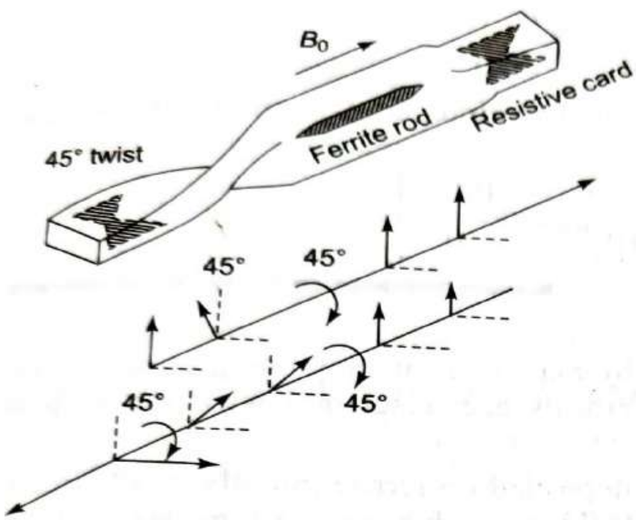


Fig 2.19 Faraday rotation Isolator

The isolator input is a 45° twist where a tapered resistive card is mounted parallel to the broad wall of the rectangular waveguide part. The dominant TE_{10} mode does not get attenuated while transmission and is rotated at 45 ° at the twist output and enters the circular waveguide through rectangular to circular waveguide transition as the TE_{11} mode. The length of the ferrite rod is selected so as to obtain Faraday rotation $\Theta = 45°$ at the output and regain its original polarisation. The plane of polarisation of the reflected wave from the load is again rotated by the same angle $\Theta = 45°$ and at the emergence through the 45° twist becomes aligned with the surface of the absorbing plate and gets absorbed. Thus, nonreciprocal isolation action takes place. Typical insertion loss and isolation are approximately l dB and 20-30 dB, respectively, for these isolators. Isolators are also available in the coaxial and microstrip forms. For an ideal lossless, matched isolator

$$|S_{21}| = 1,\ |S_{12}| = |S_{11}| = |S_{22}| = 0$$

$$[S] = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

Problem 12: A matched isolator has insertion loss of 0.5 dB and an isolation of 25 dB. Find the scattering coefficients.

Solution

$$\text{Insertion loss} = 0.5\text{ dB} = -20 \log |S_{21}|$$

or, $$S_{21} = 10^{-.5/20} = 10^{-0.025}$$

The isolation is $$25\text{ dB} = -20 \log |S_{12}|$$

or, $$S_{12} = 10^{-25/20} = 10^{-1.2}$$

Since there is no reflection, $S_{11} = S_{22} = 0$. Therefore, the S-matrix for the isolator is

$$[S] = \begin{bmatrix} 0 & 10^{-1.2} \\ 10^{-0.025} & 0 \end{bmatrix}$$

Question Bank

1. What are the limitations of Z, Y, h and ABCD parameters at microwave frequencies
2. What is S-parameter? Explain S-parameters for two port network and also explain the significance of S-parameter
3. Explain various losses in microwave network in terms of S-parameter
4. State and explain the properties of S-matrix
5. A series reactance Z=jX is connected between two lines with different characteristic impedances Z1 and Z2. Find the S-matrix of the junction
6. A series reactance Z=jX is inserted in an infinite length transmission line with characteristic impedance Zo. Find the S-parameter of the junction
7. 2 transmission lines of characteristic impedance Z1 and Z2 are joined at plane *pp'*. Express S-parameters in terms of impedance when each line is matched terminated
8. A shunt impedance Z is connected across a transmission line with characteristic impedance Zo. Find the S-matrix of the junction
9. What is mismatched load? Explain the s-parameters for a two-port mismatched load
10. Discuss different types of coaxial connectors
11. With a neat diagram, explain the working of precession type variable attenuator
12. Explain with neat sketch the construction and working principle of precession rotary phase shifter
13. With a neat sketch explain the operation of E-plane tee and also obtain its S-matrix
14. With a neat sketch explain the operation of H-plane tee and also obtain its S-matrix
15. A 20mW signal is fed into one of the collinear port1 of a lossless H-plane T-junction. Calculate the power delivered through each port when other ports are terminated in matched load
16. In a H-plane T junction, compute the power delivered to the loads of 40 ohm and 60 ohms connected to arms 1 and 2 when a 10mW power is delivered to the matched port-3
17. With a neat sketch explain the properties of Magic tee and also obtain its S- matrix
18. Discuss the applications of Magic T
19. A magic-T is terminated at collinear ports 1 and 2 and difference port 4 by impedances of reflection coefficients Γ_1= 0.5, Γ_{12}= 0.6 and Γ_4= 0.8, respectively. If 1W power is fed at sum port 3, calculate the power reflected at port 3 and power transmitted to other three ports.
20. Explain with neat sketch the construction and working principle of four port circulator
21. Explain with neat sketch the construction and working principle of three port circulator
22. Explain with neat sketch the construction and working principle of Faraday rotation Isolator
23. Prove that it is impossible to construct a perfectly matched, lossless, reciprocal three-port junction
24. A three-port circulator has an insertion loss of 1 dB, isolation 30 dB and VSWR = 1.5. Find the S-matrix.